

Pricing sovereign contingent convertible debt

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Abstract

We develop a pricing model for sovereign contingent convertible bonds (S-CoCo) with payment standstills triggered by a sovereign's credit default swap CDS spread. One innovation is the modeling of CDS spread regime switching which is prevalent during crises. Regime switching is modeled as a hidden Markov process and is integrated with a stochastic process of spread levels to obtain S-CoCo prices through simulation. The paper goes a step further and uses the pricing model in a Longstaff-Schwartz. American option pricing framework to compute state contingent S-CoCo prices at some risk horizon, thus facilitating risk management. Dual trigger pricing is also discussed using the idiosyncratic CDS spread for the sovereign debt together with a broad market index. Extensive numerical results are reported using S-CoCo designs for Greece, Italy and Germany with both the pricing and contingent pricing models.

Keywords: contingent bonds; sovereign debt; debt restructuring; state-contingent pricing; regime switching; credit default swaps.

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1 Introduction

The Eurozone crisis and the record-breaking Greek sovereign default in particular (technically, a restructuring), highlighted the need for international legal procedures to deal with sovereign defaults. In September 2015 the United Nations General Assembly adopted a resolution on “Basic Principles on Sovereign Debt Restructuring Processes”¹. With the debate on appropriate legal mechanisms ongoing, see, e.g., Li (2016), proposals have also emerged for financial innovation solutions to the problem. Sovereign contingent convertible bonds (S-CoCo) with automatic debt restructuring have been suggested in academic and policy papers as a potential solution to sovereign debt crises, Barkbu et al. (2012); Brooke et al. (2013); Consiglio and Zenios (2015). These papers advance several arguments on the merits of contingent debt for sovereigns which we do not repeat here. Our own contribution was to make these proposals concrete by suggesting a payment standstill mechanism triggered when the sovereign’s CDS spread exceed a threshold, and to develop a risk optimization model demonstrating how contingent debt improves a country’s debt risk profile using Greece as a case study.

In a similar spirit, Buchheit et al. (2013) and the German Ministry of Finance (Bastasin, 2015) suggest modifications to the European Stability Mechanism treaty to make debt restructuring a precondition for assistance. A significant difference of these proposals from those cited in the previous paragraph is that they advocate restructuring triggered by institutional decision making instead of a market-driven mechanism. An alternative proposal are GDP-linked bonds, see, e.g. Bank of England (2015); Borensztein and Mauro (2004); Kamstra and Shiller (2009). These instruments are quite distinct from S-CoCo, but they also highlight the quest for financial innovation solutions to sovereign debt crises.

For sovereign contingent debt to gain acceptance we need pricing models and this is the goal of the present paper. Our contributions are, first, to develop a pricing model for sovereign contingent debt, and, second, to develop state contingent pricing of these instrument for risk management. To achieve these objectives we model a mean-reverting stochastic process of CDS spreads. However, the risk factors underlying the variation in spreads are time-varying and the risk models could break down during a crisis when they are most needed. To address this salient issue we develop models under regime switching and this is another innovation of the paper.

Justification for using CDS spreads as a trigger can be found in existing literature. An appropriate trigger must be accurate, timely and defined so that it can be implemented in a predictable way, Calomiris and Herring (2013). CDS spreads qualify. More importantly, the trigger should be comprehensive in its valuation of the issuing entity and we review here the evidence that the CDS market is becoming the main forum for credit risk price discovery. For instance, although Duffie (1999) established the equivalence of CDS prices and credit spreads, Blanco et al. (2005) provide evidence that parity relation is an equilibrium condition and short-lived deviations from parity are explained by the “clear lead of CDS prices over credit spreads in the price discovery process”. Similarly, Castellano and Scaccia (2012) find that CDS spreads and volatility anticipate downgrading by the rating agencies. These authors analyzed corporate reference entities and although they acknowledge the existence of several sovereigns with liquid CDS and bond markets, they did not test sovereign CDS for price discovery. More recent works reach similar conclusions for sovereign CDS markets. Li and Huang (2011) study emerging markets and find that corporate CDS market is more informational efficient than the bond market, though this was not the case for sovereign CDS market up to 2003. Post-2000 growth in market size and trading volume, and regulatory efforts to improve the information transparency of CDS market and bring trading to centralized exchanges, has altered the situation. The authors find that “sovereign CDS market is playing more significant role in price information discovery than bonds market”. Similarly, Eyssell et al. (2013) find China sovereign CDS is a leading indicator providing a basis for cross-market trading and hedging and for economic

¹Resolution A/69/L.84 at <http://unctad.org/en/pages/newsdetails.aspx?OriginalVersionID=1074>

policy analysis. Zhang (2003) makes the case for Argentinean CDS spreads as early indicators of credit events. There is extensive additional empirical work in this area and the conclusions are not uniform that CDS leads price discovery. But even authors who find CDS lagging also find that the lag is reduced with market integration or increased liquidity. Augustin (2014) surveys this literature with the interpretation “that there is increasing price discovery in the credit derivative market as the market has matured”.

Having established CDS spreads as appropriate early indicators for credit risk, the question is then raised on how to model their dynamics. Investigations on what drives CDS spreads find them driven by global changes in investor risk aversion, the reference country’s macroeconomic fundamentals and liquidity conditions in the CDS market, Badaoui et al. (2013); Fabozzi et al. (2016); Longstaff et al. (2011), but the relative importance of these factors changes over time, Heinz and Sun (2014). Amato and Remolona (2003) observe that yield spreads of corporate bonds tend to be many times wider than what would be implied by expected default losses alone—a “credit spread puzzle”—so that research has been focusing on modeling CDS spread returns directly than modeling their response to market fundamentals. This approach is advocated by Cont and Kan (2011) who provide modeling guidance by analyzing stylized facts of corporate CDS spreads and spread returns. Their work identified important properties of the dynamics of CDS spread returns—stationarity, positive auto-correlations, and two-sided heavy tailed distributions—and the authors propose a heavy-tailed multivariate time series model to reproduce the stylized properties. Brigo and Alfonsi (2005) develop a shifted square-root diffusion (SSRD) model for interest rate and credit derivatives, and O’Donoghue et al. (2014) develop a one-factor tractable stochastic model of spread-returns with mean-reversion (SRMR) as an extension of Orstein-Uhlenbeck process with jumps.

These models were developed for corporate CDS but in principle they could be used for sovereign CDS as well. However, there is a prevalent issue with *regime switching* in the sovereign market, especially during crises. This became apparent to us while trying to calibrate the SRMR model to Greek sovereign CDS spread data for our earlier paper. Calibration was unsuccessful for the period December 2007–February 2012, but converged to plausible results when applied to different regimes identified using the test of Bai and Perron (1998).

Regime switching in CDS spreads has been studied systematically by others as an empirical feature of the market, but, to the best of our knowledge, did not receive any attention in CDS pricing literature. Fontana and Scheider (2010) find that euro area credit markets witnessed significant repricing of credit risk in several phases since 2007. They find a structural break in market pricing, which coincides with the sharp increase in trading of CDS and declining risk appetite of investors since summer 2007, and attribute these changes to flight-to-liquidity, flight-to-safety and limits to arbitrage. Regime switching in the corporate CDS market was identified by Cont and Kan (2011) who find the behavior of spreads “clearly divided into two regimes: before and after the onset of the subprime crisis in 2007”. These observations are consistent with the analysis of Augustin (2014) who finds that CDS spreads change abruptly in response to major financial events, such as, for instance, the Bear Stearns bailout and Lehman Brothers bankruptcy, and are very persistent otherwise, over a sample of 38 countries in the period 9 May 2003–19 August 2010. Alexander and Kaeck (2008) examine the empirical influence of a broad set of determinants of CDS spreads listed in iTraxx Europe, and find that while most theoretical variables do contribute to the explanation of spread changes, their influence depends on market conditions. CDS spreads may behave differently during volatile periods compared to their behavior in tranquil periods. Using a Markov switching model they find evidence supporting the hypothesis that determinants of credit spreads are regime specific. Castellano and Scaccia (2014) find that, for corporate CDS, it is the volatility of returns that carries the signal, and they model regime switching using a hidden Markov matrix. Not only there is ample empirical evidence of regime switching, there are also theoretical arguments to support the observations. Arghyrou and Kontonikas (2016) use earlier models by Krugman and Obstfeld

to argue that Greece can be in one of three regimes: one with credible commitment to stay in the eurozone with guarantees of fiscal liabilities, one that guarantees fiscal liabilities for as long the country stays in the eurozone but uncertainty about the country’s commitment to do so, and one without fully credible commitment to the eurozone.

Regime switching is a salient feature for our work because of the payment standstill triggered in case of a crisis, and crises typically signal a regime switch. For instance, during the eurozone crisis, a sharp drop of CDS spreads was noted across the board in the second half of 2012 following the ECB OMT announcement, and this was primarily due to switch in investor sentiment as country specific fundamentals remained broadly unchanged, Heinz and Sun (2014). Hence, we develop our model with regime switching.

The rest of the paper develops the pricing model, and uses it to develop state-dependent prices at some risk horizon and simulate holding period returns. We start in Section 2 with statistical analysis of CDS spreads and spread returns for sovereigns in the eurozone periphery and core countries, and identify regime switching. This section informs our modeling work by giving a descriptive analysis of the eurozone sovereign CDS market. Section 3 develops the scenario generating stochastic processes for both regime switching and steady state for CDS spreads, spread returns and risk free rates. Section 4 develops the pricing model, state-contingent pricing, and holding period return scenarios. We illustrate numerically for a eurozone crisis country (Greece) and core countries (Germany and Italy). Section 5 concludes. The asymptotic modeling of CDS spreads —as opposed to spread returns addressed in existing literature— is given in Appendix A.

2 Some observations on sovereign CDS spreads

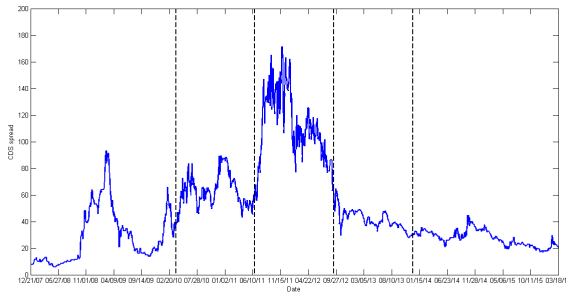
A pricing model should be guided by the stylized facts of the observed series. The simulation window for pricing S-CoCo is 20 to 30 years, and the risk horizon for state contingent S-CoCo pricing is 10 to 20 years, so we focus on long term characteristics of the data generating process. We model and calibrate the limiting dynamics of spreads (Appendix A), so we need those quantities attached to time-dependent equilibria of the process. These equilibria are the *regimes*.

We analyze the 5-yr CDS spread for a sample of European countries using daily data from February 2007 to March 2016. The test of Bai and Perron (1998) applied² to the spread level identifies the regime changes illustrated in Figure 1. France, Italy, Portugal, Spain and Cyprus are synchronized in their regime switching, whereas Germany, Ireland and Greece have idiosyncratic regime changes. Note, for instance, that only Germany has a regime switch associate with the subprime crisis and the collapse of Lehman Brothers in September 2008, while the onset of the eurozone crisis in spring 2010 signals regime switching for all countries. Figure 2 displays the 5-yr CDS spreads for Greece and highlights the events that impact spreads. April 2010 signals switching from a *tranquil* to a *turbulent* regime of the Greek economy, and the events clustered around the change of regime are summarized in Table 1. The change of regime in July 2011 is the run up to the Greek PSI signaling the start of the Greek debt crisis. The events highlighted involve an open letter to European and international authorities by German finance minister Wolfgang Schäuble about “fair burden sharing between taxpayers and private investors” in providing financial support to Greece, and Jean-Claude Juncker’s backing Germany’s proposal arguing for “soft debt restructuring” with private sector participation.

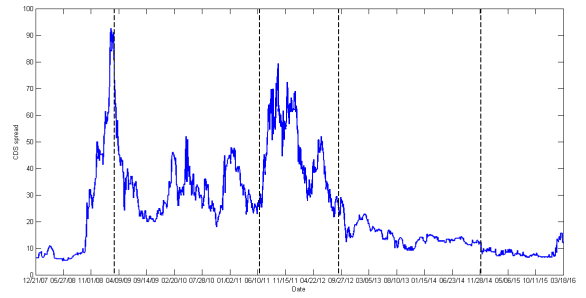
The mean and standard deviation of CDS spreads and spread returns for different regimes are in Table 2 for Germany, Greece and Italy. These quantities are needed to calibrate the simulation model. We focus on regime switching for the spreads, but regime breaks can also be identified for the volatilities as suggested by Castellano and Scaccia (2014)³.

²We use the Bai-Perron test in the free software system R.

