MORAL HAZARD IN MUTUAL FUND MANAGEMENT:
THE QUALITY-ASSURING ROLE OF FEES

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March 6, 2013

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Abstract

We model the role of premium fees in assuring the quality of active mutual fund management. Active management is a prototype of an experience good subject to moral hazard. Investors cannot tell high quality management from low quality management until after the fact. An active manager might promise to incur costly effort researching profitable portfolio selection in exchange for a fee sufficient to compensate for his higher research costs. If investors were to find this promise credible, they would buy shares until their expected returns, net of fees, just equalled investing in, say, the market index. The manager might then shirk by forgoing costly research ('closet index') and pocket the excess fee, leaving investors worse off than if they had simply invested in the index. We model this moral hazard and show how it can be mitigated by paying the manager a premium fee sufficiently high that the one-time gain from shirking is less than the capitalized value of the premium stream the manager earns from maintaining his promise to provide high quality. Investors benefit from higher fees, rather than lower fees, which act as a 'quality assuring bond', or 'efficiency wage'. Our model has a number of revealing extensions and comparative statics.

1   Introduction

For over fifty years financial economists, securities market regulators, and federal courts have debated whether mutual fund management fees are excessive. Despite evidence of robust industry competition — low industrial concentration, high rates of entry, and a record of innovation — critics argue that fund fees are largely immune from competitive forces. They cite evidence that fees have failed to fall as fund assets have risen despite presumed scale economies in management, that fund boards almost invariably renew annual management contracts, and that the fee per dollar of assets retail investors pay is substantially higher than what institutional investors pay, sometimes to the same manager. They also cite early empirical work showing that returns to high-fee actively managed funds fell short of the market index (Jensen, 1968) and more recent work showing that fees and returns are negatively correlated in the cross section of active funds (Malkiel, 1995; Carhart, 1997).

The conclusion fee critics draw is that management fees reduce mutual fund investor returns dollar-for-dollar. Were all else equal, this conclusion would be true by definition, but all else is not equal. We show that the evidence is consistent with a competitive equilibrium of rational investors, who rely on premium fees to avert moral hazard in fund management. Premium fees induce managers to provide high-quality effort researching profitable portfolio selection. As a result, they lead to gross fund returns sufficiently high to compensate investors for paying them. Because investors compete to capture any excess return, the best they can expect is a return equal to the rate they could earn investing in the index. Premium fees therefore do not reduce fund investor returns dollar-for-dollar.

At its heart the issue of excessive fees is about the inevitable conflicts of interest that arise in a specialized intermediary economy. Conflicts of interest no doubt exist in the mutual fund industry, but it is equally true that no form of complex organization is free from conflicts of interest and that competitive market forces tend to ameliorate any adverse effects lest money be left on the table. Welfare triangles (and rectangles) provide market participants with a profit opportunity from adopting more efficient forms of organization to reduce or eliminate them (Barzel, 1997). With total U.S. mutual fund assets exceeding \$11.6 trillion dollars as of this writing (ICI Factbook, 2012), the issue is one of enormous importance, among other reasons because suits for excessive fees and calls for further regulation are on the rise.

We begin our analysis by showing under plausible assumptions that the investor-fund-adviser relationship is consistent with informational efficiency and rational wealth maximization. Our

analysis builds on seminal work by Berk and Green (2004), who show that in an informationally efficient market the best rational investors can expect from buying actively-managed mutual fund shares is a normal return, and, by implication, that fees are irrelevant to investor returns. We add moral hazard by fund managers regarding their promise to actively research profitable portfolio selection and show how the traditional asset-based fee structure in mutual funds solves the problem. Our analysis is noteworthy because it clearly shows what a model based on investor rationality predicts about fees and demonstrates the power of the rationality assumption, albeit in a world subject to informational frictions.

To see these points it is helpful to identify exactly what fund investors own. Mutual funds stand ready to issue and redeem shares daily at per share net asset value (total portfolio assets net of fees and other expenses). From investors' standpoint a mutual fund is an open-access commons subject to virtually unrestricted entry and exit save for a small periodic fee per dollar of total assets paid to the adviser. Investors share the remaining assets in common with other investors. While it is true that fund investors own their proportionate share of net asset value at any given moment, they have no exclusive claim to unrealized excess returns from the fund manager's active portfolio selection. As long as investment management is subject to diminishing marginal productivity, any expectation that the manager will outperform the market in the future will be met with fund inflows until investors' expected returns are normalized. The fund manager owns his human capital, and, owing to competition between investors as suppliers of capital, he captures any Ricardian rents in total fees on a larger asset base.

Absent further assumptions, management fees are irrelevant to investor returns. Given two identical funds whose managers have equal skill but charge different fees, the fund with the lower fee will simply have more assets under management than the fund with the higher fee. Crowding by investors dilutes their returns, in essence generating a negative externality for other investors. This is because investors, as common owners, capture the average return rather than the marginal return. With zero fees, fund inflows would drive excess returns to zero. A fee equal to the cost of research effort will lead to excessive inflows and manager shirking. The optimal fee internalizes the externality between investors by setting total assets where the marginal return on assets equals investors' outside opportunity and value created — reflected in the managers' compensation — is maximized.

One complication under open access is that, with diminishing marginal productivity, total assets invested may exceed what the manager can profitably invest through active management. If so, some amount of indexing is desired. We distinguish between active and closet indexing. With

active indexing, the manager incurs costly research effort to determine, in part, the extent to which active stock selection is likely to generate returns in excess of the index. With closet indexing the manager simply indexes the entire portfolio, forgoing the opportunity to incur costly research effort that could generate excess returns while nevertheless collecting a premium fee.

Consistent with Berk and Green (2004), we show that with active indexing the management fee is irrelevant to value created and total manager compensation as long as it is low enough to ensure that the manager receives the amount of assets he can profitably invest. Lower fees increase assets just enough to leave the manager's compensation unchanged. Since these excess assets are indexed they do not cause a reduction in the return to actively-managed assets and the externality from investor crowding is avoided. What, then, determines the optimal fee?

We posit that active fund management is the prototype of an experience good subject to moral hazard. Even if investors know their manager's inherent portfolio selection skill, it takes time for them to determine whether he has spent costly effort researching portfolio selection. A manager might promise to incur costly research effort in exchange for a fee equal to marginal cost and then cheat by closet indexing to avoid these costs. To the extent investors can be temporarily fooled in this way, the manager stands to earn a one-time surplus from shirking. Knowing this, investors refuse to pay a fee that covers the manager's research cost. The result is a low-quality equilibrium. Our solution is to pay the manager a quality-assuring premium (Klein and Leffler, 1981; Shapiro, 1983), or efficiency wage (Akerlof and Yellen, 1986), in excess of his marginal research cost. This premium could persist indefinitely. Because the manager's compensation for performance in any period is paid out over time, the per period fee can be much lower than a one-time fee while ensuring that fund assets meet or exceed what the manager can profitably invest.

Our model fits the form of manager compensation in mutual funds remarkably well. What few scholars have recognized is that standard asset-based management fees are recurring. Assuming a 50 basis point annual fee, a manager who increases total assets through investment performance by $100 can expect to earn 50 cents this year, 50 cents the next year, and so on, as long as the wealth increase persists.[1] Management fees are therefore back-end loaded but conditional on continuing satisfactory performance. Either board termination or shareholder withdrawal can truncate the stream. To the extent reputational markets work, the threat of termination deters the manager from shirking if the capitalized value of the premium fee exceeds the one-time gain from shirking. Investors are assured they will earn the normal return they bargained for and thereby benefit from

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[1]In an efficient market the wealth increase can be expected to persist indefintely. The capitalized value of the manager's marginal share of returns is therefor much higher than 50 basis points; closer to ten percent at plausible discount rates.

higher fees.

We make use of Cremers and Petajisto's (2009) notion of "active share," defined as the ratio of actively managed assets to total assets. Comparative statics show that the level of actively managed assets falls as the cost of research effort per actively-managed dollar rises, but that the effect of research costs on fees and total assets is ambiguous and depends on the magnitude of offsetting effects. Although fees and total assets tend to be inversely related, it is possible that both total assets and the management fee decline as the cost of research effort rises. Similarly, actively managed assets increase in manager skill, while the effect of skill on fees and total assets is ambiguous. Higher skill managers may have both more total assets and higher fees.

By extending the model we identify the circumstances under which investors find it in their interest to subsidize the manager's costly research effort and under which the manager might want to limit fund inflows by closing the fund to new investors. Soft dollar brokerage, in which brokers who execute portfolio trades for the fund provide the manager with research bundled into brokerage commissions (paid by the fund), appears to be an ideal subsidy.

We parameterize monitoring costs and find that fees increase and total assets decrease as investors' difficulty detecting shirking increases. This explains why institutional investors pay lower fees than retail investors, often to the same manager. Being better able and better motivated to detect shirking, institutional investors have no need to pay premium fees for quality assurance. Our results are consistent with the observation that fees on bond funds are lower than fees on equity funds owing to the lower noise in bond fund returns and the greater ease investors have detecting manager shirking. Investors in closed-end funds, who are unable to withdraw their capital, have greater difficulty punishing a shirking manager than mutual fund investors, and so our analysis predicts that higher fees are necessary in closed end funds to deter shirking, as reported by Deli (2002).

The picture that emerges is one in which the mutual fund form of organization provides managers with strong incentives to perform research and investors with liquid, state-of-the art savings on the efficient frontier. Although this is exactly what the model of perfect-competition-subject-to-frictions suggests, fee critics appear to have neglected the obvious. This is not to say further regulation is necessarily out of order, only that any intrusion must address an industry in competitive equilibrium rather than disequilibrium. Sound policy requires that the merits of regulation be evaluated based on its likely competitive effects.

Section 2 of the paper provides a review of relevant literature. Our analysis draws from literature on market efficiency, the economics of property rights, principal-agent relations, and

the effect of fees and fund flows on returns to active management. Section 3 presents the basic model, demonstrating the irrelevance of fees. Section 4 introduces moral hazard in the form of difficult-to-detect research effort and shows that paying the manager a premium, back-end-loaded fee assures high-quality management and thereby solves the shirking problem. It also identifies the circumstances in which investors might want to provide a limited research subsidy, as in the form of soft dollar brokerage. In Section 5 we generate comparative statics for the effect of research costs and manager skill on total assets, actively-managed assets, fees, and active share. Section 6 highlights otherwise anomalous evidence found in the literature that is consistent with our quality-assurance hypothesis. Section 7 provides a summary and concluding remarks.

