# Portfolio diversification in the sovereign credit swap markets

Andrea Consiglio *
Somayyeh Lotfi †
Stavros A. Zenios ‡

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**Abstract**

We develop models for portfolio diversification in the sovereign credit default swap (CDS) markets and show that, despite literature findings that sovereign CDS spreads are affected by global factors, there is sufficient idiosyncratic risk to be diversified away. However, we identify regime switching in the times series of CDS spreads, and the portfolio diversification strategies may differ between regimes. The models trade off the CVaR risk measure against expected return. They are tested in an active management setting for Eurozone core, periphery, and Central, Eastern and South-Eastern Europe (CESEE) countries. Models are developed for investors with long positions in CDS, speculators that hold uncovered long and short positions, and hedgers with covered long and short exposures. The results compare favorably with the broad S&P/ISDA Eurozone Developed Nation Sovereign CDS index. We also identify several issues that remain unexplored on the way to developing integrated risk management models for CDS portfolios.

**Keywords:** Credit derivatives; portfolio diversification; Eurozone crisis; CDS spreads; Conditional Value-at-Risk; regime switching.

*Department of Economics, Management and Statistics, University of Palermo, Palermo, IT. andrea.consiglio@unipa.it

†Department of Applied Mathematics, University of Guilan, Rasht, Iran. slotfi@phd.guilan.ac.ir

‡Department of Accounting and Finance, University of Cyprus, Nicosia, CY, Norwegian School of Economics, and Wharton Financial Institutions Center, University of Pennsylvania, USA. zenios.stavros@ucy.ac.cy

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# 1 Introduction

In this paper we take a portfolio view of sovereign credit default swaps, contributing to the extensive literature of credit default swaps (CDS) pricing the natural extension from individual instruments to portfolios. Sovereign CDS are insurance contracts offering protection against the default of a referenced sovereign government. They emerged in the 1990's as a significant credit derivative security in the sovereign debt market. Their raison d'être is to hedge and trade sovereign credit risks. They offer investors the opportunity to take a view purely on credit. They are used by sovereign debt investors to *hedge* against losses from a default or other credit event of the sovereign. Speculators take *naked* positions in these instruments —without buying the underlying asset— to place a bet on the credit risk of the reference entity. Arbitrageurs exploit price differences between the derivative and the underlying debt obligation(s) by taking offsetting positions in the two.

Following standardization of CDS contracts in 1998-1999 and successful execution in a few defaults —starting with Argentina in 2001— the market grew rapidly. Packer and Suthiphongchai (2003) discuss the early developments and Figure 1 illustrates the market growth since 2005 when BIS started publishing data. We report notional amounts and gross market value together with the number of dealers. Notional amounts provide a measure of market size, but gross market values —whereby open contracts are netted out at market prices— better reflect the scale of risk transfer in the market.

CDS have been criticized for facilitating market manipulations in the eurozone crisis. Naked trading was banned by the German financial regulator in May 2010 and by the EU since November 2012, despite the findings of a European Commission report, Criado et al. (2010). More recently IMF (2013) presented evidence that refutes the criticism against their use and argues that CDS have contributed to the deepening and efficiency of the sovereign markets. A nuanced view on the role of credit default swaps in the credit crisis of 2008 is offered by Stulz (2010) who argues that "financial derivatives have clearly lost any presumption of innocence" but finds it would be misguided "to turn 180 degrees from a presumption of innocence to a presumption of guilt".

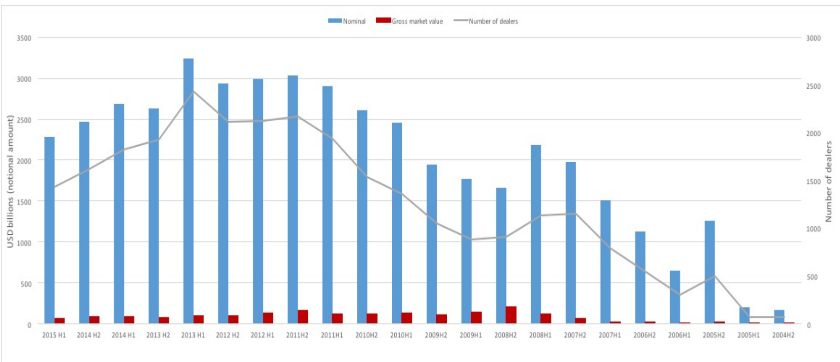



Figure 1: The growing market for sovereign Credit Default Swaps.
(Source: Bank for International Settlement and authors' calculations.)

The CDS markets, the statistical properties of CDS spreads, spread returns and spread term structures, the price discovery mechanisms and market spillovers have been studied extensively. The corporate CDS market gets the lion's share of research, but attention focused recently on the sovereign market as well. We do not review the extensive extant body of knowledge here and refer readers to the survey and discussion on future prospects, Augustin et al. (2014, 2016), and recent work on sovereign CDS, Fabozzi et al. (2016).

Conspicuously absent are studies of CDS in a portfolio context. This is where our paper makes a contribution. In summary, the paper provides some practical models for CDS investors, established the validity of diversification in sovereign credit swap market, and validated the models during the eurozone crisis. The paper also paves the way for further research. The motivation is coming from current developments:

1. While CDS are typically bought to hedge against the default of a reference sovereign bond, enterprise risk management would require sovereign bond investors to consider the insurance needs of their whole portfolio. The silo approach to hedging may lead to over-insurance when entities in the portfolio are negatively correlated.

2. A portfolio view is also required in some aspects of EU regulation on CDS (European Union, 2012). The regulatory ban on naked positions does not apply if a CDS transaction serves to hedge a long position in a portfolio of debt instruments with its pricing highly correlated with the pricing of the reference sovereign debt. In this case the CDS holder is not hedging a specific sovereign entity but their own portfolio, and a CDS portfolio may be (most likely, it will be) a better hedge than a single contract.