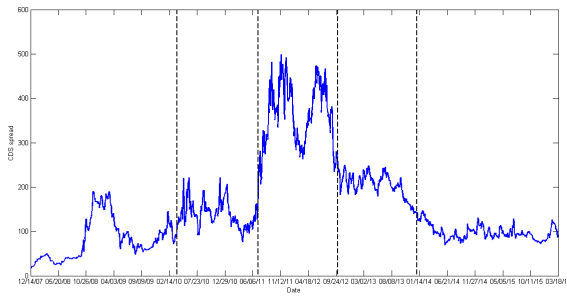
³A simulation model with joint dynamics of level and variance will be more complex with unclear marginal



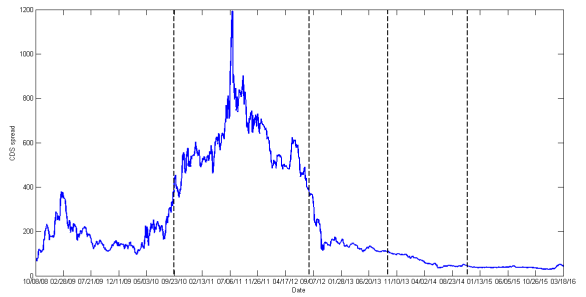
(a) France.



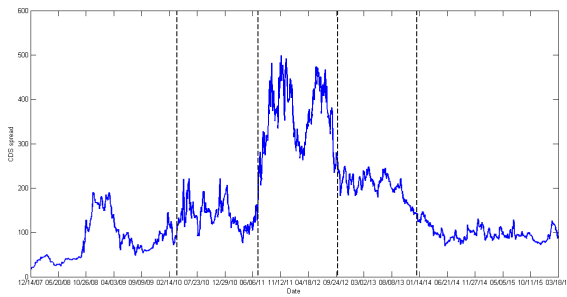
(b) Germany.



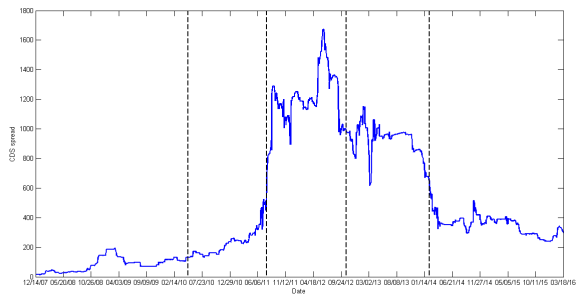
(c) Italy.



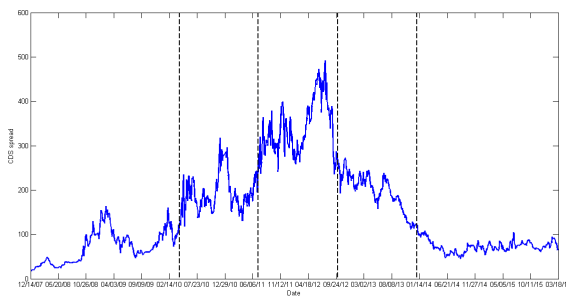
(d) Ireland.



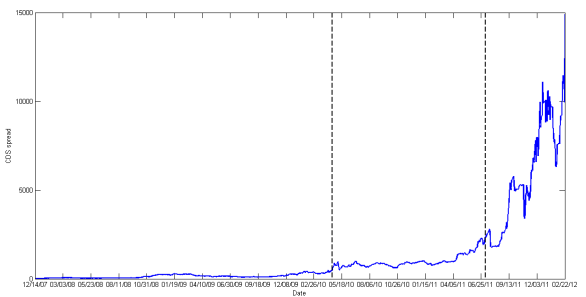
(e) Portugal.



(f) Cyprus.



(g) Spain.



(h) Greece.

Figure 1: Regime switching identified using Bai-Perron test.

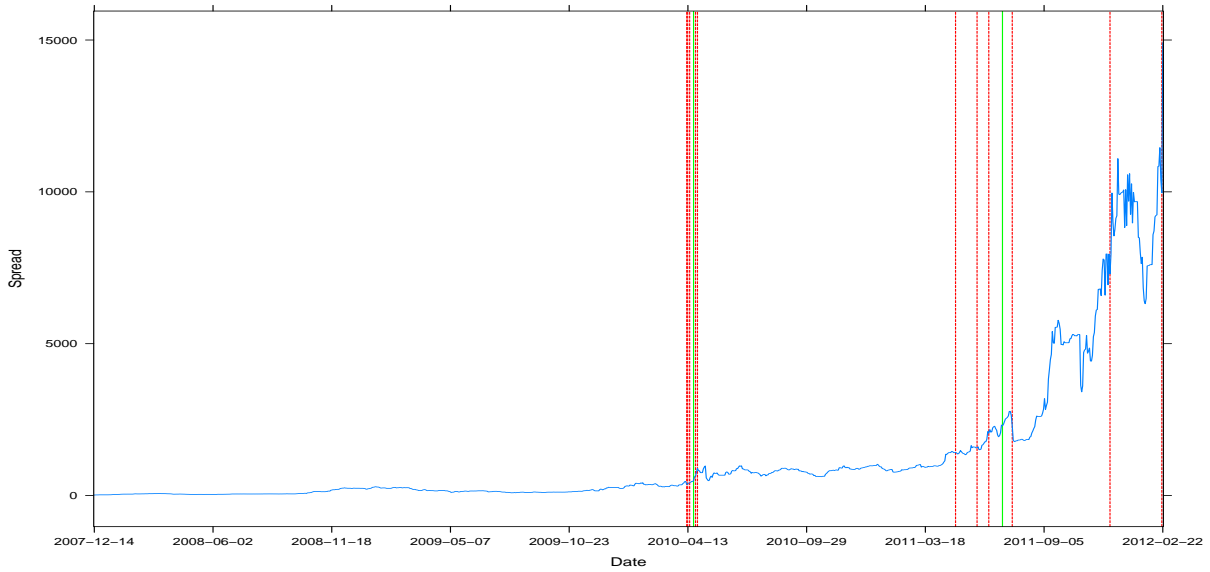


Figure 2: Time series of the 5-yr CDS for Greece. Vertical dotted lines identify events which affect CDS spreads and change the regime. Vertical solid lines denote regime switching identified using Bai-Perron test.

25/02/10	EU and IMF mission in Athens delivers grim assessment of country's finances
16/03/10	Eurozone finance ministers agree to help Greece but reveal no details
19/03/10	Prime Minister Papandreou warns Greece may have to go to the IMF
22/03/10	President Barroso urges member states to agree aid package for Greece
12/04/10	Greece announces that first trimester deficit was reduced by 39,2%
13/04/10	EU leaders agree bailout plan for Greece
14/04/10	ECB voices its support for the rescue plan of Greece

Table 1: Major events relating to the Greek sovereign crisis regime switch of July 2011.

Country	Regime	Spread mean	Spread std. dev.	Spread return std. dev.
Germany	12/21/07–03/13/09	22.22	23.54	5.48
	03/16/09–06/20/11	31.69	8.85	5.48
	06/21/11–09/12/12	45.60	13.98	6.24
	09/13/12–12/02/14	14.64	3.73	3.95
	12/03/14–03/18/16	8.25	1.93	6.31
Greece	12/14/07–04/20/10	146.09	103.90	4.45
	04/21/10–07/06/11	980.27	363.36	5.20
	07/07/11–02/22/12	5770.43	2917.45	8.05
Italy	12/14/07–03/29/10	79.71	45.48	5.01
	03/30/10–07/07/11	137.69	28.54	6.51
	07/08/11–10/02/12	361.94	68.99	4.94
	10/03/12–12/27/13	203.73	26.66	2.91
	12/30/13–03/18/16	97.31	15.82	3.67

Table 2: CDS spread and spread return statistics in each regime.

3 Scenario generating process

Our scenario generator consists of two hierarchical processes: a core process which determines regimes of the expected value of the CDS spread, and a process of the dynamics of the CDS spread superimposed on the mean value in each regime. In the next two subsections we model these sub-processes.

3.1 Regime switching process

We assume that regime transitions are driven by a discrete time-homogeneous Markov chain with finite state space $\mathcal{R} = \{1, 2, \dots, S\}$, where

$$p_{ij} = \mathbb{P}(X_k = i | X_{k-1} = j)$$

is the transition probability of switching from regime j at time $k-1$ to regime i at time k . The transition probabilities matrix $P = \{p_{ij}\}$ is a stochastic matrix, i.e., $p_{ij} \geq 0$, for all $i, j \in \mathcal{R}$, and $\sum_{j \in \mathcal{R}} p_{ij} = 1$, for all $i \in \mathcal{R}$.

Knowledge of the transition matrix P is fundamental to simulating a regime switching process. However, it cannot be estimated from observed historical series because regime breaks are rare events. We assume that the average permanence of the CDS spread in a given regime is an estimate of the limiting probability π^* (see definition below) and use this information to infer P .

We denote by $\pi_i^{(k)}$, for all $i \in \mathcal{R}$, the distribution at time k of a Markov chain X ,

$$\pi_i^{(k)} = \mathbb{P}(X_k = i).$$

Given a transition matrix P , it is possible to show that

$$\pi_j^{(k)} = \mathbb{P}(X_k = j) = \sum_{i \in \mathcal{R}} \mathbb{P}(X_k = j | X_{k-1} = i) \mathbb{P}(X_{k-1} = i) = \sum_{i \in \mathcal{R}} p_{ij} \pi_i^{(k-1)}. \quad (1)$$

If we denote by $\pi^{(k)}$ the row vector of probabilities $(\pi_1^{(k)}, \dots, \pi_S^{(k)})$, then (1) is written in matrix form as

$$\pi^{(k)} = \pi^{(k-1)} P.$$

Row vector of probabilities π^* is a *stationary distribution* for the Markov chain X_k , $k > 0$, if

$$\pi^* = \pi^* P, \text{ i.e., } \pi_j^* = \sum_{i \in \mathcal{R}} \pi_i^* p_{ij}.$$

Note that π^* does not necessarily exist, nor it is unique. If π^* exists and is unique then we can interpret π_i^* as the average proportion of time spent by the chain X in state i .

Given P , the stationary probability distribution π^* is obtained as the solution, if it exists, of the following system:

$$\pi^* = \pi^* P \quad (2)$$

$$\pi^* \mathbf{1} = 1 \quad (3)$$

$$\pi^* \geq 0. \quad (4)$$

Recall that matrix P is unknown and cannot be estimated from empirical data. Our basic assumption is that the stationary distribution can be estimated by the average number of days

improvement of the results.

the CDS spread process is in regime i , i.e.,

$$\hat{\pi}_i^* = \frac{\text{Number of days CDS spread is in regime } i}{\text{Number of total days in sample}}. \quad (5)$$

A constraint set on P is obtained by the properties of square matrices from known linear algebra theorems. In particular, let us assume that the Markov matrix $P = (p_{ij}) \in \mathbb{R}^{S \times S}$ has S distinct eigenvalues denoted by $\lambda = [\lambda_1 \lambda_2 \dots \lambda_S]^4$. Since P is a stochastic matrix, the eigenvalue with highest magnitude has absolute value equal to one, $|\lambda_1| = 1$, and according to the Perron-Frobenius theorem $1 = \lambda_1 > |\lambda_i|$, for all $i = 2, 3, \dots, S$. Denote by ξ_i the row vector which is the left eigenvector associated with the eigenvalue λ_i of P , and denote by ν_i the column vector which is the right eigenvector of the same λ_i , with ξ_i and ν_i obtained by solving

$$\xi_i P = \lambda_i \xi_i \quad (6)$$

$$P \nu_i = \lambda_i \nu_i. \quad (7)$$

Note that the left and right eigenvectors are orthonormal, so $\xi_i \cdot \nu_j = \delta_{ij}$, where δ_{ij} is the Kronecker delta.

Also observe that the right eigenvector for $\lambda_1 = 1$ is a unit vector as P is a stochastic matrix and all the rows sum up to 1, i.e.,

$$P \nu_1 = \nu_1.$$

Furthermore, if P is the transition matrix of a stationary process, then the left eigenvector for λ_1 is the steady distribution $\xi_1 = \pi^*$, and we have

$$\xi_1 P = \xi_1.$$

Denote by $U = (u_{ij})$ a matrix whose columns are the right eigenvectors of P , and by $V = (v_{ij})$ a matrix whose rows are the left eigenvectors of P . Then P can be written as

$$P = U D V,$$

where $D = (d_{ij})$ is a diagonal matrix whose entries are the eigenvalues of the transition matrix P , $D = \lambda I_S$. Recall that the eigenvectors are orthogonal so that $U V = I_S$. Moreover, the first column of V has all entries equal to 1, and if P admits a steady state the first row of U is the stationary distribution. If P is diagonalizable, it can be proved that the k -th power of P can be written as,

$$P^k = \sum_i \lambda_i^k \xi_i \nu_i.$$

Since $\lambda_1 = 1$, and $|\lambda_i| < 1$, for $i = 2, 3, \dots, S$,

$$\lim_{k \rightarrow \infty} P^k = \xi_1 \nu_1,$$

where it can be proved that the speed of convergence is given by the magnitude of λ_2 , and P converges faster to the steady state π^* for smaller values of $|\lambda_2|$.