2 Literature Review

Irwin Friend, principal investigator for the *Wharton Report*, may have been the first scholar to raise the issue of excessive fund advisory fees. His 1962 study, sponsored by the U.S. Securities and Exchange Commission, observed that industry assets under management had increased dramatically over his study period but that fees had remained steady despite what he asserted to be scale economies in fund management. He concluded that fees were immune from competitive market forces and therefore very likely excessive. His report generated calls for mandatory fee caps, leading many advisers to voluntarily adopt marginal declining fee structures. It also presaged 1970 amendments to the Investment Company Act (1940) imposing on advisers a "fiduciary duty with respect to the receipt of compensation for services" and providing fund shareholders with the right to sue advisers for up to one year's worth of excessive fees.

Since then many suits have been filed and some have generated protracted litigation. None have resulted in verdicts for the plaintiffs, although there have been settlements (Henderson, 2010). One of the main issues in these cases has been whether structural evidence of fund or adviser competition is admissible. Despite overwhelming evidence that the industry is structurally competitive (Coates and Hubbard, 2007), the Second Circuit Court of Appeals found in one of the earliest cases that evidence of competition between funds and advisers for investor dollars was inadmissible because fees are so low as a percentage of assets that investors are unlikely to react to fee differences across funds. As a result, excessive fee critics and civil litigation persist. What none have recognized is the powerful role of competition between investors as suppliers of capital in an open access commons.

Our analysis parallels early work by Frank Knight (1924). In a response to A.C. Pigou's

identification of externalities as a rationale for corrective taxation, Knight re-examined Pigou's example of two roads linking two cities. One road Pigou assumed to be slow but with sufficient capacity that it is never congested. The other road he assumed to be faster but subject to congestion. Under open access, travelers join the fast road until it becomes so congested that the marginal traveler is indifferent between which road he chooses. This leads to overuse of the fast road because the marginal traveler neglects the congestion costs he imposes on his fellow travelers, exactly Pigou's externality point.

Pigou proposed that an access tax to the fast road was necessary to prevent overuse, but Knight showed that the optimal tax would be exactly the same as the profit maximizing toll a private road owner would charge. From this Knight concluded that it was not market failure that caused overuse of the fast road but Pigou's unstated assumption that the road was unowned and therefore subject to open access. What is more, under no circumstances could travelers expect to capture the benefits of having access to the fast road. Because no traveler can exclude others, whether or not they pay a tax or toll they will invariably enter the fast road until their time cost plus tax or toll exactly equals their time cost on the slow road.

Similarly, mutual fund investors distribute their wealth across funds until the expected return in each fund is equal to their outside opportunity rate. Individually, each investor, each fund, and each fund manager is an atom in the sea of all possible investment vehicles. To the extent the mutual fund form of organization (all funds in their entirety) is more than an atom in the universe of investment alternatives, however, its presence likely leads to higher investment returns across all investments.

Early on, the property rights literature distinguished only between private and common property (Demsetz, 1967). Eventually it came to distinguish, as well, between common property and open access and generally treated open access as an undesirable form of organization akin to the complete absence of ownership. Lueck (1995) then showed that, compared to the viable alternatives, the rule of first possession — which goes hand in hand with open access — is often efficient given the cost of transacting. We show that open access is efficient in the context of open-end mutual funds, which provide investors with liquidity and state-of-the-art returns. Closed-end funds stand beside open-end funds as a closed-access form of managed portfolio, and yet closed-end funds often exhibit their own drawback in the form of large share price discounts from net asset value. Neither form of organization can be expected to achieve first-best in a world subject to frictions.

Open access does not describe the complete absence of property rights, it simply determines the moment at which ownership vests and to what. Those who race to catch fish on the open sea own

their catch, but they have no exclusive claim to the underlying fish stocks. Travelers who enter Knight's fast road own their position in the queue, even though they have no persistent ownership over the road as a capital stock. Similarly, mutual fund investors own their share of the portfolio value at any given moment, but they have no exclusive claim to the underlying capital stock — the manager's skill — that generates yet-to-be realized excess returns.

One of the ongoing controversies on the subject of informational efficiency is whether active mutual fund management can add value if the underlying portfolio securities are efficiently priced. According to early theoretical work (Fama, 1970), in an informationally efficient market mutual fund managers should be unable to consistently outperform the market, and any fee they charge for active management must therefore reduce their investors' net returns. Early empirical work by Jensen (1968) seemed to confirm this implication, finding that a large sample of active equity fund managers generated risk-adjusted net returns that, if anything, fell short of the market. Follow-on empirical work (Malkiel, 1995; Carhart, 1997) found that fees and net returns were negatively correlated in the cross section of active funds, suggesting that the fees active managers charge indeed reduce investor returns.

Later theoretical and empirical work has qualified these findings. In 1980, Grossman and Stigliz showed that markets must be subject to an efficient amount of mispricing to give market participants a reward for correcting prices. One implication of their analysis is that, in expectation, resources devoted to price discovery will generate only a normal return. Any appearance of persistent abnormal returns to an individual's price discovery efforts must be counted as Ricardian rents accruing to his superior skill.

Berk and Green (2004) show that in a world of rational shareholders and efficient markets the best mutual fund investors can expect is a normal return equal to their outside alternative. Any expectation of superior returns will attract inflows to the fund until returns are normalized. Jensen's findings that active funds do not outperform the index are therefore what we would expect in an efficient market. That investors receive normal returns does not imply that managers add no value, only that investors compete away any value the managers add. Empirically, Berk and Van Binsbergen (2012) find that the average mutual fund manager adds about $2 million per year, and that the skill that makes such value-creation possible persists.

Using an exhaustive database of all mutual funds, Wermers (2000) showed based on actual portfolio selection (rather than net asset value) that large-cap mutual fund managers outperformed the market on average by 1.3 percent before subtracting fees and various transaction costs. Net of fees and transaction costs, fund managers underperformed the market by one percent. High-

turnover (the most actively managed) funds were able to beat a popular index fund by more than enough to cover their higher fees and transaction costs, however, thus casting doubt on earlier findings of a negative relation between fees and returns.

Barras, Scaillet, and Wermers (2010) introduce a new method of accounting for false discoveries in mutual fund performance, and they find evidence of persistent manager skill. They estimate that about 10% of managers add value in the sense of delivering positive pre-expense alphas, but of course the bulk of such value accrues to managers, with only 0.6% of managers delivering positive after-expense alphas over the recent past; 75.4% of fund managers deliver zero after-expense alphas and 24% negative alphas.

On average, active management has underperformed based on net asset value returns.[2] Pástor and Stambaugh (2012) show that this is not necessarily inconsistent with investor rationality or with the prospect that active management adds value. They use the assumption of diminishing industry returns to scale in the face of finite, imperfectly estimated opportunities for profitable stock picks to explain the persistent underperformance of active management. In their model, the informational content of investor returns is dramatically reduced by investors' ability to reduce the share of fund assets that are actively managed; as a greater share of fund assets moves to index funds, the finite number of profitable stock picks is shared between fewer active assets, causing returns to increase, all else being equal. Investors update their priors regarding the returns from active management so slowly that, given the priors consistent with their past active holdings, their posteriors remain (weakly) positive despite the years of active management underperformance.

Ippolito (1992) was the first to recognize that fund management is plagued by problems of quality assurance. In his words, "[t]here is much noise in performance data across mutual funds and over time, requiring many periods to judge the ability of an investment manager with statistical confidence" (p. 46). He found a large positive relationship between fund inflows and fund quality as measured by superior net returns. The evidence allowed him to confidently reject the hypothesis that fund growth and fund performance were independent. Over sufficiently long time periods, investors appeared able to monitor and punish low quality funds by directing the flow of new money toward superior performers. He predicted that high-quality managers will seek some mechanism to assure quality such as posting a performance bond on which they would later earn a quality-assuring rent. Owing to data limitations he was unable to test this implication. Dow and Gorton (2002) also recognize the problem of moral hazard in fund management, suggesting that investors have

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[2]See Jensen (1968), Del Guercio and Reuter (2011), Fama and French (2010), Gruber (1996), Pástor and Stambaugh (2002), and others for evidence.

difficulty identifying managers' investment opportunities and that they use turnover as a rough proxy for manager effort, thereby leading managers to overtrade.

Sirri and Tufano (1998) examine various determinants of fund flows. Among other things, they find that larger fund families and funds that spent more on advertising exhibited greater inflows than their counterparts. What is more, fund families that had higher advisory fees experienced fund inflows that were double those experienced by their low-fee counterparts. Chevalier and Ellison (1997) examined the incentives created by the standard performance-flow relationship in mutual funds. Their estimates show that a fund that beat the market in 1990 by 10 percentage points experienced a 36 percent increase in fund assets, holding various other factors constant. Fund inflows persisted in subsequent years, though at a declining rate.

Based on a sample of over 4,800 advisory contracts, Deli (2002) found that managers' marginal compensation is driven by hypothesized differences in adviser marginal product (skill) and the difficulty investors have monitoring performance across funds with different characteristics. For example, high turnover funds and funds with high underlying volatility owing to investment style — both of which impose greater monitoring costs on investors — had higher marginal adviser compensation. These results are consistent with our hypothesis that fund fees mitigate agency conflicts with fund shareholders. Although Deli's results did not allow him to reject the hypothesis that manager compensation is driven by an attempt to expropriate fund investors, he argued that the ease of entry into the industry along with the large number of funds and advisors cast doubt on the expropriation hypothesis.

3 The Basic Model and the Irrelevance of Fees

The present section sets the stage for our analysis by rederiving the Berk and Green (2004) results in the context of our somewhat simplified model. To focus on the manager's moral hazard problem in the sections that follow, we diverge from Berk and Green in assuming neither stochastic returns nor asymmetric information.

3.1 The Basic Model

Let the normal rate of return available to all investors through the market index be r, with \$1 invested in the index growing to $\$(1+r)$ after one period. Consider a skilled manager who can beat the index through active management. Let shares in the manager's fund be priced at one

dollar per share, so that total invested assets T also equal the number of outstanding shares, as in a money market fund. Let the manager actively manage A shares at a cost C per share and index the remaining shares, $T - A$. Assume for the time being that the manager can borrow to invest by selling short the index, so that $T - A \lessgtr 0$. Assets invested with the manager grow to $(1 + R(A)) A + (1 + r)(T - A)$. We assume that $R(0) - C > r$, $R'(.) < 0$, $R''(.) > 0$, and $lim_{A \to \infty} R(A) = 0$. Returns per actively managed dollar decrease in actively managed assets ($R'(.) < 0$), but the decrease is attenuated as more assets are actively managed ($R''(.) > 0$).[3] Let the manager charge fees f per dollar invested. The manager has payoff $f [(1 + R(A)) A + (1 + r)(T - A)] - CA$ and investors $(1 - f) [(1 + R(A)) A + (1 + r)(T - A)]$.