3. Empirical research on sovereign CDS spread determinants finds considerable evidence that a significant fraction of the fluctuations in sovereign CDS spreads is determined by global factors, unrelated to a country's economy. The relevant literature is surveyed in (Augustin et al., 2014, Sec. 7.4). However, idiosyncratic risks remain — explaining 4% to 43% of CDS return variation depending on country and time scale— and hence a portfolio approach is useful in diversifying these risks away. Our work provides normative models to do so, and the application to eurozone core, periphery and CESEE countries shows that diversification pays.

4. It appears that hedge funds made significant profits in the CDS market. Napier group, Saba and Blue Mountain are some of the funds quoted in Augustin et al. (2016) as benefiting from their CDS positions. Activist shareholder Carl Icahn stated in November 2010 that "the risk-reward is great in the CDS" and announced that he owns credit default swaps on high yield debt against the 5-year U.S. Treasury note [1]. Increased market liquidity, standardization of contracts, the shift from OTC deals to exchange trades, facilitated the interest in CDS portfolio management.

5. Development of CDS indices allows the benchmarking of CDS investment strategies. See, for instance the Markit CDX family of tradeable CDS indices covering North America and emerging markets, and the S&P/ISDA CDS indices. Co-branding S&Ps existing CDS indices together with the International Securities and Swaps Association (ISDA) provides market participants a benchmark, designed to increase transparency and efficiency in the derivatives market. S&P Indices and ISDA currently offer the *S&P/ISDA International Developed Nation Sovereign CDS Index* and *S&P/ISDA Eurozone Developed Nation Sovereign CDS Index*, among others.

The models developed in this paper are building blocks for decision support of sovereign bond investors who buy CDS to hedge the credit risk of their portfolios, of speculators that seek to exploit perceived credit risk opportunities from deterioration (by going long)

[1] See *REUTERS SUMMIT-Investors overpaying for yield after years of low rates, Thu Nov 20, 2014*, *http://www.reuters.com/article/investment-yearend-yield-idUSL2N0T82YH20141120*

or improvements (by going short) of a sovereign's credit rating, and of dealers that both buy and sell CDS and wish to have a covered portfolio exposure. The paper is organized as follows: Section 2 reports statistical analysis of the sovereign CDS markets and identifies regime switching, Section 3 formulates the models, and Sections 4 and 5 report results with portfolio diversification and active portfolio management, respectively. Section 6 draws conclusions.

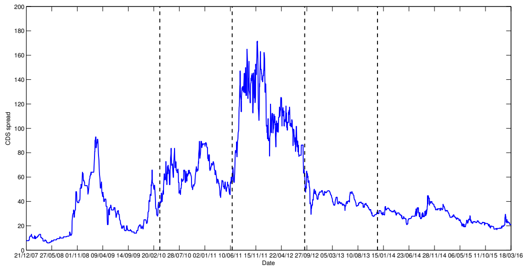
# 2 Some empirical analysis of sovereign CDS markets

The choice of a model should be guided by the relevant stylized facts of the market data. We analyze the most liquid 5-yr CDS spreads for a sample of European core and periphery countries, and a sample of CESEE countries using daily observations from 8 October 2008 to 18 March 2016. This period starts with the sub-prime crisis in the USA, covers the beginning of the Eurozone crisis and goes well into its endgame. For some countries CDS were offered after December 2007 and the Greek CDS stopped trading on 22 February 2012 after a vertiginous ascent to 14911.74.

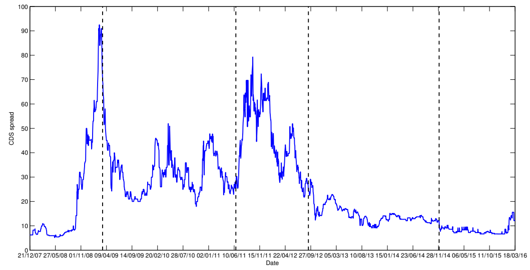
## 2.1 Regime switching

Given the significant evidence that the determinants of CDS spreads and spread returns change with time, we first identify the regimes and then calculate the statistics in each regime. Using Bai-Perron test (Bai and Perron, 1998) with the free software system R, we identified the regime changes illustrated in Figure 2 for core and periphery Eurozone countries, Figure 3 for CESEE countries, and Figure 4 for the Baltic countries. France, Italy, Portugal, Spain and Cyprus are synchronized in their regime switching, whereas Germany, Ireland and Greece have idiosyncratic regime changes. Note, for instance, that only Germany has a regime switch associated with the subprime crisis and the collapse of Lehman Brothers in September 2008, while the onset of the eurozone crisis in spring 2010 signals regime switching for all eurozone countries.

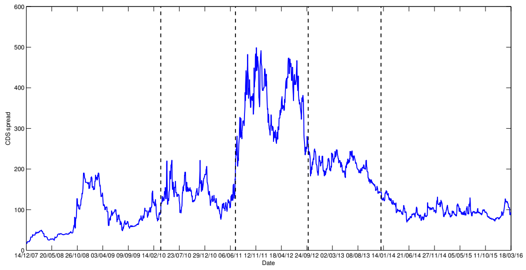
The mean, standard deviation, skewness, kurtosis, and max and min values of CDS spread returns, for different regimes, are summarized in Tables 1, 2 and 3 for core and periphery, CESEE and Baltic countries, respectively. These results verify that CDS spread returns are asymmetric with fat tails. This dictates the type of portfolio diversification model we will use next, as the classical mean variance models do not apply.