Essentially, we model P to deliver the limiting distribution $\hat{\pi}_i^*$. This is an inverse problem and, in general, there are infinitely many Markov matrices P that give a steady state distribution $\hat{\pi}_i^*$. To single out a distribution we use the maximum entropy principle. This principle has been justified in matrix estimation (and many other applications) as resulting to the least biased or maximally uncommitted matrix with respect to missing information (Censor and Zenios, 1997,

⁴We are using *diagonalization* and the same conclusions are obtained when eigenvalues are not distinct, but the corresponding eigenvectors are linearly independent. Since we can arbitrarily choose to have a transition matrix with distinct eigenvalues, we present our analysis only for this case.

ch. 9). We determine the entries of the Markov matrix P such that the above properties are satisfied while maximizing Shannon's entropy, by solving the following quadratically constrained problem in the transition probabilities:

$$\text{Maximize}_{p_{ij}} \quad - \sum_{ij} p_{ij} \log p_{ij} \quad (8)$$

s.t.

$$\sum_k u_{ik} v_{kj} = \delta_{ij}, \quad \text{for all } i, j \in \mathcal{R}, \quad (9)$$

$$\sum_k u_{ik} d_{kk} v_{kj} = p_{ij}, \quad \text{for all } i, j \in \mathcal{R}, \quad (10)$$

$$\sum_j p_{ij} = 1, \quad \text{for all } i \in \mathcal{R}, \quad (11)$$

$$p_{ij} \geq 0, \quad \text{for all } i, j \in \mathcal{R}, \quad (12)$$

where $u_{i1} = 1$, for all $i \in \mathcal{R}$, is the constraint defining the right eigenvector associated with λ_1 . Constraints $v_{1j} = \hat{\pi}_j^*$, for all $j \in \mathcal{R}$, ensure that the left eigenvector associated with λ_1 is equal to the empirically estimated steady-state distribution. Eqn. (8) is obtained from the additivity property of Shannon's entropy, i.e., the conditional entropy $H(X_k|X_{k-1})$ is calculated as

$$H(X_k|X_{k-1}) = \sum_i H(X_k|X_{k-1} = i) = \sum_i \left[- \sum_j p_{ij} \log p_{ij} \right] = - \sum_{ij} p_{ij} \log p_{ij}. \quad (13)$$

The eigenvalues of P are set to some arbitrary values, recalling that $d_{11} = \lambda_1 = 1$ and $d_{11} > d_{22} > \dots > d_{SS}$. The possibility to arbitrarily set the eigenvalues of P allows control on the expected number of time steps that the process spends consecutively in the same state. On the one hand, the trace of a matrix is invariant under rotation, which implies that

$$S \geq \sum_{i=1}^S \lambda_i = \sum_{i=1}^S p_{ii}. \quad (14)$$

On the other hand, the expected number of time steps, $E(D_i)$, that the process consecutively spends on state i is

$$E(D_i) = \sum_{k=1}^{\infty} k p_{ii}^{k-1} (1 - p_{ii}) = \frac{1}{1 - p_{ii}}. \quad (15)$$

Eqn. (14) indicates that the average of eigenvalues is equal to the average value of p_{ii} over the allowed states. Therefore, if one sets eigenvalues $\lambda_2, \lambda_3, \dots, \lambda_S$ close to 1, then also the average value of p_{ii} turns out to be close to 1, and, according to eqn. (15), the expected number of time steps that the process consecutively spends on a given state is large on average. On the contrary, if eigenvalues $\lambda_2, \lambda_3, \dots, \lambda_S$ are small, then probabilities p_{ii} are also small.

Figure 3 displays four regime scenarios for Greece. We draw this figure by simulating a Markov chain with daily frequency over a 30-yr horizon. We assign to each regime the mean spread value from Table 2, set monotonically decreasing eigenvalues close to 1 to obtain reasonable persistence in each regime, and obtain the empirical steady-state distribution using (5). Solving (8)–(12) we obtain the following transition matrix used in the simulation of Greece

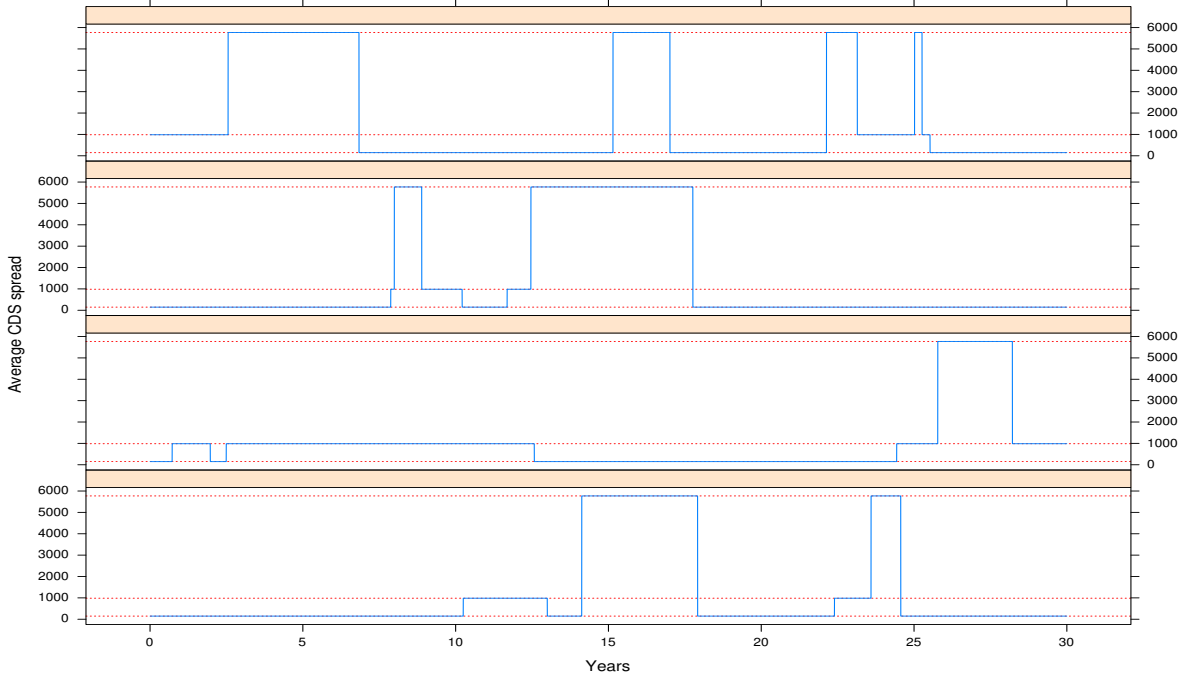


Figure 3: Simulation of regimes on a daily basis over a 30-yr horizon for Greece. Solid lines illustrate the Markov process switching regimes, and a regime is defined by the average spread (dotted lines) estimated from historical data for each regime.

regimes:

$$P = \begin{pmatrix} 0.9982 & 9.62\text{E-}4 & 7.89\text{E-}4 \\ 8.03\text{E-}4 & 0.9985 & 6.56\text{E-}4 \\ 2.62\text{E-}4 & 2.76\text{E-}4 & 0.9995 \end{pmatrix}.$$

We also calibrate the model for a country with less volatile spreads (Italy) and for a stable environment (Germany) and observe similar results, Figure 4. Note, though, that for Germany the mean levels of the regimes are very close to each other. Modeling regime switching is not always necessary.

3.2 CDS and interest rate process

We now superimpose the CDS spread process on the spread mean regimes generated by the Markov process. Broadly speaking, we generate scenarios of CDS spreads around the regime dynamics. We need a mean-reverting process that reverts to the mean CDS spread of the (simulated) regime. Furthermore, the variance should be bounded and the spread should be non-negative. The SRMR model of O'Donoghue et al. (2014) for CDS spread returns belongs to the class of Ornstein-Uhlenbeck processes and has the nice property that the variance of the log-returns is bounded with time, thus providing a process that does not deviate too much from its expected value for long intervals and remains non-negative. In Appendix A we derive the conditions on the parameters of this model so that asymptotically it converges to the regime mean values. Thus, we calibrate a stochastic process that has the desirable empirically observed properties of CDS spreads and spread returns, and conforms to the regime switches. With this approach the process dynamics capture not only the long-term mean spread but also spread and spread return volatility in each regime. Furthermore, as explained in the Appendix, this process allows to calibrate short term fluctuations and hence the smoothness of the curve.

Figure 5 illustrates a sample scenario of Greek CDS spread, around the regime scenario

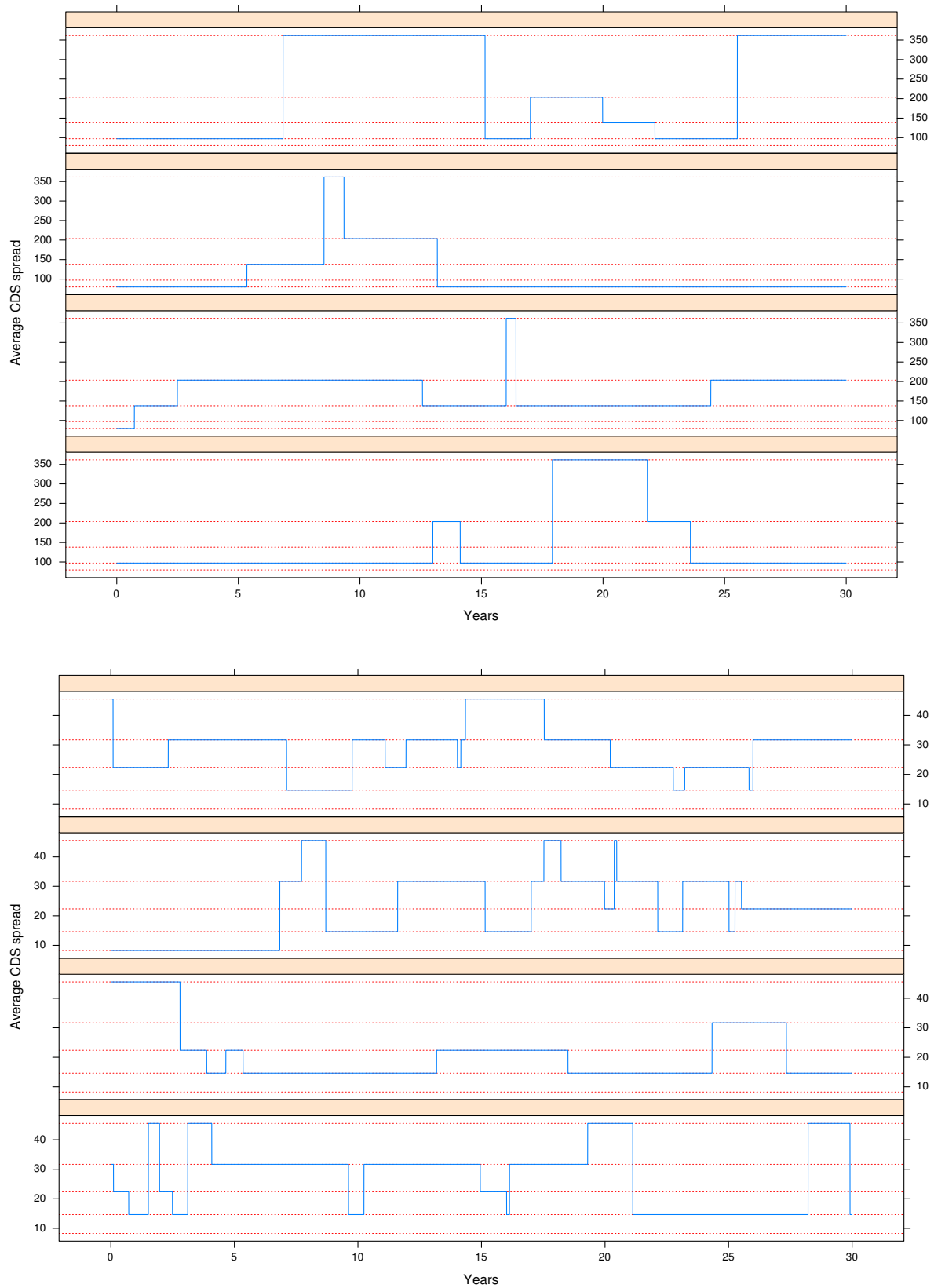


Figure 4: Simulation of regimes on a daily basis over a 30-yr horizon for Italy (top) and Germany (bottom). Solid lines illustrate the Markov process switching regimes, and a regime is defined by the average spread (dotted lines) estimated from historical data for each regime.

from Figure 3 (top panel). The simulation is run on a daily basis over a 30-yr horizon. The process follows the average CDS spread level estimated for each regime identified through the Bai-Perron test. A first impression is of a process with unrealistic jumps of the spread coinciding with regime switching. Moreover, the dynamics of the spread for the tranquil regime appear to be flat and with negligible volatility. This is due to y-axis scaling to capture the wide range of spreads for Greece over a long horizon. Zooming in at the simulated series we observe a smooth transition between regimes, with higher volatility even in the tranquil regime. Figure 6 displays the spread dynamics between years 25 and 26 where there are three consecutive regimes transitions. The transition from crisis to turbulent regime is abrupt, but the spread changes with a reasonable gradient, as seen in the inset of the same figure.