We first note that the amount of actively managed assets, A, that maximizes the combined payoff of investors and the manager depends neither on fees nor on total invested assets, T. Formally,

$$
\begin{aligned}
A^{opt} \equiv \arg\max_{A} \quad & (1 - f) [(1 + R(A)) A + (1 + r)(T - A)] \\
& + f [(1 + R(A)) A + (1 + r)(T - A)] - CA \\
= \arg\max_{A} \quad & (R(A) - r) A + (1 + r) T - CA \\
= \arg\max_{A} \quad & (R(A) - r - C) A;
\end{aligned}
$$

A^{opt} therefore satisfies

$$
R(A^{opt}) + A^{opt} R'(A^{opt}) - C = r. \tag{1}
$$

The term term $A^{opt} R'(A^{opt}) < 0$ represents the negative externality associated with an increase in actively managed assets. Because each additional actively managed dollar dilutes all other shareholders' return, the marginal return, $R(A^{opt}) + AR'(A^{opt}) - C$, is less than the average return, $R(A^{opt}) - C$.

3.2 The Irrelevance of Fees

Following Berk and Green (2004), we assume assets invested with the manager grow until such point as to equalize net of fees returns in the fund with returns on the index

$$
(1 - f) [(1 + R(A)) A + (1 + r)(T - A)] = (1 + r) T.
$$

Fees clearly do not matter for net investor returns: lower fees, f, result in higher share purchases, T, which drive down gross returns, $[(1 + R(A)) A + (1 + r)(T - A)] / T$; higher fees result in lower

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[3]Note that the contrary assumption, $R''(.) < 0$, would conflict with the natural assumption that $lim_{A \to \infty} R(A) = 0$. The diminishing marginal product assumption follows Berk and Green (2004); it is a sufficient condition for Pástor and Stambaugh's (2012) decreasing industry returns to scale assumption.

share purchases, which keep gross returns high; lower or higher fees combine with lower or higher gross returns, respectively, to equate net, after-fees returns to $1 + r$ in the two cases of low and high fees. The open-access nature of mutual funds ensures that investor returns are driven down to the normal rate.

The preceding holds true for all actively managed assets A. Still following Berk and Green (2004), we now show that fees matter neither for manager returns nor for the manager's choice of actively managed assets, which the manager equates to the optimal level A^{opt}. To see this, consider the manager's problem

$$\max_{f,A,T} f\left[(1 + R(A)) A + (1 + r) (T - A)\right] - CA \tag{2}$$

subject to

$$(1 - f)\left[(1 + R(A)) A + (1 + r) (T - A)\right] = (1 + r) T. \tag{3}$$

We have

Result 1 *Fees affect neither manager returns nor the level of actively managed assets, which equal the optimal level, A^{opt}.*

The intuition is simple: because investor returns are driven down to the normal rate available through the index, the manager receives the entire value he creates by beating the index; it is therefore in his own interest to maximize that value; he does so by choosing the optimal level of actively managed assets A^{opt}. To illustrate, consider the admittedly extreme case where $f = 1$. Clearly, no assets will be invested with the manager. Yet, the manager can reap profit $(R(A) - r - C) A$ by shorting the index at a cost r in the amount A; he will naturally maximize that value by choosing $A = A^{opt}$.

The option for the manager to invest part of the assets invested in the index is essential to Result 1. A requirement that the manager actively manage all assets, $A = T$, would require fees to equal

$$f^{opt} = \frac{-A^{opt} R'(A^{opt}) + C}{1 + R(A^{opt})}. \tag{4}$$

Only that level would induce investors to purchase A^{opt} shares in the fund.[4]

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[4]To see this, note that

$$\left(1 - f^{opt}\right)\left(1 + R\left(A^{opt}\right)\right) A^{opt} = \left(1 - \frac{-A^{opt} R'(A^{opt}) + C}{1 + R(A^{opt})}\right)\left(1 + R\left(A^{opt}\right)\right) A^{opt}$$
$$= \left(1 + R(A^{opt}) + A^{opt} R'(A^{opt}) - C\right) A^{opt}$$
$$= (1 + r) A^{opt}$$

Result 1 begs the obvious question of what determines optimal fees. We answer and elaborate on this question in the remainder of the paper. Before doing so, we establish two brief implications of Result 1.

Implication 1 *A limit to fund inflows increases investor returns above the return on the index and decreases manager returns.*

Limits to inflows prevent investors from driving returns down to the normal rate available through the index. Investors therefore receive part of the value created by the manager, whose profit is correspondingly reduced.

Implication 2 *The joint requirement that fees be set at their break-even level and that the manager actively manage all assets decreases manager returns and leaves investor returns unchanged.*

Break-even fees compensate the manager for the costs he incurs but do not account for the negative externality associated with an increase in actively managed assets, $A^{opt}R'(A^{opt}) < 0$ in (1) and (4). Shareholders consequently overinvest in the fund, thereby decreasing the value created by the manager.

4 Moral Hazard and Quality-Assuring Fees

The preceding analysis fails to consider manager shirking in the form of closet indexing, in which the manager charges a fee for active management but invests the entire amount, T, in the index. This allows him to avoid the cost of active management, CA^{opt}. He might be tempted to shirk where the total fees he earns on excess returns are less than the per dollar cost of active management:

$$f(1+r)T > f\left[\left(1 + R(A^{opt})\right)A^{opt} + (1+r)\left(T - A^{opt}\right)\right] - CA^{opt}$$

$$\Leftrightarrow f\left(R(A^{opt}) - r\right) < C.$$

Shirking can be precluded by setting fees $f \geqslant C/\left(R(A^{opt}) - r\right)$. That minimum fees, $C/\left(R(A^{opt}) - r\right)$, may generate sub-optimal inflows into the fund ($T < A^{opt}$) need not be a problem where the manager can short the index by the missing amount, $A^{opt} - T$. In the presence of short-selling constraints, however, it is no longer true that assets, T, can be supplemented by shorting the index.

where the last equality is true from (1). Optimal fees f^{opt} are the equivalent for a mutual fund of Knight's (1924) optimal toll for a private road.

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In what follows we assume the manager cannot short sell the index. Lowering fees to at least the level that induces investment of A^{opt} becomes essential for him to reap the full value of his human capital.

4.1 Reputation and 'Back-end Loading'

Managerial reputation provides one mechanism to assure fees are sufficiently low to induce optimal investment. Following Klein and Leffler (1981) and Shapiro (1983), we assume a manager who shirks by closet indexing would thereafter be denied all funding. Because of the manager's bad reputation investors will refuse to pay him anything for indexing because they can engage in indexing on their own. A shirking manager thereby would be denied the opportunity to reap the value of his human capital. The 'no shirking' condition in the presence of reputational concerns becomes

$$\frac{f\left[\left(1+R(A^{opt})\right)A^{opt}+(1+r)\left(T-A^{opt}\right)\right]-CA^{opt}}{r}\geqslant\frac{f\left(1+r\right)T}{1+r}, \tag{5}$$

where total assets invested with the manager, T, are the solution to

$$(1-f)\left[\left(1+R(A^{opt})\right)A^{opt}+(1+r)\left(T-A^{opt}\right)\right]=(1+r)T.$$

Assume for the time being that fees are such that $T\geqslant A^{opt}$. Condition (5) can be rewritten as

$$\begin{aligned}f\left(1+r\right)T-f\left[\left(1+R(A^{opt})\right)A^{opt}+(1+r)\left(T-A^{opt}\right)\right]+CA^{opt}\\\leqslant\frac{f\left[\left(1+R(A^{opt})\right)A^{opt}+(1+r)\left(T-A^{opt}\right)\right]-CA^{opt}}{r}\end{aligned} \tag{6}$$

The preceding inequality states that the one-period gain from shirking is lower than the capitalized value of future profits from active management. We have

Result 2 *The minimum level of fees necessary to preclude shirking in the presence of reputational concerns is*

$$f^{r}\equiv\frac{1}{r}\left[C\left(\frac{1+r}{R(A^{opt})-r}\right)-1\right]. \tag{7}$$

Expanding f^{r} into $C/\left(R(A^{opt})-r\right)-\left[1-C/\left(R(A^{opt})-r\right)\right]/r$, we observe that managerial reputation decreases the minimum level of fees by $\left[1-C/\left(R(A^{opt})-r\right)\right]/r$, from the level, $C/\left(R(A^{opt})-r\right)$, that would be necessary if there were no reputational constraints. To interpret f^{r}, it is helpful to rewrite (7) as

$$f^{r}\left(R(A^{opt})-r\right)A^{opt}=CA^{opt}-\frac{\left(R(A^{opt})-r-C\right)A^{opt}}{r}. \tag{8}$$

The term $\left(R(A^{opt}) - r - C\right) A^{opt}$ is the per period value of the manager's services. The value to the manager of remaining in the relationship is therefore $\left(R(A^{opt}) - r - C\right) A^{opt}/r$. This value is foregone in the event the manager shirks because it is paid out as a stream of back-end loaded fees.

4.2 Subsidies, Limits to Inflows, and Soft Dollars

Assume reputation effects fail to lower fees to the level the manager can profitably manage actively; that is, $T < A^{opt}$ at f^r. Consider a subsidy, SA^{opt}, from investors to cover a portion of the manager's costs. It is clear that such a subsidy decreases the shirking problem. By reducing the cost the manager incurs for active management, the subsidy decreases his gain from shirking and thereby decreases the minimum level of fees. Fees, f, and subsidies, S, must be such that[5]

$$(1 - f)\left(1 + R(A^{opt})\right) A^{opt} = (1 + r) A^{opt} + SA^{opt} \tag{9}$$

$$\Leftrightarrow S = (1 - f)\left(1 + R(A^{opt})\right) - (1 + r) \tag{10}$$

and

$$f(1 + r) A^{opt} - f\left(1 + R(A^{opt})\right) A^{opt} + (C - S) A^{opt}$$
$$= \frac{f\left(1 + R(A^{opt})\right) A^{opt} - (C - S) A^{opt}}{r} \tag{11}$$

$$\Leftrightarrow f\left(R(A^{opt}) - r\right) = C - S - \frac{R(A^{opt}) - r - C}{r}, \tag{12}$$

where equation (12) is obtained by substituting S from (10) into the RHS of (11). Equation (12) shows that the minimum level of fees consistent with no shirking by the manager at A^{opt} falls with the (per dollar) subsidy, S. Equation (10) equates that subsidy to the difference between the after-fees return investors receive and the normal return.

The problem, of course, is that investors must be compensated for the subsidy they pay. A low level of fees alone will fail to do so because of the open access nature of the fund. It is therefore necessary to limit inflows into the fund: recall from our discussion at the end of Section **??** that such a limitation increases investors' returns above the normal rate; it thereby provides the means to compensate investors for the subsidy they have paid. We show in Result 3 that the limitation of fund assets to A^{opt} must be contractual, because the manager otherwise would seek to increase fund assets beyond A^{opt}, to the detriment of those investors who collectively pay the subsidy SA^{opt}.