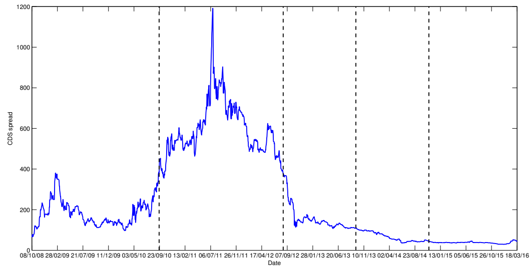
(a) France.



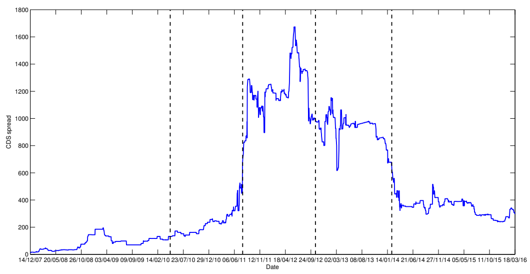
(b) Germany.



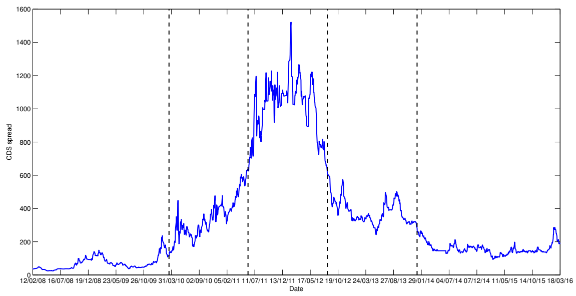
(c) Italy.



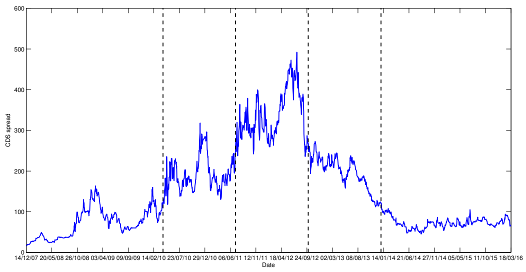
(d) Ireland.



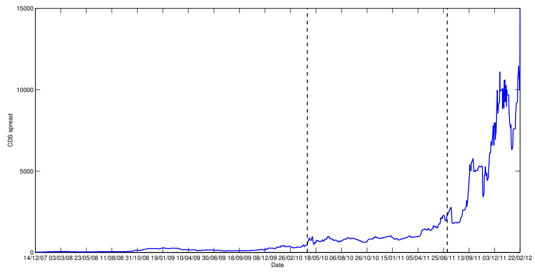
(e) Cyprus.



(f) Portugal.

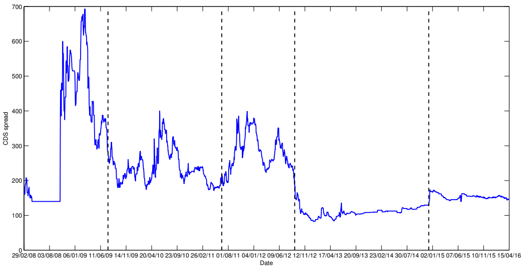


(g) Spain.

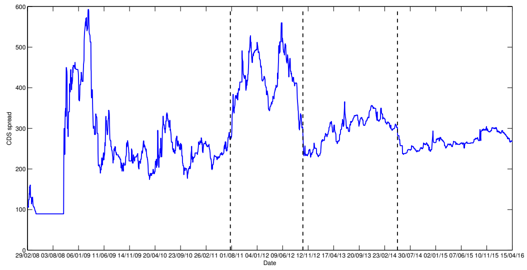


(h) Greece.

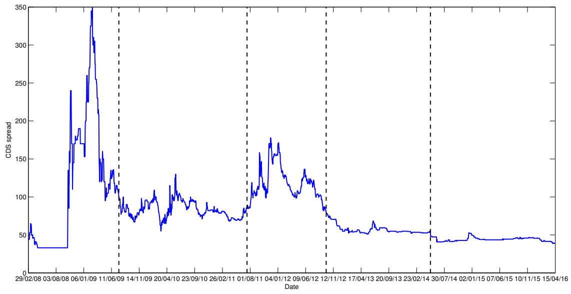
Figure 2: Regime switching for core and periphery countries.



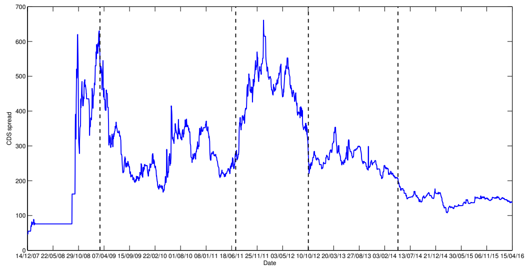

(a) Bulgaria.



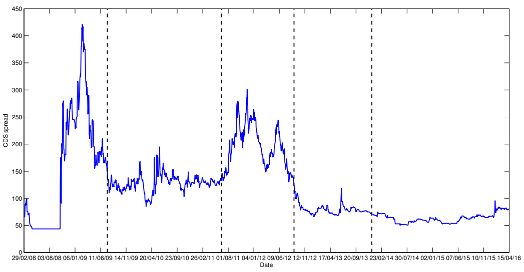

(b) Croatia.



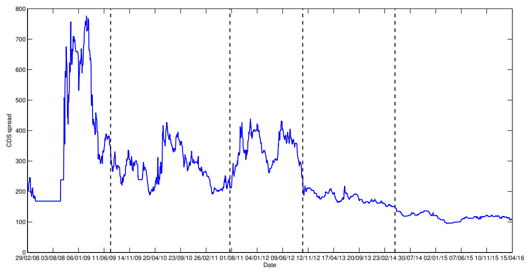

(c) Czech Republic.