One desirable property of the model is the bounded variance of the stochastic process. Figure 5 (bottom) illustrates the 5% and 95% quantiles of the CDS spreads obtained over 1000 simulations. We observe that volatility does not increase with time and is dependent only on the given regime, so that turbulent regimes have higher volatilities than tranquil regimes and crisis regimes even higher. The largest Greek CDS spread during the crisis was almost 15,000, and was generated by our simulation at the 95% quantile.

As explained in the next session, the S-CoCo cashflows are discounted using the EURO AAA-rated bond yields (E-AAA for short). We simulate the E-AAA short rate dynamics following the approach just described. To this purpose, we extract from the historical series of the E-AAA yield curve the series of the 1-month rate, and we determine the regime sub-intervals and relative statistics to calibrate the model. We remark here that such an approach is not term structure consistent, therefore, we are not able to match observed bond prices on a given date. A possible workaround to this drawback would be to introduce an exogenous, deterministic, time-dependent process matching the actual forward curve, and calibrating the parameters of the model with given volatilities (implicit or historical ones). Unlike our implementation, where the process fluctuates around the simulated regimes, we could make the short rate to mean-revert towards a deterministic, exogenously given forward curve.

4 Modeling sovereign contingent convertible debts

We develop now the pricing models using Monte Carlo simulations. Prices are obtained as the expected discounted cashflows from simulations of the Markov chain and the stochastic process of spreads and interest rates in each regime. We also obtain state contingent pricing at some risk horizon to facilitate risk management.

4.1 Pricing

We denote by $\xi = \{r_t, s_t\}$ the coupled stochastic process of the short rate r_t and CDS spread s_t , where we assume that $\text{cov}[r_t, s_t] = 0$. To simplify notation, we also use t to indicate discrete time steps whose set of indices is $\mathcal{T} = \{0, 1, 2, \dots, T\}$. We draw from the probability distribution of ξ a discrete number of sample paths (*scenarios*), $\xi^l = \{r_t^l, s_t^l\}$, where $l \in \Omega = \{1, 2, \dots, N\}$ and $t \in \mathcal{T}$. The time-discretized approximation of the stochastic process ξ , for each scenario $l \in \Omega$, is obtained from eqn. (30) by sampling the diffusion term w_t from a Gaussian distribution. All scenarios are equally likely with probability $1/N$.

Denote by \bar{s} the threshold of the CDS spread which activates the standstill. If at time t and under scenario l the CDS rate s_t^l hits \bar{s} , coupon payments are suspended for the next K periods. We define the set of time periods with standstill by $\mathcal{T}_m^l = \{t, t+1, \dots, t+K\}$, and $m = 1, 2, \dots, M$, where M is the number of times that the standstill mechanism is activated under scenario l , with the following properties:

1. For any m and n , $\mathcal{T}_m^l \cap \mathcal{T}_n^l = \emptyset$. (This precludes overlapping of standstill time sets.)

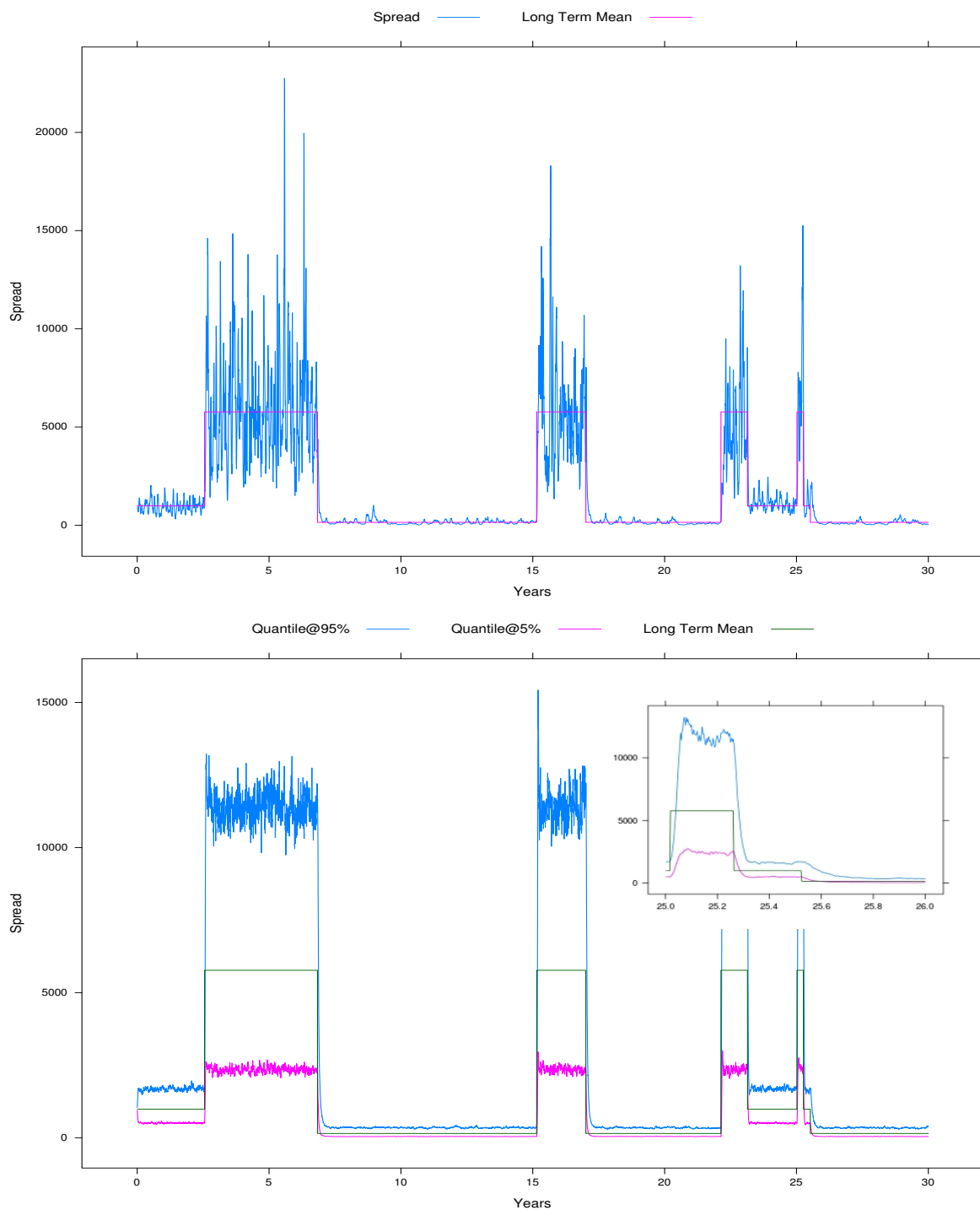


Figure 5: A typical simulation path of the daily CDS spread for Greece over a 30-yr horizon (top) and the 5% and 95% quantiles over 1000 scenarios (bottom) with a detail of the spread dynamics between years 25 and 26. The average spread of each regime is as estimated in Table 2.

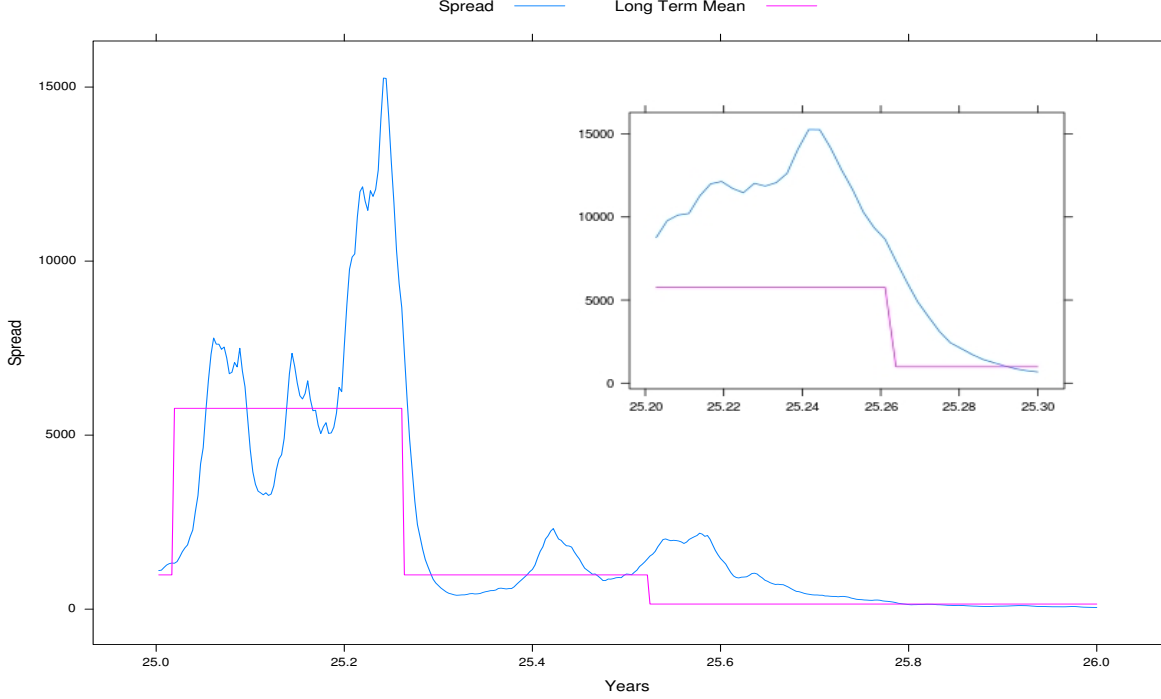


Figure 6: Simulated daily CDS spread for Greece during regime switching from year 25 to 26.

2. For any $t + 1 \in \mathcal{T}_m^l$, where $t = \min \mathcal{T}_m^l$, if $s_{t+1}^l \geq \bar{s}$ the trigger signal is ignored. (This avoids multiple triggering during a standstill interval.)

The set of time periods $t \in \mathcal{T}$ in which the payment standstill is active for scenario l is defined as

$$\Lambda^l = \bigcup_{m=1}^M \mathcal{T}_m^l, \quad (16)$$

and we define an indicator function $\mathbb{1}_{\Lambda^l} : \mathcal{T} \rightarrow \{0, 1\}$ as

$$\mathbb{1}_{\Lambda^l}(t) = \begin{cases} 0, & \text{if } t \in \Lambda^l \\ 1, & \text{if } t \notin \Lambda^l. \end{cases} \quad (17)$$

The standstill provision includes a special treatment of credit events occurring within K periods before maturity. In such cases, coupon payment standstill is accompanied by a postponement of principal payment. In particular, denoting by \mathcal{T}_Z^l the terminal standstill set under scenario l , and defining by J^l the first time step of \mathcal{T}_Z^l , $J^l = \min \mathcal{T}_Z^l$, the principal payment is delayed by $\Delta T^l = T - J^l + 1$, provided that $T - J^l < K$.