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[5]Note there are no passively managed assets because the purpose of the subsidy is to raise actively managed assets to the optimal level A^{opt}.

Result 3 *The limitation of fund assets to A^{opt} is not self-enforcing and must be contractual.*

In essence, the manager can issue new shares to increase invested assets past the level, A^{opt}, at which existing shares are compensated for the subsidy, SA^{opt}, they have paid the manager. Unlike existing shareholders, new shareholders are compensated for the subsidy they pay — despite the decreased per dollar return due to increased investment — because the subsidy is lower than what existing shareholders pay. The manager profits by earnings fees on a higher level of invested assets.

Now suppose directly subsidizing the manager's research costs is impossible, perhaps because the manager cannot commit to using the subsidy for the intended purpose. It is possible that an indirect subsidy tied to a suitable proxy for the manager's research effort will solve the problem. An example of an indirect subsidy is soft dollar brokerage, in which the cost of research is bundled into the brokerage commission the portfolio pays for securities trades. More research brings more trading and vice-versa.[6] Formally, let a broker offer the manager research subsidies SA in return for a brokerage fee premium of the same amount. The manager's payoff is then

$$f\left(1 + R(A) - S\right)A - (C - S)A \tag{13}$$

Investors' payoff remains unchanged at $(1 + r)A$ because of open access

$$(1 - f)\left(1 + R(A) - S\right)A = (1 + r)A \tag{14}$$

We show

Result 4 *The soft dollar subsidy S can be used to decrease f to the point at which $A = A^{opt}$. Subsidy and fees are such that*

$$S = 1 + R(A^{opt}) - \frac{1 + r}{1 - f} \tag{15}$$

and

$$f\left(R(A^{opt}) - r - S\right) = C - S - \frac{R(A^{opt}) - r - C}{r}, \tag{16}$$

respectively.

Note that the choice of $A = A^{opt}$ is optimal for the manager, whose payoff in (13) can be rewritten using the equation for investors' payoff in (14)

$$f\left(1 + R(A) - S\right)A - (C - S)A = \left(1 + R(A) - S\right)A - (1 + r)A - (C - S)A$$

$$= \left(1 + R(A)\right)A - (1 + r)A - CA;$$

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[6] Horan and Johnsen (2008) examine the role soft dollars play in subsidizing research while assuring the quality of the broker's execution.

the last expression is maximized at A^{opt}. Unlike the case for direct subsidies, there is no need for contractually limiting inflows for soft dollars. This is to be expected, as there can be no discrimination between existing and new shares in the case of soft dollars. There is, however, a need to limit the use of soft dollars by the manager because the use of a level above S in (15), combined with fees f in (16), would decrease investors' return below the normal rate.

Can such a limit be enforced where the manager cannot commit to limiting his use of soft dollars? Agency law appears to do so by providing shareholders with an ex post cause of action for fiduciary breach. One established statement of an agent's fiduciary duty is that he must act on behalf of the principal with the same care and prudence he would use to conduct his own affairs. This implies that the manager should use the subsidy to buy research up to the point at which a dollar of research generates no less than one dollar in returns for the fund. Any use of soft dollars beyond this point risks suit for breach of fiduciary duty. This requirement recalls Equation (15) defining the optimal level of soft dollars, which can be rewritten as

$$(1 - f) S = (1 - f) \left(1 + R(A^{opt})\right) - 1 + r.$$

In equilibrium, the fraction of soft dollar expenses borne by investors must generate net-of-fees abnormal returns of zero in percentage terms.

Note that soft dollars decrease the minimum level of fees only where they are used to subsidize research costs. This is clear from equation (16), which suggests that fees would increase rather than decrease in S if the term $C - S$ were replaced by C, i.e., if soft dollars were used to subsidize other than research costs. In accordance with that requirement, Section 28(e) of the Securities Exchange Act (1934) provides a safe harbor to managers from fiduciary suits as long as they use the subsidy strictly for "brokerage and research services." The U.S. Securities & Exchange Commission could bring civil actions against managers who use soft dollars to acquire things it believes do not qualify as research, and although the acquisition of such items does not necessarily violate state agency law or other federal laws most fund boards rely on the SEC's interpretation as a policy limitation to their managers' use of soft dollars.

5   Comparative Statics

We now derive the comparative statics of the costs of active management, C, on total assets actively managed, A, total assets under management, T, fees, f, and active share, A/T. We also extend the model to account for differences in manager skill. We assume throughout that managers are

'empire builders'; all else being equal, they prefer more assets under management to less. They therefore choose the lowest level of fees consistent with no shirking, that is, they choose $f = f^r$.

5.1 Costs of Active Management

The manager solves

$$\max_{f,A,T} f\left[(1 + R(A)) A + (1 + r) (T - A)\right] - CA$$

subject to

$$(1 - f)\left[(1 + R(A)) A + (1 + r) (T - A)\right] = (1 + r) T$$

and

$$(1 + rf)(R(A) - r) - (1 + r) C = 0.$$

This equilibrium is defined by[7]

$$(1 + rf)(R(A) - r) - (1 + r) C = 0, \tag{17}$$

$$R(A) - r + AR'(A) - C = 0, \tag{18}$$

and

$$(1 - f)(R(A) - r) A - f (1 + r) T = 0, \tag{19}$$

with f, A, and T the endogenous variables.[8] We wish to examine how these vary in costs, C. We have

Result 5 *Assets under active management decrease in costs ($\partial A/\partial C < 0$). A necessary and sufficient condition for fees to increase in costs ($df/dC > 0$) is*

$$-(1 + rf) R'(A) + (1 + r)\left(2R'(A) + AR''(A)\right) < 0. \tag{20}$$

The condition is also sufficient for total assets under management and for active share to decrease in costs ($\partial T/\partial C$ and $\partial [A/T]/\partial C$).

That assets under active management decrease in the costs of active management is to be expected. That fees do not necessarily increase in costs is perhaps unexpected. There are both

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[7]Denoting λ and μ the Lagrange multipliers associated with the two constraints, respectively, we have $\lambda = 1$ and $\mu = 0$.

[8]The solutions for A and f are A^{opt} and f^r in (1) and (7), respectively. We drop the supbscripts to simplify the notation.

a direct and an indirect effect through assets under active management of an increase in costs on fees. The former increases fees, the latter decreases them by increasing actively managed assets and thereby increasing per dollar return: the larger is per dollar return, the more the manager stands to lose from shirking and the lesser therefore the need for him to bond his performance through high fees. Intuitively, condition (20) states that the marginal return, $R'(.)$, on actively managed assets must not be so large in magnitude as to make the indirect, negative effect of an increase in C on A offset the direct, positive effect in the expression for f.[9]

Unlike the case for fees, condition (20) is only sufficient for total assets under management, T: it is possible for these to decrease ($\partial T/\partial C < 0$) even as fees decrease ($\partial f/\partial C < 0$).[10] This may occur where the decrease in actively managed assets, A, due to the increase in costs, C, is so large as to dominate any increase in passively managed assets, $T - A$, possibly due to the decrease in fees, f.

The intuition for the result on active share, A/T, can perhaps best be understood by considering equation (38), reproduced here for convenience

$$\frac{A}{T} = \left[\frac{1-f}{f}\left(R(A) - r\right)\right]^{-1}(1 + r).$$

It is active management that makes possible the abnormal returns that compensate investors for the payment of fees. Where fees decrease in costs there is less need for active management and A/T declines. This is all the more so because per-dollar return, $R(A)$, increases owing to the reduction in actively managed assets. Where, in contrast, fees increase in cost it may be that the increase in active per-dollar return, $R(A)$, fails to cover the increased payment of fees. A higher fraction of assets must be managed actively in such a case and active share must rise.

We now consider gross returns on total assets under management and assets under active management. The former are[11]

$$1 + R_{G,T} \equiv \frac{(R(A) - r)A + (1 + r)T}{T}$$
$$= \left(\frac{f}{1-f} + 1\right)(1 + r). \tag{22}$$

Note that $sgn\{\partial R_{G,T}/\partial C\} = sgn\{\partial f/\partial C\}$. Owing to open access, net returns on total assets under management equal r, and investors therefore must be compensated for the payment of higher

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[9]Formally, condition (20) can be shown to be necessary and sufficient for $\partial f/\partial C + \partial f/\partial A \times \partial A/\partial C > 0$.

[10]To see this, totally differentiate (19) with respect to C to obtain

$$f(1+r)\frac{\partial T}{\partial C} = -\left[(R(A) - r)A + (1+r)T\right]\frac{\partial f}{\partial C} + (1-f)\left[AR'(A) + R(A) - r\right]\frac{\partial A}{\partial C}. \tag{21}$$

[11]The equality is obtained by using (19) to replace T by $(1 - f)(R(A) - r)A/[f(1+r)]$.

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fees through higher gross returns. Alternatively, as abnormal returns accrue to the manager he captures higher gross returns through higher fees.

Gross returns on assets under active management are

$$1 + R_{G,A} \equiv \frac{(1 + R(A)) A}{A} = 1 + R(A). \tag{23}$$

Differentiating with respect to C, we obtain

$$\frac{\partial R_{G,A}}{\partial C} = R'(A) \frac{\partial A}{\partial C} > 0.$$

Where the cost of active management is higher and fewer assets are managed actively, the assumption of decreasing marginal returns on actively managed assets implies that per-dollar return increases.

5.2 Manager Skill

We have thus far assumed that all fund managers are identical. Yet, as Berk and Green (2004) and others have noted, there is a wide distribution of manager skill. Some managers are able to earn higher returns than others for given C. In this section we analyze how actively managed assets, A, total assets under management, T, fees, f, and active share, A/T, vary with manager skill.

Let θ index managerial skill and write per dollar return on active management as $R(A, \theta)$. We assume $\partial R(., \theta)/\partial \theta > 0$, $\partial R'(., \theta)/\partial \theta > 0$, and $\partial R''(., \theta)/\partial \theta > 0$. The first inequality indicates that higher quality managers obtain higher return, the second that higher quality managers attenuate more the decrease in return that results from an increase in invested assets, $(R'(., \theta) < 0)$, and the third that the attenuation of the decrease in return due the increase in assets $(R''(., \theta) > 0)$ is faster for higher quality managers.