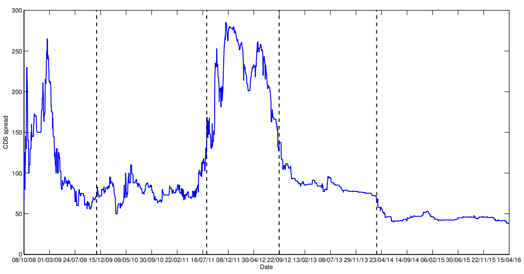

(d) Hungary.



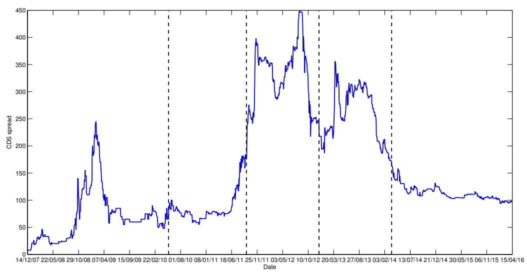

(e) Poland.




(f) Romania.

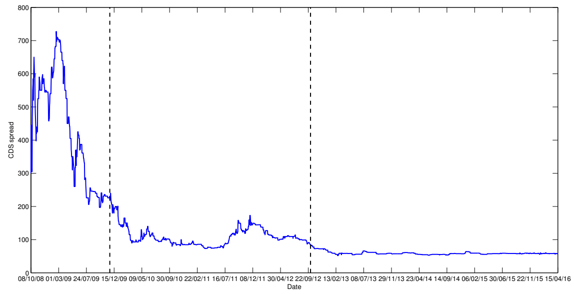



(g) Slovakia.

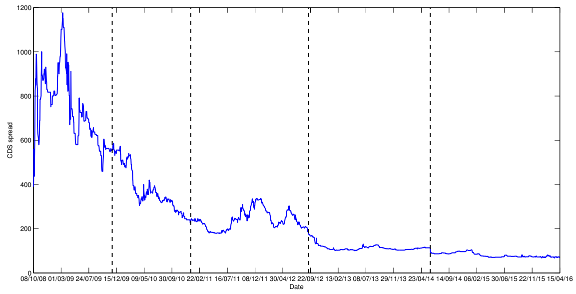



(h) Slovenia.

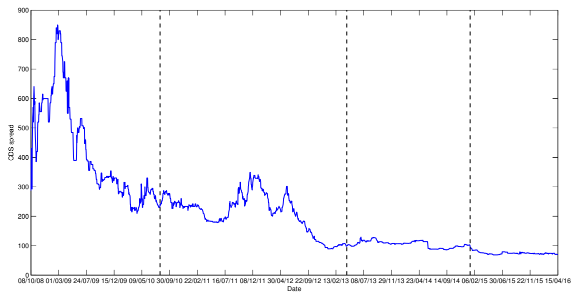
Figure 3: Regime switching for CESEE countries.




(a) Estonia.




(b) Latvia.




(c) Lithuania.

Figure 4: Regime switching for the Baltic countries.

| Country-Regime | Mean | SD | Skewness | Kurtosis | Max | Min |
|---|---|---|---|---|---|---|
| France | | | | | | |
| 21/12/07–26/03/10 | 0.25 | 5.72 | 0.98 | 11.20 | 37.47 | -21.85 |
| 29/03/10–21/06/11 | 0.15 | 4.87 | -0.04 | 5.64 | 18.44 | -20.53 |
| 22/06/11–13/09/12 | 0.03 | 6.22 | -0.01 | 4.46 | 19.18 | -23.08 |
| 14/09/12–09/12/13 | -0.21 | 3.49 | -1.06 | 16.86 | 17.14 | -23.39 |
| 10/12/13–18/03/16 | -0.07 | 4.01 | 0.47 | 10.55 | 25.36 | -18.49 |
| Germany | | | | | | |
| 21/12/07–13/03/09 | 0.78 | 5.48 | 2.82 | 28.86 | 52.48 | -17.44 |
| 16/03/09–20/06/11 | -0.18 | 5.48 | -0.57 | 10.58 | 24.92 | -37.14 |
| 21/06/11–12/09/12 | -0.02 | 6.24 | 0.54 | 6.26 | 31.87 | -21.04 |
| 13/09/12–02/12/14 | -0.13 | 3.95 | -0.23 | 12.16 | 23.12 | -23.12 |
| 03/12/14–18/03/16 | -0.00 | 6.31 | -0.02 | 7.39 | 25.76 | -25.07 |
| Italy | | | | | | |
| 14/12/07–29/03/10 | 0.29 | 5.01 | 0.02 | 13.95 | 33.86 | -37.12 |
| 30/03/10–07/07/11 | 0.18 | 6.51 | -0.84 | 10.39 | 20.97 | -45.20 |
| 08/07/11–02/10/12 | 0.13 | 4.94 | 0.34 | 5.55 | 22.20 | -16.27 |
| 03/10/12–27/12/13 | -0.19 | 2.91 | 0.38 | 8.20 | 15.68 | -14.68 |
| 30/12/13–18/03/16 | -0.08 | 3.67 | 0.38 | 7.98 | 23.39 | -16.15 |
| Ireland | | | | | | |
| 08/10/08–20/09/10 | 0.34 | 5.41 | 0.20 | 8.32 | 25.37 | -33.11 |
| 21/09/10–15/08/12 | -0.00 | 3.08 | -0.26 | 6.08 | 13.11 | -13.47 |
| 16/08/12–26/09/13 | -0.43 | 3.93 | -1.11 | 15.49 | 17.48 | -22.15 |
| 27/09/13–07/11/14 | -0.30 | 2.19 | -2.64 | 21.33 | 6.13 | -18.08 |
| 10/11/14–18/03/16 | -0.01 | 1.69 | 1.57 | 13.96 | 11.43 | -7.12 |
| Cyprus | | | | | | |
| 14/12/07–30/04/10 | 0.37 | 4.40 | 0.51 | 22.11 | 26.24 | -33.65 |
| 03/05/10–27/07/11 | 0.51 | 4.55 | 3.41 | 32.69 | 40.68 | -20.91 |
| 28/07/11–22/10/12 | 0.12 | 3.72 | 2.90 | 49.28 | 36.27 | -27.07 |
| 23/10/12–07/02/14 | -0.11 | 3.00 | -0.49 | 31.65 | 19.79 | -26.14 |
| 10/02/14–18/03/16 | -0.15 | 3.19 | 1.28 | 44.46 | 33.46 | -29.74 |
| Portugal | | | | | | |
| 12/02/08–16/03/10 | 0.20 | 4.61 | 0.28 | 5.62 | 17.70 | -17.65 |
| 17/03/10–02/06/11 | 0.54 | 6.59 | -2.28 | 24.99 | 27.42 | -59.00 |
| 03/06/11–20/08/12 | -0.00 | 4.09 | 0.37 | 7.65 | 21.28 | -17.61 |
| 21/08/12–03/01/14 | -0.22 | 2.85 | 0.86 | 11.45 | 18.95 | -11.79 |
| 06/01/14–18/03/16 | -0.08 | 3.84 | 0.62 | 6.07 | 17.32 | -14.28 |
| Spain | | | | | | |
| 14/12/07–12/04/10 | 0.32 | 5.17 | 0.10 | 10.34 | 33.85 | -30.13 |
| 13/04/10–07/07/11 | 0.24 | 6.26 | -0.63 | 10.17 | 28.26 | -41.75 |
| 08/07/11–02/10/12 | 0.04 | 4.83 | 0.03 | 5.92 | 21.66 | -19.35 |
| 03/10/12–27/12/13 | -0.24 | 2.94 | -0.01 | 6.89 | 11.79 | -15.28 |
| 30/12/13–18/03/16 | -0.11 | 3.97 | 0.54 | 8.26 | 25.10 | -18.41 |
| Greece | | | | | | |
| 14/12/07–20/04/10 | 0.54 | 4.45 | 0.33 | 9.03 | 24.54 | -23.65 |
| 21/04/10–06/07/11 | 0.51 | 5.20 | -2.51 | 37.50 | 22.26 | -52.20 |
| 07/07/11–22/02/12 | 0.89 | 8.05 | -0.41 | 6.73 | 26.78 | -37.83 |