The S-CoCo price is obtained as the expectation, over scenarios $l \in \Omega$, of the present value of coupon and principal payments, contingent on the standstill provision triggered by threshold breach events. That is,

$$P_0 = \frac{1}{N} \sum_{l \in \Omega} \sum_{t \in \mathcal{T}} B^l(0, t) \mathbb{1}_{\Lambda^l}(t) c + B^l(0, T + \Delta T^l), \quad (18)$$

where c is the coupon and $B^l(t, s)$ is the discount factor between time periods t and t'

$$B^l(t, t') = \exp \left\{ - \sum_{u=t}^{t'} r_u^l \right\}. \quad (19)$$

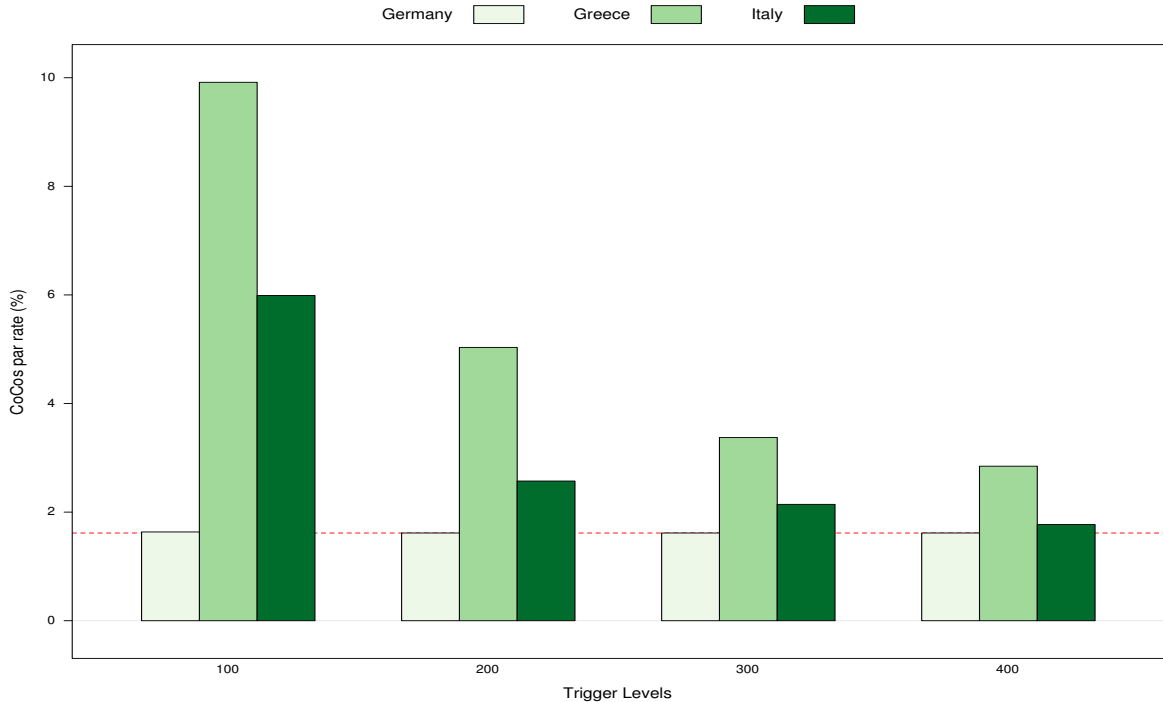


Figure 7: Par rate of the S-CoCo *vs* trigger thresholds \bar{s} . The red dashed line indicates the par yield of a AAA-rated bond (1.6%).

There can be several variants of the standstill provision, such as payment standstill with an associated maturity extension for as long the spread exceeds the threshold. Also, there are various alternative ways to treat coupon payments missed during the standstill, such as resumption of nominal value payments until the (extended) maturity —this was our original S-CoCo suggestion— or resumption of payments on an accrual basis or total write-down of missed payments. The pricing formula still applies but modifications are needed of the definition of the triggering set Λ^l or a different accounting of cashflows in (18). Modifications are conceptually straightforward but complicate the notation and we do not give them here.

Using numerical line search we solve pricing formula (18) for c such that

$$P_0(c) = 1. \quad (20)$$

The difference between c and the par rate of a AAA sovereign bond is the premium charged by investors to buy the S-CoCo.

Figure 7 displays par rates of a 20-yr S-CoCo for the three countries of our study with threshold $\bar{s} = 100, 200, 300, 400$. The CDS processes are calibrated on daily historical series from January 2007 to the end of 2016, except for Greece whose CDS trading was suspended at the end of 2012. The parameters of the short rate dynamics are inferred from the daily historical series of the E-AAA 1-month bond yield. Each par rate is computed by solving eqn. (20) over a set of 100 regime scenarios and 1000 interest rates and spread scenarios for each regime, for a total of 100,000 paths of length 20 years and semi-annual time step. Also shown in the figure is the par rate of a plain AAA-rated bond (1.6%). Greece has the highest premium over the AAA-rated yield due to the very high average level of CDS spreads of the recent past. The premium increases as \bar{s} is reduced since the probability of breaching the threshold increases with a commensurate increase in the number of standstill time periods. German S-CoCo is priced at par with AAA-rated bonds as the likelihood of German CDS spreads breaching even a very low threshold is virtually nil. The Italian spread is, naturally, between Greece and Germany.

Note that, the convergence of the par rate to the AAA level is due to the unique short rate dynamics used for all countries, which is calibrated on the AAA-rated bond yields historical series. Differentiation of the minimum par rate, and therefore convergence to different minimum levels as \bar{s} increases would be observed if the short rate dynamics are calibrated separately for each country.

4.2 State contingent pricing and holding period returns

For risk management we need the price (equivalently, return) probability distribution of financial instruments at the risk horizon to compute risk measures or for portfolio optimization or credit value adjustments. Such distributions are conditioned on the relevant risk factors and are needed under the true, objective, probability measure. See Mulvey and Zenios (1994) for generation and use of these distributions for fixed income securities and Consiglio and Zenios (2016) for use in risk management for sovereign debt restructuring.

Given the stochastic dynamics of a risk factor, a closed form expression of the expected value of the pricing function is not always available, especially when there are more than one risk factors. Hence, we resort to the numerical *Least Square Monte Carlo* —LSM in short— of Longstaff and Schwartz (2001). This method was developed to price American options and can be suitably modified to compute the conditional expectation of the S-CoCo bond contingent on the short rate r_t and the trigger binary function $\mathbb{1}_\Lambda(t)$.

LSM is based on backward induction whereby the expected value of the (discounted) asset payoff at $t + 1$ is approximated by a function of the realizations of the random variable at t :

$$E[V_{t+1}(X_{t+1})|X_t = x_t] \approx f_t(x_t, \beta_t), \quad x_t \in \mathbb{R}^d. \quad (21)$$

In the S-CoCo context, $X_t = (r_t, \mathbb{1}_\Lambda)$ is the 2-dimensional vector which takes values $x_t = (r_t^l, \mathbb{1}_{\Lambda^l})$ obtained by the Monte Carlo pricing simulation.

The payoff function $V_{t+1}(X_{t+1})$ has to account for the cashflow occurring at $t + 1$. This is made up by the possible coupon payment, plus the expected value of the S-CoCo at $t + 1$. For a given realization of the random variable X_{t+1} , we have

$$V_{t+1}(r_{t+1}^l, \mathbb{1}_{\Lambda^l}(t+1)) = \left[\mathbb{P}_{t+1}(r_{t+1}^l, \mathbb{1}_{\Lambda^l}(t+1)) + c\mathbb{1}_{\Lambda^l}(t+1) \right] B^l(t, t+1), \quad (22)$$

where, $\mathbb{P}_{t+1}(r_{t+1}^l, \mathbb{1}_{\Lambda^l}(t+1))$ is the regression function approximating the expected S-CoCo price in the next period and $B^l(t, t+1)$ is the discount factor.

Starting from $V_T(x)$ (see discussion below about the terminal payoff function), we estimate backwards the parameters $\beta_t \in \mathbb{R}^M$ and the error term $\epsilon_t \in \mathbb{R}$ that best fit the expected value

$$\mathbb{P}_t(x_t, \beta_t) + \epsilon_t = E[V_{t+1}(X_{t+1})|X_t = x_t], \quad (23)$$

where $\mathbb{P}_j(\cdot)$ is obtained as a linear combination of basis functions

$$\mathbb{P}_t(x_t, \beta_t) = \sum_{k=1}^M \beta_{tk} \phi_k(x_t). \quad (24)$$

The choice and the number of basis functions ϕ_k depend on the characteristics of the problem under review. Most authors suggest a trial-and-error approach, starting from simple basis functions and then increase their complexity (for example, using power function with dampening factors, Hermite or Laguerre polynomials), together with statistical selection procedures to find the optimal number of functions. Following (Glasserman, 2003, p. 462) we set $\phi_k(r_t) = r_t^k$ and $\phi_k(\mathbb{1}_\Lambda(j)) = \mathbb{1}_\Lambda(j)$. For the short rate we tried different sets of basis functions $\{r_t^k\}_0^{M-1}$, where $M = 3, 4, 5$. For the binary variable, we only considered $k = 1$ since any power of $\mathbb{1}_\Lambda(t)$ will

deliver the same value.

Given the sample values for r_t^l , $\mathbb{1}_{\Lambda^l}(t)$ and V_{t+1}^l , starting from $j = T$ and proceeding backwards until $t = 1$, we estimate $\{\beta_{tk}\}_{k=0}^M$ through standard OLS. The price of the S-CoCo at $t = 0$ is given by

$$P_0^{\text{LSM}} = \frac{1}{N} \sum_{l \in \Omega} \left\{ \left[\mathbb{P}_1(r_1^l, \mathbb{1}_{\Lambda^l}(1)) + c \mathbb{1}_{\Lambda^l}(1) \right] B(0, 1) \right\}. \quad (25)$$

As discussed in Section 4.1, the standstill provision allows for principal payment to be postponed if the triggering event occurs within K periods before maturity. Therefore, at $j = T$ the value of the S-CoCo is contingent on the scenario l and is given by

$$V_T^l(x_T^l) = \begin{cases} \mathbb{B}(T, T + \Delta T^l), & \text{if } T \in \Lambda^l \\ 1 + c, & \text{if } T \notin \Lambda^l, \end{cases} \quad (26)$$

where $\mathbb{B}(T, T + \Delta T^l)$ is the expected value of a zero coupon bond maturing at $T + \Delta T^l$. To compute $\mathbb{B}(T, T + \Delta T^l)$, we apply again LSM with r_t^l the only conditioning variable, for $t = T, T + 1, \dots, T + \Delta T^l$, and terminal value $V_{T+\Delta T^l} = 1$.

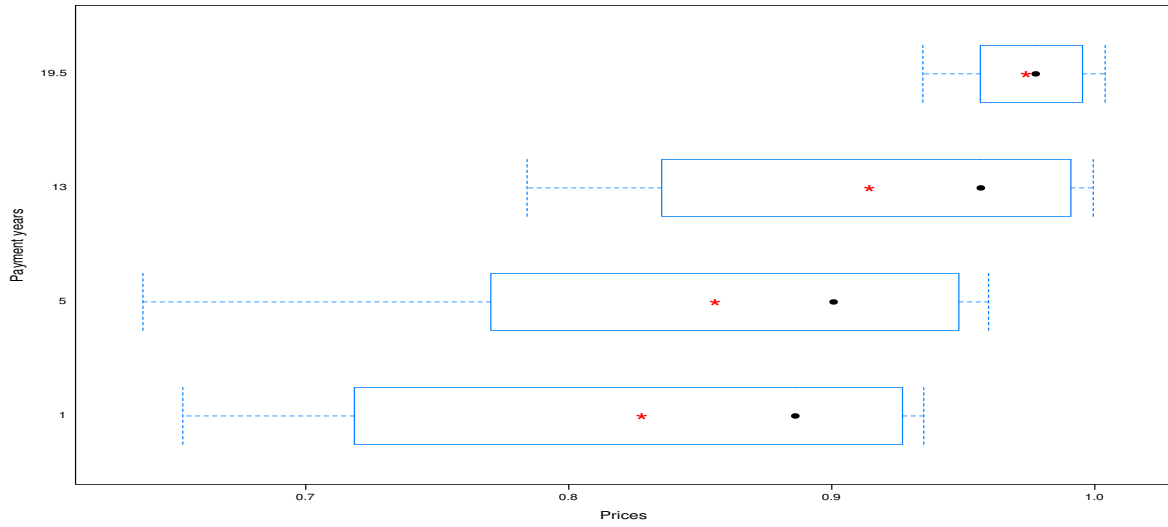
Table 3 compares the results obtained using different sets of basis functions $\{r_t^k\}_0^{M-1}$ and the dummy variable $\mathbb{1}_{\Lambda}(j)$. The prices \bar{P}_0^{LSM} in Table 3 are average prices of an S-CoCo with 10 year maturity⁵, obtained by changing the seed of the random engine to generate 5000 sample paths of length 10 years of short rates and CDS spreads, keeping the regimes from Section 4.1. In the same table we show the mean absolute percentage error with respect to the Montecarlo price. (For a true comparison we need a price obtained through a completely different approach, which, at the moment, is not available for S-CoCo.) The experiment highlights that basis functions up to degree two deliver satisfactory approximations.