The system of equations (17)-(19) that defines equilibrium remains valid, with return on active management now including the additional argument θ. We have

Result 6 *Assets under active management increase in manager skill ($\partial A/\partial \theta > 0$). A necessary and sufficient condition for fees and active share to decrease in skill ($\partial f/\partial \theta < 0$ and $\partial [A/T]/\partial \theta < 0$) is*

$$R'(A, \theta) \left(\frac{\partial R(A, \theta)}{\partial \theta} + A \frac{\partial R'(A, \theta)}{\partial \theta} \right) - \left(2R'(A, \theta) + AR''(A, \theta) \right) \frac{\partial R(A, \theta)}{\partial \theta} > 0. \tag{24}$$

The condition is also sufficient for total assets under management to increase in manager skill ($\partial T/\partial \theta > 0$).

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That higher skill managers should actively manage more assets is to be expected. The intuition for the other results is as follows. Condition (24) is equivalent to[12]

$$\frac{dR(A,\theta)}{d\theta} = \frac{\partial R(A,\theta)}{\partial\theta} + R'(A,\theta)\frac{\partial A}{\partial\theta} > 0.$$

For a given level of cost, $R(A,\theta)$ determines the manager's active per-dollar return. The higher that return, the more the manager stands to lose from shirking and the lower the level of fees necessary to deter the manager from shirking. The manager's active per-share return naturally increases in skill, θ ($\partial R(A,\theta)/\partial\theta > 0$). There is, however, an offsetting effect because actively managed assets also increase in skill ($\partial A/\partial\theta > 0$). This decreases active per-dollar return ($R'(.,\theta) < 0$), all else being equal. The former effect is direct, the latter indirect through actively managed assets. Which of the direct or indirect effect dominates determines whether active per-dollar return increases or decreases in skill and thereby determines whether fees decrease or increase in skill.

Somewhat symmetrically to $\partial f/\partial C < 0$ and $\partial T/\partial C < 0$, it is possible that $\partial T/\partial\theta > 0$ even as $\partial f/\partial\theta > 0$. That is, high-skill managers may have both higher fees and more total assets under management than their low-skill counterparts.[13] The intuition is essentially symmetrical to that for C: higher quality managers have more actively managed assets, which have a direct positive effect on total assets under management and an indirect negative effect that may be so large as to dominate any decrease in passively managed assets, $T - A$, possibly due to the increase in fees, f.

To understand the intuition for the result $sgn\left\{\partial\left[A/T\right]/\partial\theta\right\} = sgn\left\{\partial f/\partial\theta\right\} = -sgn\left\{dR(A,\theta)/d\theta\right\}$, recall (38)

$$\frac{A}{T} = \left[\frac{1-f}{f}\left(R(A,\theta)-r\right)\right]^{-1}(1+r).$$

Essentially, this states that active share is inversely proportional to active per-dollar return (recall that fees are themselves inversely proportional to active per-dollar return). Active share makes possible the abnormal returns that compensate investors for the payment of fees. Because fees are inversely related to active per-dollar return, managers that have lower active per-dollar return charge higher fees, which require them to have higher active share.

We now consider gross returns on total assets under management, T, and assets under active

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[12]As with (20), condition (24) is also equivalent to $\partial f^b/\partial\theta + \partial f^b/\partial A \times \partial A/\partial\theta < 0$.

[13]This can easily be seen by totally differentiating (19) with respect to θ to obtain

$$f(1+r)\frac{\partial T}{\partial\theta} = -\left[(R(A,\theta)-r)A+(1+r)T\right]\frac{\partial f}{\partial\theta}+(1-f)\left[AR'(A,\theta)+R(A,\theta)-r\right]\frac{\partial A}{\partial\theta}+(1-f)\frac{\partial R(A,\theta)}{\partial\theta}A.$$

Note the term $(1-f)A\partial R(A,\theta)/\partial\theta$, which has no counterpart in the corresponding equation (21) for C; this term represents the direct effect of managerial quality on the equilibrium condition for total funds under management.

management, A. By analogy to Equations (22) and (23), these are

$$1 + R_{G,T} = \left(\frac{f}{1-f} + 1 \right)(1+r)$$

and

$$1 + R_{G,A} \equiv \frac{(1 + R(A,\theta))\, A}{A} = 1 + R(A,\theta).$$

As for C and for the same reason, we have $sgn\,\{\partial R_{G,T}/\partial\theta\} = sgn\,\{\partial f/\partial\theta\}$. Regarding $R_{G,A}$, it follow immediately that the condition (24) that determines the sign of $dR(A,\theta)/d\theta$ — as well as those of $\partial f/\partial\theta$ and $\partial\,[A/T]\,/\partial\theta$ — also determines the sign $\partial R_{G,A}/\partial\theta$. Specifically,

$$sgn\,\{\partial R_{G,A}/\partial\theta\} = sgn\,\{dR(A,\theta)/d\theta\}\,(= -sgn\,\{\partial f/\partial\theta\} = -sgn\,\{\partial\,[A/T]\,/\partial\theta\} = -sgn\,\{\partial R_{G,T}/\partial\theta\})\,.$$

Gross returns on assets under active management vary in manager skill in step with active per-dollar returns: gross returns on assets under active management are higher where active returns are higher and lower where active returns are lower.

6 Extensions and Evidence

6.1 Imperfect and Costly Detection of Shirking

Up to this point we have assumed that fund shareholders detect shirking with certainty after one period. We now assume they detect shirking with a probability $\gamma \leqslant 1$. The no shirking condition, (5), then becomes

$$\frac{f\,[(1+R(A))\,A + (1+r)\,(T-A)] - CA}{r}$$
$$\geqslant \frac{f\,(1+r)\,T}{1+r} + (1-\gamma)\frac{f\,(1+r)\,T}{(1+r)^2} + (1-\gamma)^2\frac{f\,(1+r)\,T}{(1+r)^3} + \dots$$
$$= \frac{f\,(1+r)\,T}{1+r}\left(\frac{1}{1 - \frac{1-\gamma}{1+r}} \right). \tag{25}$$

which we rewrite as

$$f \geqslant \frac{C}{R(A)-r} - \left(1 - \frac{C}{R(A)-r}\right)\frac{\gamma}{r} \equiv f^r \tag{}$$
$$\Leftrightarrow f^r\,(R(A)-r) = C - \gamma\frac{R(A)-r-C}{r}. \tag{26}$$

Equation (26) differs from its analog in the case where shirking is detected with certainty, (8), in that the value to the manager of remaining in the relationship with investors, $\left(R(A^{opt}) - r - C\right)A^{opt}/r$,

is lost only in case of detection, which occurs with probability γ. Setting f equal to its lower bound, the system of equations (17)-(19) that defines the equilibrium becomes

$$(\gamma + rf)(R(A) - r) - (\gamma + r)C = 0, \tag{27}$$

$$R(A) - r + AR'(A) - C = 0, \tag{28}$$

and

$$(1 - f)(R(A) - r)A - f(1 + r)T = 0. \tag{29}$$

Result 7 *Assets under active management are unaffected by the probability of detection* ($\partial A/\partial \gamma = 0$). *Fees and active share decrease* ($\partial f/\partial \gamma < 0$ *and* $\partial [A/T]/\partial \gamma < 0$) *and total assets under management increase in the probability of detection* ($\partial T/\partial \gamma > 0$).

Assets under active management are unaffected by the probability of detection, γ, because unlike costs, C, and skill, θ, that probability has no bearing on the value of the manager's human capital, which determines $A = A^{opt}$. An increase in the probability of detection decreases fees because there is less need for bonding through high fees. The decrease in fees increases total funds under management because, unlike in the previous two cases of costs and skill, there is no effect through actively managed funds possibly to offset the effect of decreased fees.

We have thus far assumed that monitoring is done at no cost. Suppose this remains true for some probability of detection γ_0, but that it is possible to increase the probability of detecting shirking to $\gamma > \gamma_0$ at a cost $C(\gamma)A$, such that $C'(.) > 0$, $C''(.) > 0$, with $C(\gamma_0) = 0$ and $\lim_{\gamma \to \gamma_0} C'(\gamma) = 0$. Would the parties wish to spend resources to increase the probability of detection?

The answer is no where $T \geqslant A^{opt}$ at the level of fees f_0 such that equation (27) holds with probability of detection γ_0, i.e., at f_0 such that

$$(\gamma_0 + rf_0)(R(A^{opt}) - r) = (\gamma_0 + r)C.$$

There is no need to incur the cost of detection $C(\gamma)A^{opt}$ where quality assurance can be achieved at no cost through a premium fee, f_0. Because shareholders receive the normal rate of return they must be compensated by the manager for any detection costs they might bear, and the manager's returns are thereby reduced by the amount such costs, from $(R(A^{opt}) - r - C)A^{opt}$ to $(R(A^{opt}) - r - C - C(\gamma))A^{opt}$.

Suppose however that $T < A^{opt}$ at f_0. The premium fee required for quality assurance is now so high that it reduces funds under management, T, below A^{opt}. As in Section 4.2, the lower level of

funds under management precludes the manager from receiving the full value of his human capital. Instead, he receives

$$\left(R(A_0) - r - C\right) A_0 < \left(R(A^{opt}) - r - C\right) A^{opt},$$

where A_0 solves $(1 - f_0)\,(1 + R(A))\,A = (1 + r)\,A$. We show

Result 8 *Where $T < A^{opt}$ at f_0, the manager will compensate investors for bearing the cost, $C\left(\gamma\right)$, of increasing the probability of detection γ beyond γ_0.*

The manager is willing to compensate investors for bearing the cost of detecting shirking, to some extent at least, because he can increase funds under management towards A^{opt} in so doing: recall from Result 7 that $\partial T/\partial \gamma > 0$ and note that all funds are actively managed, $A = T$, in as long as funds under management are less than A^{opt}.

Under the quality assurance hypothesis for fund fees, the lower cost of detecting shirking institutional investors face explains why they pay lower fees, even to the same fund manager who issues to retail investors. The problem with this hypothesis is that there is a very good alternative explanation, namely that there are economies of scale in servicing accounts (Deli, 2002). The larger and fewer accounts of institutions are cheaper to service per dollar invested in the fund than are the smaller accounts of individual investors. In any event, neither hypothesis is consistent with the claim that management fees are so high as to expropriate shareholders.

6.2 Expropriation

Mutual fund fees are higher in countries in which the rule of law is relatively weak (Khorana, Servaes, and Tufano, 2009). One explanation is that unaccountable managers in these countries are able to expropriate investors by arbitrarily setting higher fees. But assuming managers compete on reported fees, reported fees in these countries should be lower to reflect managers' ability to surreptitiously expropriate investor wealth. Our analysis predicts that fees in low rule of law countries are higher to allow managers to bond themselves against expropriation.