Table 1: CDS spread return statistics during each regime for core and periphery countries.

| **Country-Regime** | **Mean** | **SD** | **Skewness** | **Kurtosis** | **Max** | **Min** |
|---|---|---|---|---|---|---|
| Bulgaria | | | | | | |
| 29/02/08–24/07/09 | 0.15 | 5.70 | 6.45 | 84.09 | 74.45 | -21.19 |
| 27/07/09–22/06/11 | -0.07 | 3.69 | 0.76 | 14.52 | 29.19 | -22.31 |
| 23/06/11–10/09/12 | -0.02 | 3.02 | -0.18 | 6.61 | 11.92 | -15.45 |
| 11/09/12–09/12/14 | -0.07 | 2.22 | -0.36 | 29.36 | 17.34 | -20.94 |
| 10/12/14–15/04/16 | 0.03 | 1.63 | 3.65 | 53.57 | 17.54 | -9.73 |
| Croatia | | | | | | |
| 29/02/08–22/07/11 | 0.11 | 4.94 | 6.28 | 116.34 | 87.50 | -24.89 |
| 25/07/11–10/10/12 | 0.03 | 2.46 | 1.14 | 11.18 | 17.06 | -7.46 |
| 11/10/12–12/05/14 | -0.00 | 1.69 | -1.31 | 17.24 | 8.33 | -13.54 |
| 13/05/14–15/04/16 | -0.02 | 1.16 | -0.26 | 19.70 | 8.38 | -8.00 |
| Czech Republic | | | | | | |
| 29/02/08–22/07/09 | 0.23 | 8.05 | 1.85 | 26.80 | 68.94 | -46.26 |
| 23/07/09–14/07/11 | -0.04 | 4.13 | 0.13 | 17.54 | 30.23 | -31.85 |
| 15/07/11–02/10/12 | -0.02 | 3.61 | 1.11 | 12.90 | 21.74 | -17.92 |
| 03/10/12–12/05/14 | -0.09 | 1.43 | 0.21 | 17.35 | 9.61 | -8.36 |
| 13/05/14–15/04/16 | -0.07 | 1.35 | 0.21 | 47.75 | 13.97 | -12.41 |
| Hungary | | | | | | |
| 14/12/07–12/03/09 | 0.78 | 6.77 | 5.54 | 56.67 | 75.62 | -20.06 |
| 13/03/09–13/07/11 | -0.12 | 3.63 | 0.85 | 15.06 | 27.25 | -24.05 |
| 14/07/11–11/10/12 | 0.02 | 2.77 | 0.04 | 4.60 | 10.59 | -9.47 |
| 12/10/12–25/04/14 | -0.08 | 2.61 | 0.69 | 43.84 | 26.01 | -23.29 |
| 28/04/14–15/04/16 | -0.08 | 1.46 | 0.17 | 8.94 | 7.61 | -5.93 |
| Poland | | | | | | |
| 29/02/08–22/07/09 | 0.25 | 8.64 | 4.52 | 74.83 | 105.40 | -65.39 |
| 23/07/09–20/06/11 | -0.03 | 4.29 | -0.71 | 14.20 | 24.54 | -33.65 |
| 21/06/11–06/09/12 | -0.04 | 3.46 | -0.21 | 4.89 | 10.77 | -13.58 |
| 07/09/12–26/12/13 | -0.16 | 2.64 | 1.79 | 21.19 | 20.72 | -13.09 |
| 27/12/13–15/04/16 | 0.01 | 1.93 | -1.39 | 43.89 | 16.27 | -21.26 |
| Romania | | | | | | |
| 29/02/08–21/07/09 | 0.13 | 5.57 | 2.13 | 25.53 | 39.67 | -33.93 |
| 22/07/09–21/07/11 | -0.08 | 3.23 | 0.15 | 12.75 | 22.53 | -22.29 |
| 22/07/11–09/10/12 | 0.03 | 2.72 | 0.37 | 5.85 | 11.39 | -9.11 |
| 10/10/12–25/04/14 | -0.12 | 1.61 | -0.06 | 22.63 | 12.27 | -11.91 |
| 28/04/14–15/04/16 | -0.07 | 1.15 | -0.44 | 11.91 | 6.54 | -5.82 |
| Slovakia | | | | | | |
| 08/10/08–20/11/09 | 0.02 | 7.77 | 0.48 | 14.47 | 46.05 | -44.63 |
| 23/11/09–05/08/11 | 0.11 | 4.78 | 0.14 | 10.92 | 24.85 | -28.77 |
| 08/08/11–20/09/12 | 0.03 | 4.34 | 0.88 | 12.33 | 27.57 | -19.60 |
| 21/09/12–26/03/14 | -0.16 | 1.66 | -1.69 | 31.64 | 9.30 | -14.45 |
| 27/03/14–15/04/16 | -0.11 | 1.43 | -2.40 | 26.66 | 7.35 | -13.56 |
| Slovenia | | | | | | |
| 14/12/07–17/05/10 | 0.33 | 7.11 | 1.17 | 17.37 | 55.96 | -36.10 |
| 18/05/10–16/09/11 | 0.29 | 3.82 | 1.19 | 10.51 | 20.76 | -11.78 |
| 19/09/11–17/12/12 | 0.10 | 2.99 | 0.12 | 13.94 | 14.47 | -18.83 |
| 18/12/12–18/03/14 | -0.11 | 2.49 | 1.11 | 13.84 | 14.38 | -11.03 |
| 19/03/14–15/04/16 | -0.11 | 1.49 | 0.10 | 13.96 | 8.76 | -8.33 |