Basis functions	\bar{P}_0^{LSM}	MAPE
1, r , $\mathbb{1}_{\Lambda}$	0.95558736	0.09163%
1, r , r^2 , $\mathbb{1}_{\Lambda}$	0.96610208	0.08913%
1, r , r^2 , r^3 , $\mathbb{1}_{\Lambda}$	0.95644892	0.33191%
1, r , r^2 , r^3 , r^4 , $\mathbb{1}_{\Lambda}$	0.97058076	1.67800%

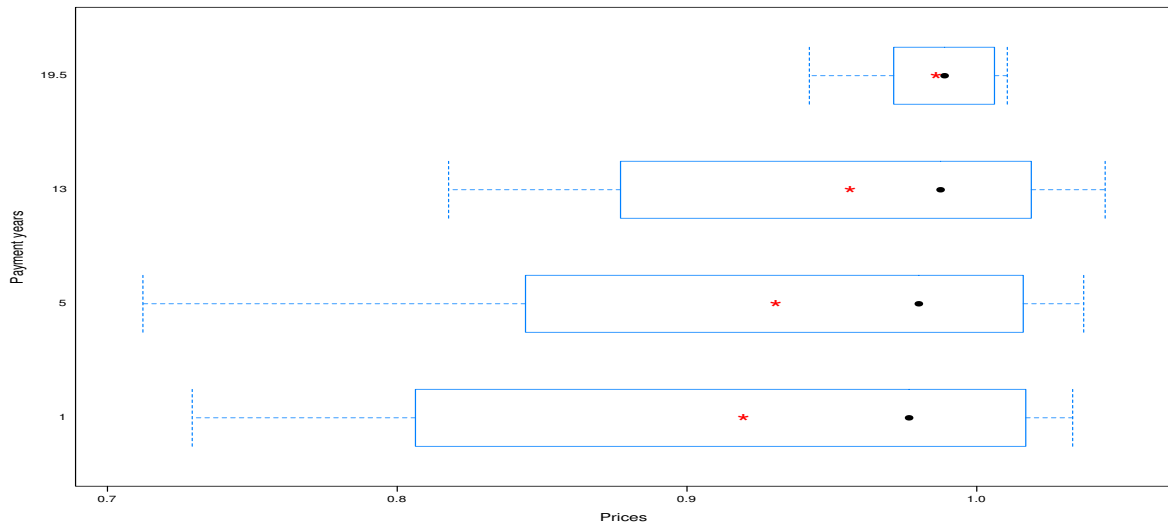
Table 3: Average LSM price at the root node and mean absolute percentage error (MAPE) with respect to Montecarlo pricing for different basis function sets.

However, the objective is not to provide an alternative pricing method, but to determine the future distribution of prices for risk management. We apply LSM to price a 20-yr S-CoCo for Greece, Italy and Germany, and obtain price distributions at 1, 5, 13 and 19.5 years. CDS spread and short rate dynamics are calibrated on the same set of data as in Section 4.1. Experiments are carried out for 100 regime scenarios, and 1000 CDS spread scenarios for each regime scenario. Box-Whiskers plots illustrate in Figure 8 the distributions for $\bar{s} = 200$, where a red star indicates the average and a black dot the median of the price distributions. Price distributions converge to an expected price of par at maturity and this pull-to-par phenomenon shrinks the variability of price distributions near maturity. The distributions are skewed and bimodal (bi-modality is not seen from the Box-Whiskers plot but is evident when plotting the histogram). These results are intuitive and the contribution of our paper is to quantify them. These price distributions can be used to compute holding period returns at different horizons for risk management (Consiglio and Zenios, 2015; Mulvey and Zenios, 1994).

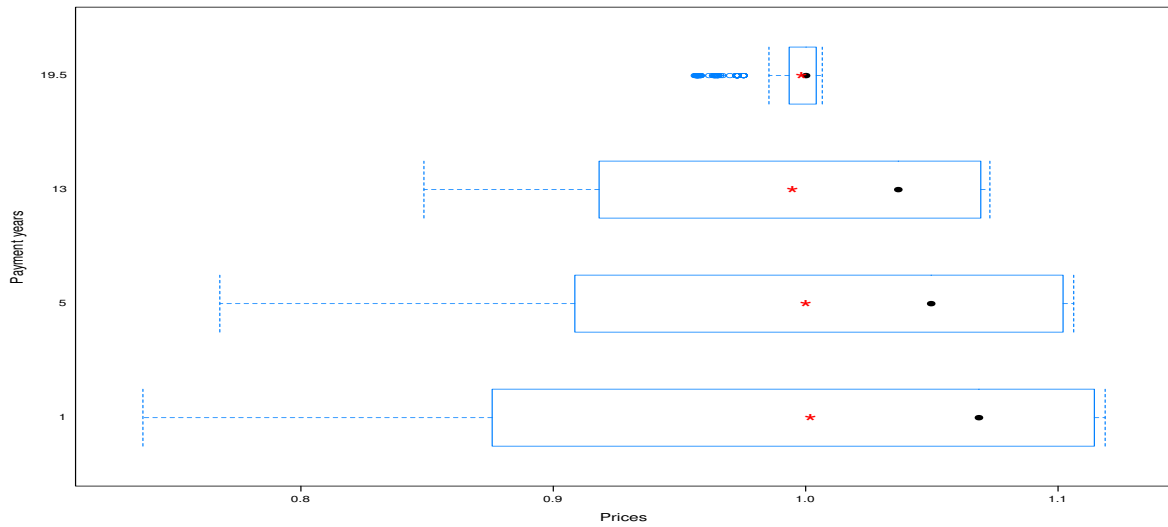
⁵A shorter maturity is used to reduce computational time, but similar results are obtained when running the experiment for a 20-yr bond on a single set of basis functions.



(a) Greece



(b) Italy.



(c) Germany

Figure 8: Price distribution of 20-yr S-CoCo with threshold 200 at 1, 5, 13 and 19.5 years.

4.3 The effect of regime switching on state contingent prices

To gain further insights in the performance of S-CoCo, we numerically test the effects of regime switching. Italy is used in all experiments, with thresholds 200 and 500. In the former case the standstill is activated and the results are qualitatively similar to what one would expect for Greece as well. In the later case the standstill is very rarely triggered and the results are very different from those of Greece. Results are reported again for a 20-yr S-CoCo price distribution at 1, 5, 13 and 19.5 years, but under different scenario test beds with and without regime switching. In particular:

R-OFF No regime switching, with the parameters used to calibrate the CDS spread model set to their historical average and simulating 5000 CDS spread and interest rate scenarios.

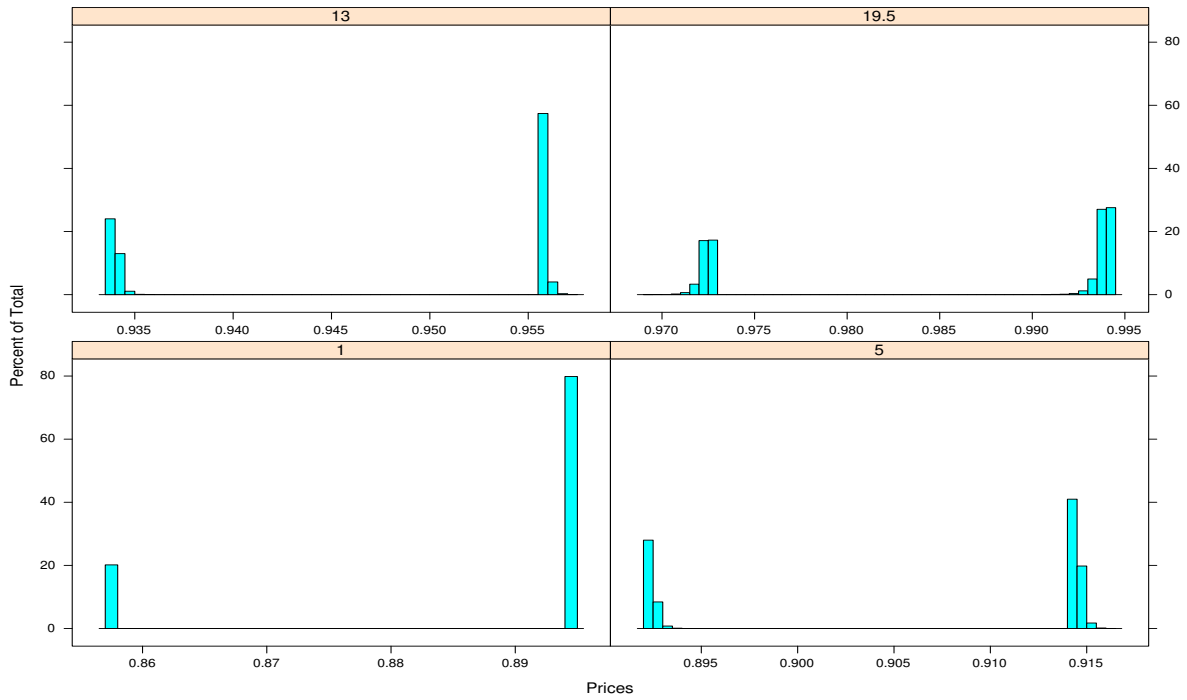
R-1 Only one scenario of regime switching between the identified regimes with 5000 CDS spread and interest rate scenarios.

R-100 100 simulations of regime switching between the identified regimes and 1000 scenarios of CDS spread and interest rates for each regime scenario.

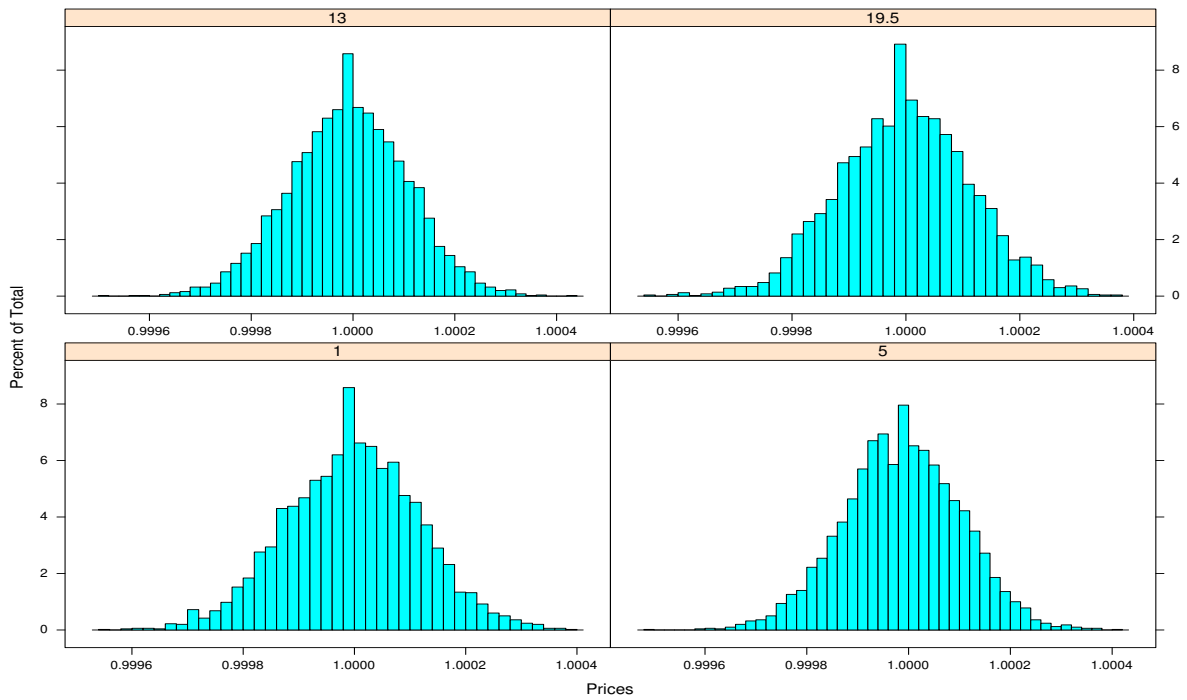
Figures 9–11 show the distribution of the prices for the three scenario test beds. The following observations can be made:

1. With the regime scenario simulation switched off and the CDS spread calibrated to the historical average, the S-CoCo with threshold 200 exhibits an (almost) binary distribution, while at threshold 500 its prices are just like a straight bond. Under the historical average regime the Italian CDS spreads do not exhibit sufficient variability to trigger the S-CoCo. Payment standstill becomes an extremely rare event, but with big impact.
2. When introducing even one regime scenario, capturing the observation of the recent past that Italy may move from a tranquil regime into turbulence and even a crisis, then the distribution of prices at threshold 200 exhibits more variability. There is also a non-trivial effect for threshold 500, although significantly lower than at the 200 threshold.
3. Finally, when simulating properly both regime switching and CDS spreads we obtain at the risk horizon multi-modal distributions. These modalities result from a combination of regime switching and standstill triggers.