First note that, with competition, if fees were intended simply to compensate managers for the costs incurred in managing their funds, fees would be *lower* in countries in which property rights are weaker because the gains from expropriation would constitute part of a fund manager's compensation. They would substitute for fees. Formally, under the zero profit condition for fund management, and in the absence of expropriation, we have

$$f\left[(1 + R(A))\,A + (1 + r)\,(T - A)\right] = CA.$$

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With expropriation possible at rate η per dollar of asset under management, the preceding equation becomes

$$(f + \eta)\left[(1 + R(A))A + (1 + r)(T - A)\right] = CA.$$

Clearly, $\partial f / \partial \eta < 0$: fees decrease in the rate of expropriation. This is not unlike the observation that civil servants' salaries are lower in countries in which there is more corruption, *ceteris paribus*.

Now consider the case where fees are intended to assure manager quality, which in this case consists not only of refraining from shirking but also of refraining from expropriating investors. Fees must be such that

$$\frac{f\left[(1 + R(A))A + (1 + r)(T - A)\right] - CA}{r} \geqslant \frac{(f + \eta)(1 + r)T}{1 + r}.$$

We can rewrite the preceding condition as

$$f \geqslant \frac{C}{R(A) - r} - \left(1 - \frac{C}{R(A) - r}\right)\frac{1}{r} + \frac{\eta(1 + r)}{R(A) - r}\frac{T}{A} \equiv f^r,$$

with $\partial f^r / \partial \eta > 0$, exactly the finding of Khorana, et al. (2009). Intuitively, fees must be higher where they must deter the manager from expropriating investors. To understand the intuition for the inequality, rewrite it as

$$f(R(A) - r)A \geqslant CA - \frac{(R(A) - r - C)A}{r} + \eta(1 + r)T.$$

The beneficial effect of the value to the manager of remaining in the relationship is now counteracted by the negative effect of the gains from expropriation. These apply to total funds under management, T, as opposed to the value of the relationship and the costs of active management, which apply to actively managed assets, A. Both actively and passively managed funds are vulnerable to expropriation whereas the value of the relationship lies in making it possible for the manager to realize the value of his human capital through active fund management.

It may be worth saying a few words about the similarities between our analysis and that on efficiency wages in labor economics. As with quality-assuring fees in our model, efficiency wages deter shirking by workers (Akerlof and Yellen, 1986). Workers paid efficiency wages have more to lose from being fired for shirking. Two predictions (and findings) of efficiency wage theory are of particular interest to us: (i) wages are lower where there is closer supervision (Krueger, 1991), (ii) wages are lower where there are fewer opportunities for shirking. Prediction (i) is consistent with observations on fees paid by institutional investors, which can be presumed to monitor fund managers more closely than can individual investors; prediction (ii) is consistent with the Khorana, et al. (2009) findings regarding higher fees in countries with weaker property rights, in which there are more opportunities for expropriation.

6.3 Closed-End Funds

The central difference between open- and closed-end funds is that total funds under management, T, cannot be decreased through investors' redemptions in closed-end funds. We show

Result 9 *Fees must be higher in the case of closed-end funds, ceteris paribus. Formally*

$$f \geqslant \frac{C}{R\left(A\right) - r} > f^r \ \ in \ (7)$$

The minimum level of quality-assuring fees is higher for closed-end funds than it is for open-end funds (Deli, 2002), precisely because the threat denying the manager the value of his human capital in case he should shirk is virtually inoperative.

6.4 Further Evidence

Brown, Harlow, and Starks (1996) empirically assess the behavior of private money managers paid performance fees. They find that those whose interim performance is poor have a tendency to inefficiently increase portfolio risk as the date of reporting looms. Yet they also find that this tendency is smaller for managers with a strong past performance record. This is likely because these managers face the loss of back-end loaded fees in the asset-based component of their compensation.

Elton, Gruber, and Blake (2003) analyze the effect of so-called 'incentive' fees, in which the manager earns a higher one-off fee based on current-period fund returns rather than on total assets. Their analysis suggests that incentive fees provide fund managers with incentives superior to asset-based fees, even though the use of incentive fees in the mutual fund setting is rare. Our analysis suggests that asset-based fees have a strong positive effect on manager incentives because, being recurrent rather than one-off, they are back-end loaded and conditional on the manager's satisfactory performance. In fact, as the authors point out, managers paid an incentive fee are typically paid an asset-based fee as well. The downside to incentive fees is that they are one-off, possibly giving the manager a perverse incentive to 'bet the farm' in the event a bad performance report looms. This explains why the performance component of the manager's fee is often paid out on a deferred basis, presumably conditional on some contractible metric of satisfactory performance, thereby mimicking the incentives created by asset-based fees. It also explains why incentive fees are much more common for private money managers, who contract their services to institutional investors. These investors have the wherewithal to closely monitor the manager to identify and punish misbehavior, and in fact they routinely pay substantial fees to consultants to help them do

so.[14]

7 Summary and Concluding Remarks

Some excessive fee critics have relied on behavioral theory to explain why investors are persistently duped by excessive fund fees. Following the seminal work of Berk and Green (2004), in which mutual fund fees are irrelevant to investor returns, we have shown using a simple moral hazard model how higher fees benefit fund investors by assuring the quality of active managers' costly research effort. Our analysis fully explains many of the criticisms leveled at fund managers for charging excessive fees, most importantly the seemingly damning criticism that institutional investors pay far lower fees for what appear to be the same management services. Although this observation is also consistent with scale economies in the administration of accounts, neither supports the inference that fund managers are able to take persistent advantage of investors by charging excessive, out-of-equilibrium fees.

One criticism of our model is that if all investors index they can earn r, so why invest in active funds in which they can earn, at best, r? Any competitive model finds that the marginal consumers earns excess returns. Only those with special talents — fund managers in our model — will earn Ricardian rents. Since managers are members of society, society is better off. Our view is that active fund management in its entirely is a sufficiently large share of the investment universe that it draws funds from alternative investments and very likely moves investors, as suppliers of capital, along an upward sloping supply curve. Thus, expected returns across the investment universe should be higher with mutual funds than without them.

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[14]We abstract from the distinction between the adviser and the manager. Most mutual fund portfolio managers are either employees of a sub-adviser or an advisory firm, which typically administers a family of funds. The adviser is paid an asset-based fee, while the employee-manager is often paid in part on a performance fee basis. This is consistent with our monitoring hypothesis.

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Proofs

Proof of Result 1: Problem (2) has first-order conditions

$$(1 + R(A)) A + (1 + r) (T - A) - \lambda [(1 + R(A)) A + (1 + r) (T - A)] = 0$$

$$[f + \lambda (1 - f)] [1 + R(A) + AR'(A) - (1 + r)] - C = 0$$

$$f (1 + r) + \lambda [(1 - f) (1 + r) - (1 + r)] = 0$$

and

$$(1 - f) [(1 + R(A)) A + (1 + r) (T - A)] = (1 + r) T.$$

where λ denotes the Lagrange Multiplier associated with (3).

From the first equation, we have $\lambda = 1$. Substituting into the second equation, it is clear that fees have no effect on the amount that is actively managed, which equals the optimal amount, A^{opt}, by comparison with Equation (1). Substituting into the third equation, that equation becomes $0 = 0$. The fourth equation implies that there are no optima for f and T: an increase in one is offset by a decrease in the other. These offsetting changes leave the manager's profit unchanged regardless of fees.■

Proof of Implication 1: Assume the manager chooses to limit inflows to $T^{li} < T$, with T being the level of assets induced by fees absent the limit on inflows. Investor returns are

$$\frac{(1 - f) [(1 + R(A)) A + (1 + r) (T^{li} - A)]}{T^{li}} = \frac{f (1 + r) T}{T^{li}} + (1 - f) (1 + r) > 1 + r$$

$$\Rightarrow (1 - f) [(1 + R(A)) A + (1 + r) (T^{li} - A)] > (1 + r) T^{li}.$$

The manager's returns therefore are

$$f [(1 + R(A)) A + (1 + r) (T^{li} - A)] - CA$$

$$= [(1 + R(A)) A + (1 + r) (T^{li} - A)] - CA - (1 - f) [(1 + R(A)) A + (1 + r) (T^{li} - A)]$$

$$< [(1 + R(A)) A + (1 + r) (T^{li} - A)] - CA - (1 + r) T^{li}$$

$$= (R(A) - r) A - CA$$

$$\leqslant (R(A^{opt}) - r) A^{opt} - CA^{opt},$$

where the last inequality is true by the definition of A^{opt}.■

Proof of Implication 2: Break-even fees, f^{be}, and assets under management, A^{be}, are such that

$$f^{be} \left(1 + R(A^{be})\right) A^{be} = CA^{be} \tag{30}$$

and

$$\left(1 - f^{be}\right)\left(1 + R(A^{be})\right) A^{be} = (1+r) A^{be}. \tag{31}$$

Equations (30) and (31) together imply

$$R(A^{be}) - C = r. \tag{32}$$

Comparing (32) with (1) and recalling that $R'(.) < 0$ in turn imply $A^{be} > A^{opt}$ and, from the definition of A^{opt}

$$\left(R(A^{be}) - r - C\right) A^{be} < \left(R(A^{opt}) - r - C\right) A^{opt}. \tag{33}$$

Using (31) to write

$$r = \left(1 - f^{be}\right)\left(1 + R(A^{be})\right) - 1, \tag{34}$$

likewise writing[15]

$$r = \left(1 - f^{opt}\right)\left(1 + R(A^{opt})\right) - 1, \tag{35}$$

and substituting (34) and (35) into the LHS and the RHS of (33), respectively, we obtain

$$f^{be}\left(1 + R(A^{be})\right) A^{be} - CA^{be} < f^{opt}\left(1 + R(A^{opt})\right) A^{opt} - CA^{opt}.$$

Note that (34), (35), $R'(.) < 0$, and $A^{be} > A^{opt}$ together imply $f^{be} < f^{opt}$.∎

Proof of Result 2: Using the result that managers earn the value of their human capital, we can replace the numerator in the RHS of (6) by $\left(1 + R(A^{opt})\right) A^{opt} - (1+r) A^{opt} - CA^{opt}$ and then solve for f to obtain

$$f \geqslant \frac{1}{r}\left[C\left(\frac{1+r}{R(A^{opt}) - r}\right) - 1\right] \equiv f^r. \blacksquare \tag{36}$$

Proof of Result 3: Equation (9) implies that there is some $\Delta A(s) \geqslant 0$ such that, for $s \leqslant S$,

$$(1 - f)\left(1 + R(A^{opt} + \Delta A(s))\right) \Delta A(s) = (1+r) \Delta A(s) + s\Delta A(s). \tag{37}$$