Table 2: CDS spread return statistics during each regime for CESEE countries.

| Country-Regime | Mean | SD | Skewness | Kurtosis | Max | Min |
|---|---|---|---|---|---|---|
| Estonia | | | | | | |
| 08/10/08–20/11/09 | -0.10 | 5.84 | 1.56 | 18.12 | 42.17 | -28.39 |
| 23/11/09–03/10/12 | -0.13 | 3.25 | -0.02 | 12.59 | 20.07 | -19.31 |
| 04/10/12–15/04/16 | -0.04 | 1.16 | 0.02 | 27.38 | 9.41 | -8.92 |
| Latvia | | | | | | |
| 08/10/08–20/11/09 | 0.12 | 5.46 | 0.34 | 10.86 | 30.50 | -24.67 |
| 23/11/09–06/01/11 | -0.29 | 2.64 | -0.43 | 7.72 | 12.22 | -14.11 |
| 07/01/11–12/09/12 | -0.06 | 2.27 | 0.73 | 8.05 | 11.35 | -10.42 |
| 13/09/12–06/06/14 | -0.10 | 1.57 | -2.61 | 42.88 | 10.63 | -17.02 |
| 09/06/14–15/04/16 | -0.10 | 1.88 | -2.90 | 44.89 | 11.68 | -21.62 |
| Lithuania | | | | | | |
| 08/10/08–11/08/10 | -0.04 | 5.06 | 0.86 | 25.61 | 43.08 | -38.53 |
| 12/08/10–10/04/13 | -0.12 | 2.20 | -0.17 | 7.38 | 10.80 | -10.11 |
| 11/04/13–13/01/15 | 0.00 | 1.51 | -6.82 | 125.10 | 10.65 | -23.02 |
| 14/01/15–15/04/16 | -0.11 | 1.35 | 1.10 | 17.16 | 9.53 | -6.34 |

Table 3: CDS spread return statistics during each regime for the Baltic countries.

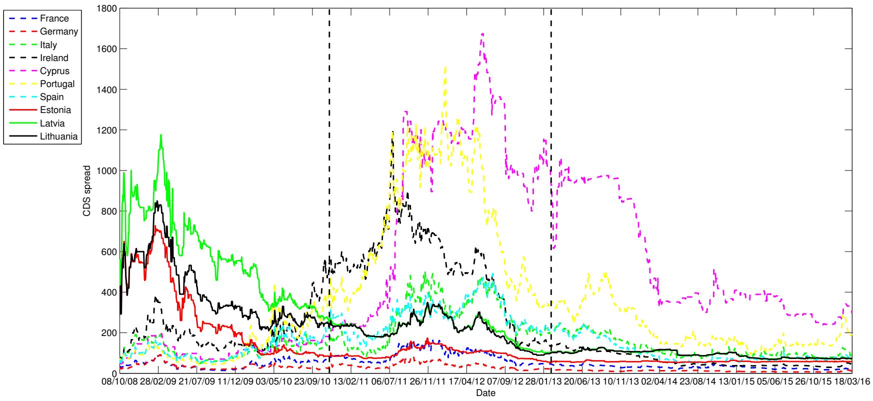



Figure 5: Regime switching for multiple time series identified using Bai-Qu-Perron test with VSTOXX as a common risk factor.