The multi-modality of the distributions, when simulated properly, may be disconcerting. This is inescapable when modeling events with large impact —such as regime switching— and limited historical data to calibrate. If we could offer a criticism to our modeling approach is that a regime derived from expert opinion —such as “following the Brexit referendum, Italian CDS spreads will switch to levels seen at the peak of the Eurozone crisis and stay there until the Brexit issue is resolved”— maybe more appropriate than a statistical model. If an expert opinion regime is available the pricing model applies unaltered with **R-OFF**.

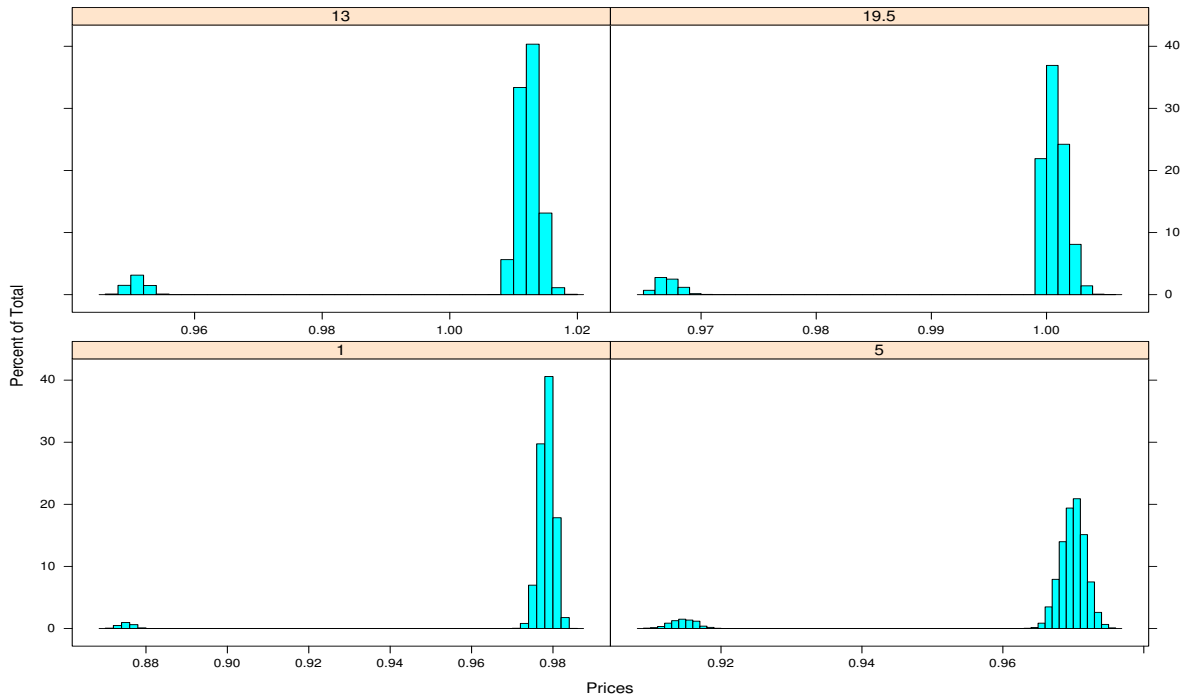


(a) Threshold 200

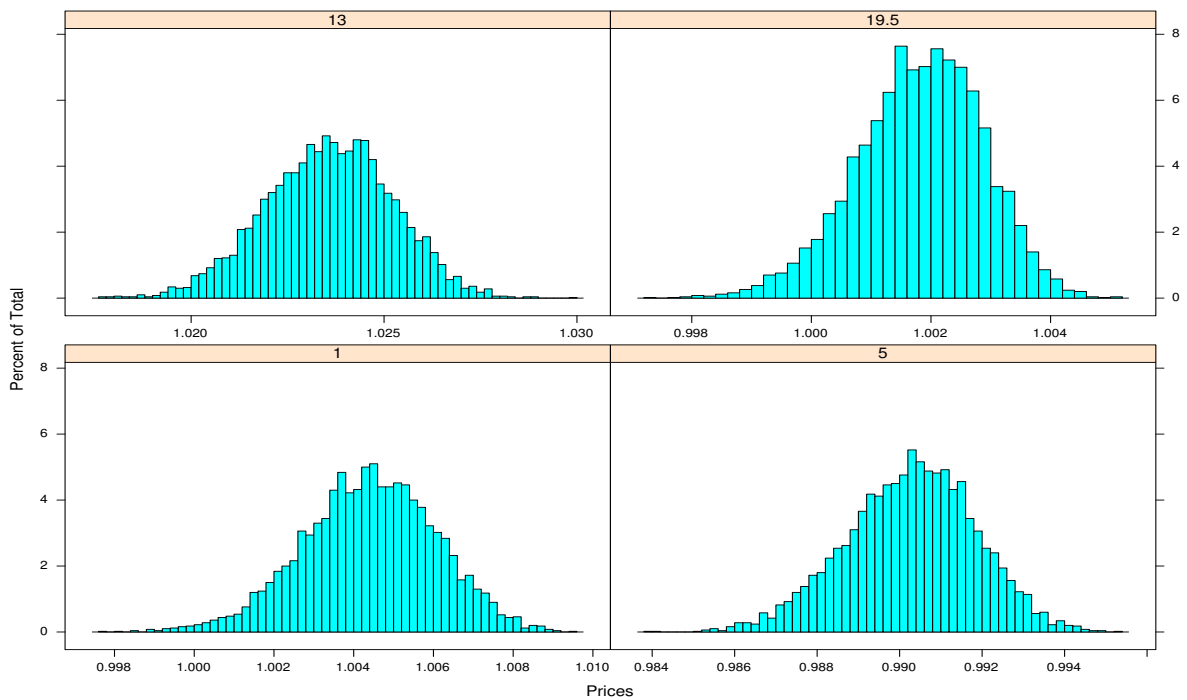


(b) Threshold 500

Figure 9: Price distribution of 20-yr Italian S-CoCo at different risk horizons without regime switching (test bed R-OFF).

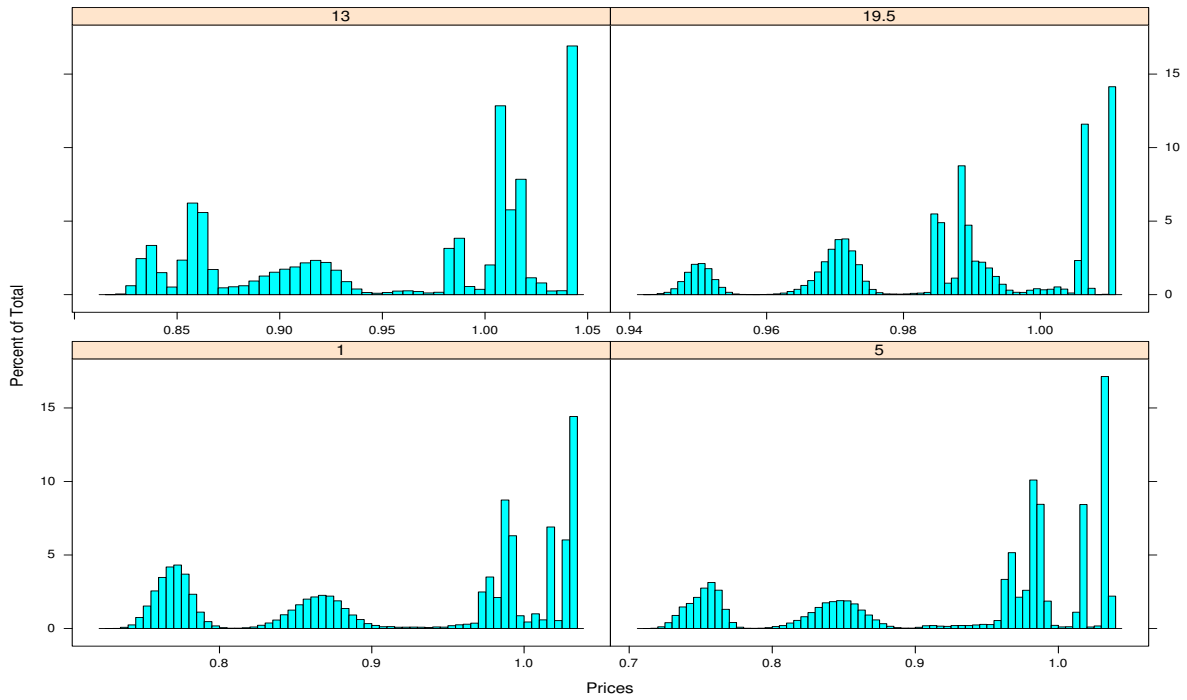


(a) Threshold 200

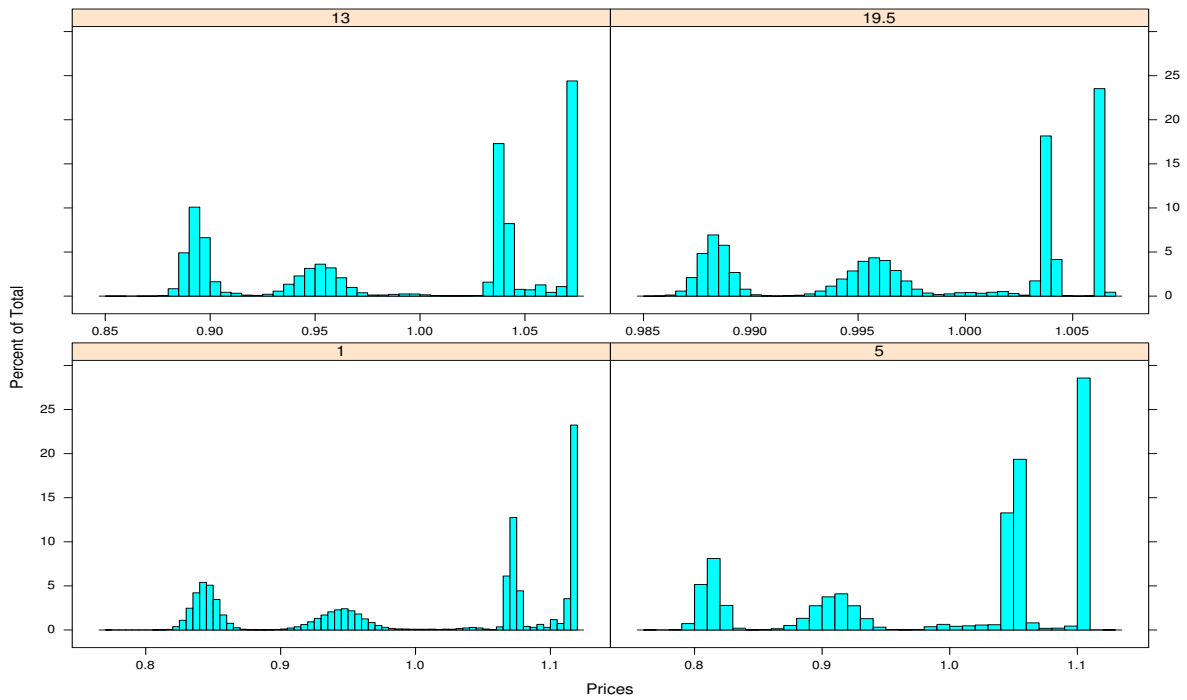


(b) Threshold 500

Figure 10: Price distribution of 20-yr Italian S-CoCo at different risk horizons with only one regime switching scenario (test bed R-1).



(a) Threshold 200



(b) Threshold 500

Figure 11: Price distribution of 20-yr Italian S-CoCo at different risk horizons with multiple regime switching scenarios (test bed R-100).

4.4 Dual trigger pricing

McDonald (2013) articulates the opinion that bank CoCo should not be converted for idiosyncratic problems, but only when the entity's difficulties come with market-wide problems. He illustrates dual trigger structures with a simple pricing example. The arguments for dual triggers seem well accepted for corporate debt where firms should be allowed to fail when they face difficulties in a benign market, but not when they face problems during a market-wide crisis. Sovereigns, on the other hand, do not fail, and Consiglio and Zenios (2015) "do not see any arguments in favor of a dual trigger, although an additional market-specific indicator could be introduced to allow for potential [sovereign] default". However the debate on sovereign contingent debt is at an early stage, and should dual price trigger be considered necessary we could use a systemic market index such as the CBOE volatility index VIX, or the emerging markets EMBI index, or, for eurozone countries, the CDS spreads on AAA-rated sovereigns. If a sovereign's CDS threshold is breached during a systemic crisis as indicated by the market index, then the payment standstill is triggered, but for idiosyncratic crises there would be a different treatment.