'New' investors may profitably invest an amount $\Delta A(s) \geqslant 0$ over and above the amount A^{opt} if these new investors are not required to subsidize the manager to the same extent as the 'old' investors: $s \leqslant S$. The manager's payoff becomes in such case

$$f\left(1 + R(A^{opt} + \Delta A)\right)\left(A^{opt} + \Delta A(s)\right) - C\left(A^{opt} + \Delta A(s)\right) + SA^{opt} + s\Delta A(s)$$
$$= \left(1 + R(A^{opt} + \Delta A(s))\right)\left(A^{opt} + \Delta A(s)\right) - (1+r)\left(A^{opt} + \Delta A(s)\right)$$
$$- s\left(A^{opt} + \Delta A(s)\right) - C\left(A^{opt} + \Delta A(s)\right) + SA^{opt} + s\Delta A(s)$$
$$= \left(R(A^{opt} + \Delta A(s)) - r - C\right)\left(A^{opt} + \Delta A(s)\right) + (S - s) A^{opt};$$

---

[15] As does (34), (35) follows from the driving down of investor returns to the normal rate r.

this last term equals $\left(R(A^{opt}) - r - C\right) A^{opt}$ for $s = S$ and $\Delta A(S) = 0$.[16] The manager's problem is therefore

$$\max_{s \leqslant S} \left(R(A^{opt} + \Delta A(s)) - r - C\right) \left(A^{opt} + \Delta A(s)\right) + (S - s) A^{opt}$$

subject to

$$(1 - f)\left(1 + R(A^{opt} + \Delta A(s))\right) = (1 + r) + s.$$

The problem has FOC

$$-A^{opt} + \left(A^{opt} R'(A^{opt} + \Delta A(s)) + R(A^{opt} + \Delta A(s)) - r - C\right) \frac{\partial \Delta A(s)}{\partial s} - \lambda = 0,$$

where $\lambda \geqslant 0$ is the Lagrange multiplier associated with the inequality constraint $s \leqslant S$ and

$$\frac{\partial \Delta A(s)}{\partial s} = \frac{1}{(1 - f) R'(A^{opt} + \Delta A(s))} < 0.$$

At $s = S$ and $\Delta A(S) = 0$, recalling that $A^{opt} R'(A^{opt}) + R(A^{opt}) - r - C = 0$, the FOC becomes

$$\lambda = -A^{opt} < 0,$$

which is a contradiction. The manager will choose $s < S$ and $\Delta A(s) > 0$, thereby increasing his wealth above the value of his human capital. The limitation of invested assets to A^{opt} therefore is not self-enforcing. The manager's profit comes at a loss to existing shareholders, who receive

$$(1 - f)\left(1 + R(A^{opt} + \Delta A(s))\right) A^{opt} = (1 + r) A^{opt} + s A^{opt} < (1 + r) A^{opt} + S A^{opt}. \blacksquare$$

Proof of Result 4: Fees f are now such that

$$f(1 + r) A - f(1 + R(A) - S) A + (C - S) A$$
$$= \frac{f(1 + R(A) - S) A - (C - S) A}{r}$$
$$\Leftrightarrow f(R(A) - r - S) = C - S - \frac{R(A) - r - C}{r}.$$

As $f < 1$, it is clear that $\partial f / \partial S < 0$, keeping A constant. The level of fees necessary for bonding decreases in subsidies. The solution is therefore characterized by $A = A^{opt}$ and S and f such that[17]

$$(1 - f)\left(1 + R(A^{opt}) - S\right) A^{opt} = (1 + r) A^{opt}$$

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[16]We have used (37) with $\Delta A(s)$ replaced by $A^{opt} + \Delta A(s)$ to obtain the first equality.

[17]Note that the LHS of the equation that follows is increasing in S. To see this, differentiate and use

$$\frac{\partial f}{\partial S} = -\frac{1 - f}{R(A^{opt}) - r - S}.$$

$$\Leftrightarrow S = 1 + R(A^{opt}) - \frac{1+r}{1-f}$$

and

$$f\left(R(A^{opt}) - r - S\right) = C - S - \frac{R(A^{opt}) - r - C}{r}.\blacksquare$$

Proof of Result 5: Totally differentiating the system of equations (17)-(19) with respect to C, we have

$$\mathbf{AX} = \mathbf{B},$$

where

$$\mathbf{A} = \begin{bmatrix} (1+rf)\,R'(A) & r\,(R(A)-r) & 0 \\ 2R'(A) + AR''(A) & 0 & 0 \\ (1-f)\,[AR'(A) + R(A) - r] & -[(R(A)-r)\,A + (1+r)\,T] & -f\,(1+r) \end{bmatrix},$$

$$\mathbf{X} = \begin{bmatrix} \frac{\partial A}{\partial C} \\ \frac{\partial f}{\partial C} \\ \frac{\partial T}{\partial C} \end{bmatrix},$$

and

$$\mathbf{B} = \begin{bmatrix} 1+r \\ 1 \\ 0 \end{bmatrix}.$$

Note, initially, that

$$|\mathbf{A}| = \left[2R'(A) + AR''(A)\right] r\,(R(A)-r)\,f\,(1+r) < 0,$$

by the second order condition for actively managed assets. We use Cramer's rule to obtain

$$\frac{\partial A}{\partial C} = \frac{r\,(R(A)-r)\,f\,(1+r)}{|\mathbf{A}|} < 0,$$

$$\frac{\partial f}{\partial C} = \frac{[-(1+rf)\,R'(A) + (1+r)\,(2R'(A) + AR''(A))]\,f\,(1+r)}{|\mathbf{A}|} \lessgtr 0,$$

and

$$\frac{\partial T}{\partial C} = \frac{\begin{array}{c} [(1+rf)\,R'(A) - (1+r)\,(2R'(A) + AR''(A))]\,[(R(A)-r)\,A + (1+r)\,T] \\ + (1-f)\,[AR'(A) + R(A) - r]\,r\,(R(A)-r) \end{array}}{|\mathbf{A}|} \lessgtr 0.$$

A necessary and sufficient condition for $df/dC > 0$ and sufficient condition for $\partial T/\partial C < 0$ is

$$-(1+rf)\,R'(A) + (1+r)\,(2R'(A) + AR''(A)) < 0.$$

33

Although it is possible to compute the derivative of active share, A/T, directly, we compute it by using (19) to write

$$\frac{A}{T} = \left[\frac{1-f}{f}\left(R(A) - r\right)\right]^{-1}(1+r).\qquad(38)$$

We have just shown A to decrease and therefore $R(A)$ to increase in C. A sufficient condition for active share to decrease in C is that fees, f, decrease in costs, C.∎

Proof of Result 6: Totally differentiating the system of equations with respect to θ, we have

$$\mathbf{C}\mathbf{Y} = \mathbf{D},$$

where

$$\mathbf{C} = \begin{bmatrix} (1+rf)\,R'(A,\theta) & r\left(R(A,\theta) - r\right) & 0 \\ 2R'(A,\theta) + AR''(A,\theta) & 0 & 0 \\ (1-f)\left[AR'(A,\theta) + R(A,\theta) - r\right] & -\left[\left(R(A,\theta) - r\right)A + (1+r)\,T\right] & -f\,(1+r) \end{bmatrix},$$

$$\mathbf{Y} = \begin{bmatrix} \frac{\partial A}{\partial \theta} \\ \frac{\partial f}{\partial \theta} \\ \frac{\partial T}{\partial \theta} \end{bmatrix},$$

and

$$\mathbf{D} = \begin{bmatrix} -(1+rf)\,\frac{\partial R(A,\theta)}{\partial \theta} \\ -\left(\frac{\partial R(A,\theta)}{\partial \theta} + A\frac{\partial R'(A,\theta)}{\partial \theta}\right) \\ -(1-f)\,\frac{\partial R(A,\theta)}{\partial \theta}\,A \end{bmatrix}.$$

Note that

$$|\mathbf{C}| = \left[2R'(A,\theta) + AR''(A,\theta)\right]r\left(R(A,\theta) - r\right)f\,(1+r) < 0,$$

by the second order condition for actively managed funds A. Use Cramer's rule to obtain

$$\frac{\partial A}{\partial \theta} = -\frac{r\left(R(A,\theta) - r\right)f\,(1+r)}{|\mathbf{C}|}\left(\frac{\partial R(A,\theta)}{\partial \theta} + A\frac{\partial R'(A,\theta)}{\partial \theta}\right) > 0,$$

$$\frac{\partial f}{\partial \theta} = \frac{f\,(1+r)\,(1+rf)}{|\mathbf{C}|} \times$$
$$\left[R'(A,\theta)\left(\frac{\partial R(A,\theta)}{\partial \theta} + A\frac{\partial R'(A,\theta)}{\partial \theta}\right) - \left(2R'(A,\theta) + AR''(A,\theta)\right)\frac{\partial R(A,\theta)}{\partial \theta}\right] \lessgtr 0,$$

$$\frac{\partial T}{\partial \theta} = \frac{1}{|\mathbf{C}|} \times$$

$$\begin{bmatrix} - (1 + rf)\, R'(A,\theta)\, [(R(A,\theta) - r)\, A + (1 + r)\, T] \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) \\ + (2R'(A,\theta) + AR''(A,\theta))\, r\, (R(A,\theta) - r)\, (1 - f)\, \frac{\partial R(A,\theta)}{\partial \theta}\, A \\ + (2R'(A,\theta) + AR''(A,\theta))\, [(R(A,\theta) - r)\, A + (1 + r)\, T]\, (1 + rf)\, \frac{\partial R(A,\theta)}{\partial \theta} \\ - (1 - f)\, [AR'(A,\theta) + R(A,\theta) - r]\, r\, (R(A,\theta) - r) \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) \end{bmatrix}.$$

$$= \frac{1}{|\mathbf{C}|} \times$$

$$\left\{ \begin{array}{c} - \left[\left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) R'(A,\theta) - (2R'(A,\theta) + AR''(A,\theta)) \frac{\partial R(A,\theta)}{\partial \theta} \right] \times \\ [(1 + rf)\, [(R(A,\theta) - r)\, A + (1 + r)\, T] + (1 - f)\, r\, (R(A,\theta) - r)\, A] \\ - (1 - f)\, r\, (R(A,\theta) - r)^2 \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) \end{array} \right\} \lessgtr 0,$$

and

$$\frac{\partial [A/T]}{\partial \theta} = \frac{1}{T^2} \left[\frac{\partial A}{\partial \theta} T - A \frac{\partial T}{\partial \theta} \right]$$

$$= \frac{1}{T^2} \frac{1}{|\mathbf{C}|} \times$$

$$\begin{bmatrix} -r\, (R(A,\theta) - r)\, f\, (1 + r) \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) T \\ + (1 + rf)\, R'(A,\theta)\, [(R(A,\theta) - r)\, A + (1 + r)\, T] \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) A \\ - (2R'(A,\theta) + AR''(A,\theta))\, r\, (R(A,\theta) - r)\, (1 - f)\, \frac{\partial R(A,\theta)}{\partial \theta}\, A^2 \\ - (2R'(A,\theta) + AR''(A,\theta))\, [(R(A,\theta) - r)\, A + (1 + r)\, T]\, (1 + rf)\, \frac{\partial R(A,\theta)}{\partial \theta}\, A \\ + (1 - f)\, [AR'(A,\theta) + R(A,\theta) - r]\, r\, (R(A,\theta) - r) \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) A \end{bmatrix}$$

$$= \frac{1}{T^2} \frac{1}{|\mathbf{C}|} \times$$

$$\begin{bmatrix} -r\, (R(A,\theta) - r) \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) \{f\, (1 + r)\, T - (1 - f)\, [AR'(A,\theta) + R(A,\theta) - r]\, A\} \\ + (1 + rf)\, R'(A,\theta)\, [(R(A,\theta) - r)\, A + (1 + r)\, T] \left(\frac{\partial R(A,\theta)}{\partial \theta} + A \frac{\partial R'(A,\theta)}{\partial \theta} \right) A \\ - (2R'(A,\theta) + AR''(A,\theta))\, r\, (R(A,\theta) - r)\, (1 - f)\, \frac{\partial R(A,\theta)}{\partial \theta}\, A^2 \\ - (2R'(A,\theta) + AR''(A,\theta))\, [(R(A,\theta) - r)\, A + (1 + r)\, T]\, (1 + rf)\, \frac{\partial R(A,\theta)}{\partial \theta}\, A \end{bmatrix}.$$