## 2.2 Common regime breaks

For portfolio management we identify common regimes for the CDS series. An extension of the Bai-Perron test to multivariate regressions is Qu and Perron (2007) and we run what we denote as Bai-Qu-Perron test using GAUSS 16.0.3 on the series of Eurozone core, periphery and Baltic countries. As common regressor we used the Euro Stoxx 50 Volatility Index (VSTOXX) which is the most watched European volatility index. The test on the full set of core, periphery and CESEE countries would not converge, but we found various combinations of core, periphery and Baltic countries giving stable results when adding additional countries. Hence, we use the regimes identified for core-periphery-Baltic as applying to the whole set of series. Results are illustrated in Figure 5. These regimes are referred to later as *I. Turbulent*, *II. Crisis* and *III. Post crisis* regimes. We use these regimes to test for regime specific diversification results.

# 3 The CVaR portfolio optimization models

We consider now the model for building a diversified portfolio of credit default swap positions using CVaR portfolio optimization, which is well suited for the skewed and fat-tailed returns of CDS. CVaR optimization has its origins in the work of Rockafellar and Uryasev (2000) and its properties are well understood, see, e.g., Zenios (2007) for discussion and references. The CVaR optimization models we develop are single-period. We measure returns during the risk horizon by spread changes, but do not account for collected premia or payments in the case of default. This is a reasonable approximation for short horizons or when dealing with sovereigns without potential default (e.g, Germany or the US). In the conclusions we discuss extensions of the models beyond these limiting assumptions.

The expected value of the $\alpha$-tail[2] of the distribution of portfolio loss $X$, $\mathrm{CVaR}_\alpha(X)$, and its minimization formula are given in the following theorem:

**Theorem 3.1** ***Fundamental minimization formula*** (Rockafellar and Uryasev, 2002) *As a function of $\gamma \in \mathbb{R}$, the auxiliary function*

$$F_\alpha(X,\gamma) = \gamma + \frac{1}{1-\alpha}\mathbb{E}\{[X-\gamma]^+\},$$

*where $\alpha \in (0,1]$ is the confidence level and $[t]^+ = max\{0,t\}$, is finite and convex, with*

$$CVaR_\alpha(X) = \min_{\gamma\in\mathbb{R}} F_\alpha(X,\gamma).$$

Consider an investor operating in a market with $n$ risky assets with rates of return denoted by random vector $\xi$. For an investment vector $x \in \mathbb{R}^n$ of notional proportional allocations to the risky assets, the loss function is given by $f(x,\xi) = -x^\top\xi$. When dealing with portfolio optimization, loss is a function of the portfolio $x$ and we write the auxiliary function and CVaR as functions of $x$. According to Theorem 3.1 the conditional value-at-risk of the loss function is the solution of

$$\mathrm{CVaR}_\alpha(x) = \min_{\gamma\in\mathbb{R}} F_\alpha(x,\gamma), \tag{1}$$

where

$$F_\alpha(x,\gamma) = \gamma + \frac{1}{1-\alpha}\mathbb{E}\{[f(x,\xi)-\gamma]^+\}.$$

Hence, models for selecting a portfolio with minimum CVaR and a target expected return constraint can be posed as:

$$\begin{aligned} \min_{\gamma\in\mathbb{R}, x\in\mathbb{X}} \quad & F_\alpha(x,\gamma) \\ s.t. \quad & \mu^\top x \geq \bar{\mu}. \end{aligned} \tag{2}$$

$\mathbb{X}$ is the constraint set on the investment variables which specifies feasible portfolios, $\mu$ is a vector of mean returns of risky assets, and $\bar{\mu} \in \mathbb{R}$ is the target expected return.

[2] $\alpha \in (0,1]$ and all numerical experiments in this paper are carried out for $\alpha = 0.95$.

From historical data we generate an $S \times n$ matrix $R$ of return scenarios for the $n$ risky assets —see Steps 0 to 3 in Section 5— and $\mu$ is the vector of mean returns of $R$. From (2) we formulate CVaR optimization models for three investment strategies:

1. Long exposures (`L`). This is for using CDS as they were originally intended to hedge credit risk, but never, so far, employed in a portfolio context.
2. Uncovered long and short exposures (`LS`). This is the strategy followed by speculators that seek to exploit credit risk opportunities from deterioration (using long positions) or improvements (using short positions) of a sovereign's rating.
3. Covered long and short exposures (`NZ`). This is the strategy of dealers that both buy and sell CDS but do not wish to have uncovered exposures. This could be a "net zero" position with no net cash outflows.

The notion of covered and uncovered position differs depending on the context. Typically a short CDS position is covered if the investor has borrowed the share or has entered into an agreement to borrow the share, or has an arrangement with a third party that guarantees that the share can be made available. In our work we consider a position covered if, on the aggregate, there are as many long positions as there are short.

## 3.1 Long exposures

The constraint set on the investment variables stipulates that all variables are non-negative (i.e., no short sales allowed) with a proportionality condition that nominal asset allocations add up to an initial endowment of 1, i.e.,

$$\mathbb{X}_L = \{x \in \mathbb{R}^n \mid x \geq 0, \sum_{i=1}^{n} x_i = 1\}. \tag{3}$$

The CVaR portfolio optimization model is given by

$$\begin{aligned} \min_{x \in \mathbb{X}_L,\, u \in \mathbb{R}^S,\, \gamma \in \mathbb{R}} \quad & \gamma + \frac{1}{S(1-\alpha)} e^\top u \\ s.t. \quad & \\ & -Rx - u - e\gamma \leq 0 \\ & \mu^\top x \geq \bar{\mu} \\ & u \geq 0, \end{aligned} \tag{4}$$

where $e$ is a vector of all 1.