We develop here the S-CoCo pricing model with a dual trigger to illustrate that such a structure is possible. The model of Section 4.1 is extended by augmenting the stochastic process ξ with the market index v_t , $\xi = \{r_t, s_t, v_t\}$. (We assume that the stochastic components of ξ are uncorrelated. More complex patterns of correlation between the market index v_t and r_t and s_t can be introduced, but they are beyond the scope of the present paper.) A trigger threshold \bar{v} relates to the market index v_t , and \mathcal{T}_m^l , $m = 1, 2, \dots, M$, denotes those time sets in which, conditioned on scenario l , there is a coupon payment standstill for K_1 periods.

For the standstill to be triggered two conditions must be satisfied at time t : both s_t^l and v_t^l breach their respective thresholds \bar{s} and \bar{v} . But we also need to model situations where the market and the country specific indices decouple, to account for idiosyncratic crises. There are different patterns of aid that can be envisioned for such eventualities, which are represented using a different standstill period, K_2 . We embrace the view that countries in financial distress for purely idiosyncratic reasons need more help and therefore $K_2 > K_1$.

If at time t , and under scenario l , the CDS rate s_t^l hits \bar{s} , and the market index v_t^l is below \bar{v} , coupon payments are suspended for K_2 periods. Denote by $\mathcal{V}_q^l = \{t, t+1, \dots, t+K_2\}$, $q = 1, 2, \dots, Q$, such time sets with the same properties as \mathcal{T}_m^l . The time sets defining the dual trigger mechanism are then given by

$$\Lambda^l = \bigcup_{m=1}^M \mathcal{T}_m^l, \quad \Upsilon^l = \bigcup_{q=1}^Q \mathcal{V}_q^l, \quad (27)$$

and the new indicator function $\mathbb{1}_{\Upsilon^l} : \mathcal{T} \rightarrow \{0, 1\}$ is

$$\mathbb{1}_{\Upsilon^l}(t) = \begin{cases} 0, & \text{if } t \in \Upsilon^l \\ 1, & \text{if } t \notin \Upsilon^l. \end{cases} \quad (28)$$

The S-CoCo price function (18) with a dual trigger becomes

$$P_0 = \frac{1}{N} \sum_{l \in \Omega} \sum_{t \in \mathcal{T}} B^l(0, t) (\mathbb{1}_{\Lambda^l}(t) \cdot \mathbb{1}_{\Upsilon^l}(t)) c + B^l(0, T + \Delta T^l). \quad (29)$$

Note that, since $\mathcal{T}_m^l \cap \mathcal{V}_q^l = \emptyset$, for any $m = 1, 2, \dots, M$, and $q = 1, 2, \dots, Q$, then also $\Lambda^l \cap \Upsilon^l = \emptyset$, and the product of the two indicator functions correctly represents the dual trigger mechanism.

5 Conclusions

We developed a pricing model for sovereign contingent convertible bonds with payment standstill captures the regimes switching nature of the triggering process. We modify an existing one-factor tractable stochastic model of spread-returns with mean-reversion, to model spread levels converging to a long-term steady state value estimated from market data, and superimpose it on a novel regime switching Markov process model of the steady states. The Monte Carlo simulation pricing model is embedded in a Longstaff-Schwartz framework to compute state contingent prices at some risk horizon. This facilitates risk management.

Extensive numerical experiments illustrate the performance of the models and shed light on the performance of sovereign contingent debt. In particular, we observe the skewed distribution of prices at the risk horizon, the pull-to-par phenomenon as securities approach maturity and the multi-modality of the price distribution as the underlying CDS process switches regimes and/or the payment standstill is triggered.

The models are applied to S-CoCo designs for Greece, Italy and Germany to illustrate how these instruments would be priced for countries in different economic conditions. The results are intuitive and the contribution of the paper is in providing a model to quantify prices and holding period returns. Such a model is an essential tool if sovereign contingent debt is to receive attention and eventual acceptance as a practical financial innovation response to the problem of debt restructuring in the case of sovereign debt crises. Indeed, in a companion paper we have shown how these models can be used to develop a sovereign debt risk optimization model and highlight how S-CoCo improve a country's risk profile, Consiglio and Zenios (2015).

A limitation of the model is that it is not developed in an risk neutral context but at present, since no such instruments are issued, this limitation remains of academic than practical interest and a topic for future research.

A Appendix. Asymptotic modeling of the scenario generating process

We determine the parameters of the model for CDS spread return to identify its asymptotic dynamics. We start from the discrete time version of O'Donoghue et al. (2014) model (cf. eqn. (2), or eqn. (6) without the jump term) and derive a set of conditions on the asymptotic moments to be matched with their empirical counterparts. To simplify the notation in their eqns. (2) or (6), set $k_0 = \gamma$, $k_1 = \alpha + \beta$ and $k_2 = \alpha\beta$, to get

$$\Delta r_t = \left(k_0 - k_1 r_t - k_2 \sum_{s=0}^t r_s \Delta t \right) \Delta t + \sigma w_t, \quad (30)$$

where r_t is the return at time t and $w_t \sim \mathcal{N}(0, \Delta t)$. Following O'Donoghue et al. (2014), for $t \rightarrow \infty$, we have

$$\mathbb{E}[r_t] = 0 \quad (31)$$

$$\text{var}[r_t] = \frac{\sigma^2}{2k_1} \quad (32)$$

$$\mathbb{E}[C_t] = \frac{k_0}{k_2} \quad (33)$$

$$\text{var}[C_t] = \frac{\sigma^2}{2k_1 k_2}, \quad (34)$$

where $C_t = \sum_{s=0}^t r_s \Delta t$ and C_t is normally distributed.

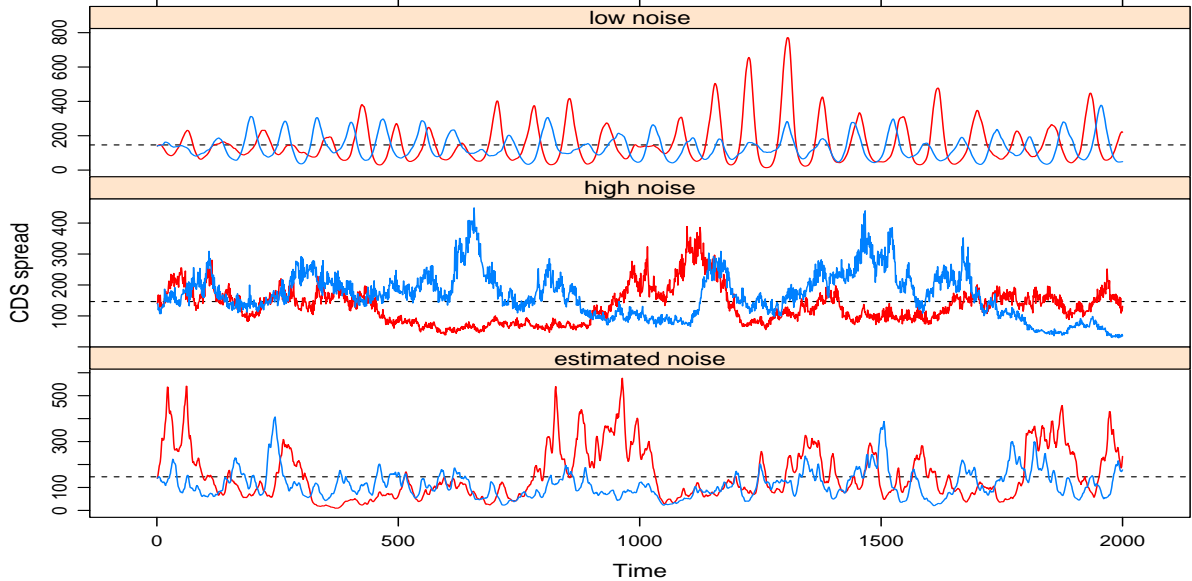


Figure 12: The effect of the parameter σ . In the upper panel the CDS spread dynamics is smoothed with a low value of the parameter σ . The center panel shows the effect of a high value of σ . In the bottom panel we obtained σ from eqn. (37), where \hat{s}^2 is estimated from the data and k_1 is given. The dotted lines show the asymptotic level of the CDS spread around which the two selected scenarios fluctuate.

The spread process $S_t = S_0 \exp(C_t)$ is log-normally distributed with

$$\mathbb{E}[S_t] = S_0 \exp\left(\frac{k_0}{k_2} + \frac{\sigma^2}{4k_1k_2}\right) \quad (35)$$

$$\text{var}[S_t] = S_0^2 \exp\left(2\frac{k_0}{k_2} + \frac{\sigma^2}{2k_1k_2}\right) \left[\exp\left(\frac{\sigma^2}{2k_1k_2}\right) - 1\right]. \quad (36)$$

To identify the stochastic dynamics in (30) we need one additional condition which is related to the noise term σ . We derive it from the ergodic value of $\mathbb{E}[(\Delta r_t)^2]$. To support our statement, we show in Figure 12 the effect of the noise term σ on two selected paths obtained from (30). Observe that the lower the σ , the smoother is the generated curve, so we can relate σ to $\mathbb{E}[(\Delta r_t)^2]$, and take as sample measure the expression

$$\hat{s}^2 = \frac{1}{N} \sum_{t=1}^N (r_t - r_{t-1})^2. \quad (37)$$

With some standard assumptions for stochastic processes, namely that $\mathbb{E}[w_t C_t] = 0$, $\mathbb{E}[r_t C_t] = 0$ and $\mathbb{E}[r_t w_t] = 0$, and using simple algebra, it can be shown that

$$\mathbb{E}[(\Delta r_t)^2] = \frac{\sigma^2}{2} \left(k_1 + \frac{k_2}{k_1} + 2\right). \quad (38)$$

The theoretical moments defined by (32), (35), (36) and (38) are then used to match the empirical observations. In particular, we denote by \hat{S} the asymptotic CDS spread level, by $\hat{\sigma}_S$ the asymptotic variance of CDS spread level, by $\hat{\sigma}_r$ the asymptotic variance of CDS spread returns, and by \hat{s}^2 the smoothness of the CDS spread level. These quantities are estimated for each regime separately if regime switching is present in the empirical data, e.g., Table 2. Denoting by T_τ the set of time periods in regime τ , we obtain the following moment estimates:

$$\hat{S} = \frac{1}{|T_\tau|} \sum_{t \in T_\tau} S_t \quad (39)$$

$$\hat{\sigma}_S^2 = \frac{1}{|T_\tau|} \sum_{t \in T_\tau} (S_t - \hat{S})^2 \quad (40)$$

$$\hat{\sigma}_r^2 = \frac{1}{|T_\tau|} \sum_{t \in T_\tau} (r_t - \hat{r})^2 \quad (41)$$

$$\hat{s}^2 = \frac{1}{|T_\tau|} \sum_{t \in T_\tau} (r_t - r_{t-1})^2. \quad (42)$$

We can now match the theoretical moments to estimated (or assumed) values using the system of nonlinear equations in k_0 , k_1 , k_2 and σ :

$$\exp\left(\frac{k_0}{k_2} + \frac{\sigma^2}{4k_1k_2}\right) = \frac{\hat{S}}{S_0} \quad (43)$$

$$\exp\left(2\frac{k_0}{k_2} + \frac{\sigma^2}{2k_1k_2}\right) \left[\exp\left(\frac{\sigma^2}{2k_1k_2}\right) - 1\right] = \frac{\hat{\sigma}_S^2}{S_0^2} \quad (44)$$

$$\frac{\sigma^2}{2k_1} = \hat{\sigma}_r^2 \quad (45)$$

$$\frac{\sigma^2}{2} \left(k_1 + \frac{k_2}{k_1} + 2\right) = \hat{s}^2. \quad (46)$$

S_0 is the starting value of the CDS spread for the selected regime. The closed form solution to the system of equations (43)–(46) is given by

$$k_0 = \frac{\hat{\sigma}_r^2}{\log\left(1 + \frac{\hat{\sigma}_S^2}{\hat{s}^2}\right)} \log\left(\frac{\hat{S}}{S_0}\right) - \frac{1}{2}\hat{\sigma}_r^2 \quad (47)$$

$$k_1 = \frac{\sigma^2}{2\hat{\sigma}_r^2} \quad (48)$$

$$k_2 = \frac{\hat{\sigma}_r^2}{\log\left(1 + \frac{\hat{\sigma}_S^2}{\hat{s}^2}\right)} \quad (49)$$

$$\sigma^2 = 2\hat{\sigma}_r^2 \left[-1 + \sqrt{1 + \frac{\hat{s}^2 - k_2\hat{\sigma}_r^2}{4\hat{\sigma}_r^2}}\right]. \quad (50)$$

Finally, to ensure that $\sigma \in \mathbb{R}^+$ we need $\hat{s}^2 - k_2\hat{\sigma}_r^2 > 0$.

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