Now, note that

$$f\, (1 + r)\, T - (1 - f)\, \left[AR'(A,\theta) + R(A,\theta) - r \right] A = (1 - f)\, (R(A) - r)\, A - (1 - f)\, CA$$

$$= (1 - f)\, (R(A) - r - C)\, A > 0,$$

so that

$$\frac{\partial\left[A/T\right]}{\partial\theta} = \frac{1}{T^2}\frac{1}{|\mathbf{C}|} \times$$

$$\left[\begin{array}{c} -r\left(R(A,\theta)-r\right)\left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)(1-f)\left(R(A)-r-C\right)A \\ +\left(1+rf\right)R'(A,\theta)\left[(R(A,\theta)-r)A+(1+r)T\right]\left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)A \\ -\left(2R'(A,\theta)+AR''(A,\theta)\right)r\left(R(A,\theta)-r\right)(1-f)\frac{\partial R(A,\theta)}{\partial\theta}A^2 \\ -\left(2R'(A,\theta)+AR''(A,\theta)\right)\left[(R(A,\theta)-r)A+(1+r)T\right](1+rf)\frac{\partial R(A,\theta)}{\partial\theta}A \end{array}\right].$$

In turn note that

$$-r\left(R(A,\theta)-r\right)\left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)(1-f)\left(R(A)-r-C\right)A$$

$$-\left(2R'(A,\theta)+AR''(A,\theta)\right)r\left(R(A,\theta)-r\right)(1-f)\frac{\partial R(A,\theta)}{\partial\theta}A^2$$

$$=-r\left(R(A,\theta)-r\right)(1-f)A\left[\begin{array}{c} \left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)\left(R(A)-r-C\right) \\ +\left(2R'(A,\theta)+AR''(A,\theta)\right)\frac{\partial R(A,\theta)}{\partial\theta}A \end{array}\right]$$

$$=-r\left(R(A,\theta)-r\right)(1-f)A\left[\begin{array}{c} -\left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)AR'(A,\theta) \\ +\left(2R'(A,\theta)+AR''(A,\theta)\right)\frac{\partial R(A,\theta)}{\partial\theta}A \end{array}\right]$$

$$=r\left(R(A,\theta)-r\right)(1-f)A^2\left[\begin{array}{c} \left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)R'(A,\theta) \\ -\left(2R'(A,\theta)+AR''(A,\theta)\right)\frac{\partial R(A,\theta)}{\partial\theta} \end{array}\right].$$

We can therefore write

$$\frac{\partial\left[A/T\right]}{\partial\theta} = \frac{1}{T^2}\frac{1}{|\mathbf{C}|}\left[\begin{array}{c} \left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)R'(A,\theta) \\ -\left(2R'(A,\theta)+AR''(A,\theta)\right)\frac{\partial R(A,\theta)}{\partial\theta} \end{array}\right] \times$$

$$\left[r\left(R(A,\theta)-r\right)(1-f)A^2+(1+rf)\left[(R(A,\theta)-r)A+(1+r)T\right]A\right].$$

A necessary and sufficient condition for $\partial f/\partial\theta < 0$ and $\partial\left[A/T\right]/\partial\theta < 0$ is

$$R'(A,\theta)\left(\frac{\partial R(A,\theta)}{\partial\theta}+A\frac{\partial R'(A,\theta)}{\partial\theta}\right)-\left(2R'(A,\theta)+AR''(A,\theta)\right)\frac{\partial R(A,\theta)}{\partial\theta}>0.$$

The condition is also sufficient for $\partial T/\partial\theta > 0.\blacksquare$

Proof of Result 7: Totally differentiating the system of equations (27)-(29) with respect to γ, we have

$$\mathbf{EZ} = \mathbf{F},$$

where

$$\mathbf{E} = \left[\begin{array}{ccc} (\gamma+rf)\,R'(A) & r\left(R(A)-r\right) & 0 \\ 2R'(A)+AR''(A) & 0 & 0 \\ (1-f)\left[AR'(A)+R(A)-r\right] & -\left[(R(A)-r)A+(1+r)T\right] & -f\left(1+r\right) \end{array}\right],$$

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$$\mathbf{Z} = \begin{bmatrix} \frac{\partial A}{\partial \gamma} \\ \frac{\partial f}{\partial \gamma} \\ \frac{\partial T}{\partial \gamma} \end{bmatrix},$$

and

$$\mathbf{F} = \begin{bmatrix} -(R(A) - r - C) \\ 0 \\ 0 \end{bmatrix}.$$

Initially note that

$$|\mathbf{E}| = \left[2R'(A) + AR''(A)\right] r \left(R(A) - r\right) f \left(1 + r\right) < 0,$$

by the second order condition for actively managed funds A. Now use Cramer's rule to obtain

$$\frac{\partial A}{\partial \gamma} = \frac{0}{|\mathbf{E}|} = 0,$$

$$\frac{\partial f}{\partial \gamma} = -\frac{\left[2R'(A) + AR''(A)\right] \left(R(A) - r - C\right) f \left(1 + r\right)}{|\mathbf{E}|} < 0,$$

and

$$\frac{\partial T}{\partial \gamma} = \frac{\left[2R'(A) + AR''(A)\right] \left[\left(R(A) - r\right) A + \left(1 + r\right) T\right] \left(R(A) - r - C\right)}{|\mathbf{E}|} > 0.$$

That $\partial \left[A/T\right]/\partial \gamma < 0$ is immediate from the two results $\partial A/\partial \gamma = 0$ and $\partial T/\partial \gamma > 0$. ∎

Proof of Result 8: Note that f and A corresponding to $\gamma > \gamma_0$ are such that

$$(1 - f)(1 + R(A)) A = (1 + r) A + C(\gamma) A$$

and

$$\frac{f(1 + R(A)) A - CA}{r} = \frac{f(1 + r) A}{1 + r} \left(\frac{1}{1 - \frac{1 - \gamma}{1 + r}}\right).$$

The latter equation is obtained by writing inequality (25) as an equality and setting $T = A$; the former by adjusting equation (29) to allow for investor compensation for the cost $C(\gamma) A$ of increasing the probability of detection to $\gamma > \gamma_0$. These two equations can be rewritten as, respectively,

$$(1 - f)(1 + R(A)) - (1 + r) - C(\gamma) = 0 \tag{39}$$

and

$$(r + \gamma)\left[f(1 + R(A)) - C\right] - rf(1 + r) = 0. \tag{40}$$

Totally differentiating with respect to γ and solving for $\partial A/\partial \gamma$, we have

$$\frac{\partial A}{\partial \gamma} = \frac{\begin{vmatrix} -(1+R(A)) & C'(\gamma) \\ (r+\gamma)(1+R(A)) - r(1+r) & -[f(1+R(A)) - C] \end{vmatrix}}{\begin{vmatrix} -(1+R(A)) & (1-f)R'(A) \\ (r+\gamma)(1+R(A)) - r(1+r) & (r+\gamma)fR'(A) \end{vmatrix}}$$

$$= \frac{(1+R(A))[f(1+R(A)) - C] - [(r+\gamma)(1+R(A)) - r(1+r)]C'(\gamma)}{-R'(A)[(1+R(A))(r+\gamma)f + [(r+\gamma)(1+R(A)) - r(1+r)](1-f)]}.$$

Note that $\partial A/\partial \gamma > 0$ at $\gamma = \gamma_0$.[18]   Assuming for simplicity that investment in detection is contractible, the manager's problem is

$$\underset{\gamma \geqslant \gamma_0}{Max}\, (R(A) - r - C - C(\gamma))\, A,$$

where the manager does of course recognize the dependence of A (and f) on γ. This problem has FOC

$$\left(R(A) - r - C - C(\gamma) + A'R'(A)\right)\frac{\partial A}{\partial \gamma} - C'(\gamma) + \lambda = 0,$$

where λ is the Lagrange multiplier associated with the inequality constraint $\gamma \geqslant \gamma_0$. At $\gamma = \gamma_0$ and $A = A_0$, the FOC becomes

$$\lambda = -\left(R(A_0) - r - C + A_0 R'(A_0)\right)\left.\frac{\partial A}{\partial \gamma}\right|_{\gamma=\gamma_0} < 0,$$

which is a contradiction.[19] The constraint is therefore slack. That is, $\gamma > \gamma_0$.∎

Proof of Result 9: In closed-end funds as in open-end funds, there is the need to preclude shirking, where shirking is again defined as closet indexing. Fees, f, therefore must be such that

$$\frac{f[(R(A) - r)A + (1+r)T] - CA}{r} \geqslant \frac{f(1+r)T}{r} \tag{41}$$

$$\Leftrightarrow f \geqslant \frac{C}{R(A) - r}$$

$$> \frac{C}{R(A) - r} - \frac{1}{r}\left(1 - \frac{C}{R(A) - r}\right) = f^r,$$

where we have used the definition of f^r in equation (7).[20] Note that the RHS of inequality (41) recognizes that total funds under management, T, remain in the fund even if the manager should shirk.∎

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[18]Use equation (40) to conclude that both $f(1+R(A)) - C$ and $(r+\gamma)(1+R(A)) - r(1+r)$ are positive.

[19]Use $R(A_0) - r + A_0 R'(A_0) - C > 0$ for $A_0 < A^{opt}$.

[20]We again drop superscripts for simplicity.