## 3.2 Uncovered long and short exposures

We introduce non-negative variables $x^+$ and $x^-$ to represent long and short positions, respectively. We assume a starting wealth of 1 unit to be invested in long positions, but the long positions can be augmented from capital raised by short sales. We set an (arbitrary) limit that no single short position can be higher than our original endowment, but overall there is no guarantee that the aggregate short positions will not be significantly higher that the original endowment. The difference between the original endowment and the aggregate short position (if negative) is a proxy for the margin that the investor needs to put down in order to sell CDS protection.

The constraint set on the investment variables is given by

$$\mathbb{X}_{LS} = \{x \in \mathbb{R}^n \mid x = x^+ - x^-,\ 0 \leq x^+,\ x^- \leq 1,\ \sum_{i=1}^{n}(x_i^+ - x_i^-) = 1\}, \tag{5}$$

and the CVaR portfolio optimization model by

$$\begin{aligned} \min_{x^+, x^- \in \mathbb{R}^n,\, u \in \mathbb{R}^S,\, \gamma \in \mathbb{R}} \quad & \gamma + \frac{1}{S(1-\alpha)} e^\top u \\ s.t. \quad & \\ & -Rx^+ + Rx^- - u - e\gamma \leq 0 \\ & \mu^\top x^+ - \mu^\top x^- \geq \bar{\mu} \\ & \sum_{i=1}^{n}(x_i^+ - x_i^-) = 1 \\ & 0 \leq x^+,\ x^- \leq 1 \\ & u \geq 0. \end{aligned} \tag{6}$$

### 3.3 Covered long and short exposures

We impose now a constraint that total short position is equal to total long position. This is a net zero position with no net cash outflow required, with the endowment of 1 unit considered as collateral. The constraint set on the investment variables is given by

$$\mathbb{X}_{NZ} = \{x \in \mathbb{R}^n \mid x = x^+ - x^-,\ 0 \leq x^+,\ x^- \leq 1,\ \sum_{i=1}^{n} x_i^+ = 1,\ \sum_{i=1}^{n} x_i^- = 1\}, \tag{7}$$

and the CVaR portfolio optimization formulation by

$$\begin{aligned} \min_{x^+, x^- \in \mathbb{R}^n,\, u \in \mathbb{R}^S,\, \gamma \in \mathbb{R}} \quad & \gamma + \frac{1}{S(1-\alpha)} e^\top u \\ s.t. \quad & \\ & -Rx^+ + Rx^- - u - e\gamma \leq 0 \\ & \mu^\top x^+ - \mu^\top x^- \geq \bar{\mu} \\ & \sum_{i=1}^{n} x_i^+ = 1,\ \sum_{i=1}^{n} x_i^- = 1 \\ & 0 \leq x^+,\ x^- \leq 1 \\ & u \geq 0. \end{aligned} \tag{8}$$

All models can be easily modified to incorporate linearly proportional transaction costs, (Zenios, 2007, pp. 80–81), and all numerical experiments are carried out with transaction cost 0.5%.

# 4 Portfolio diversification

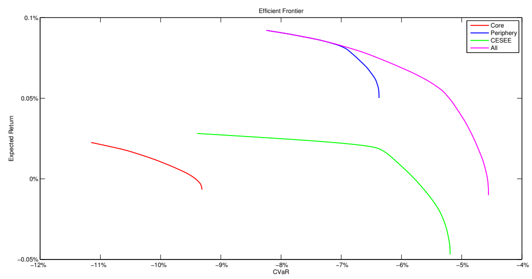
The $S$ scenarios of returns are taken to be all possible realization of historical data observed during the time period of interest. To make our empirical testing consistent with our modeling setup, Greece was excluded form the portfolio experiments since it defaulted during the testing period. We develop efficient frontiers for the whole time period and for each one of the three regimes separately. Frontiers are developed for the three investment strategies we modeled, `L`, `LS`, and `NZ`.

## 4.1 Diversification pays

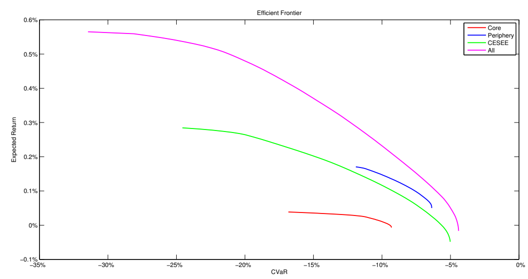
Figure 6 shows the efficient frontiers for the whole time period October 2008 to March 2016 with the three strategies. We make the following observations:

1. The efficient frontiers improve —expected returns increase and risk measures decrease— as we go from core to CESEE and/or periphery countries.
2. The efficient frontiers improve even further when considering all countries together.
3. There exists a natural zero risk strategy for the covered long-short position portfolio.

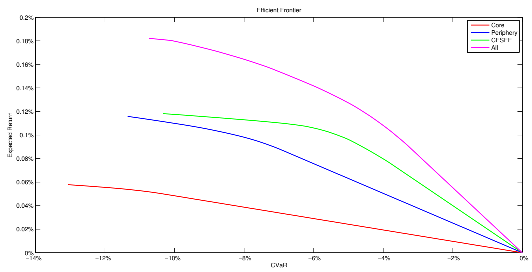
Drawing the efficient frontiers for the different strategies together in Figure 7 we observe that, naturally, position `L` is the least efficient whereas `LS` achieves the highest expected return but at higher risk. Position `NZ` dominates the other two at low risk levels, but cuts off the high expected return (and high downside potential) of the `LS` position. Overall, diversification pays. However, we noticed that the relative positions of the frontiers change with the regimes, as issue we examine next.




(a) Long exposures




(b) Uncovered long and short exposures




(c) Covered long and short exposures

Figure 6: Efficient frontiers with different investment strategies over the whole period October 2008 to March 2016.

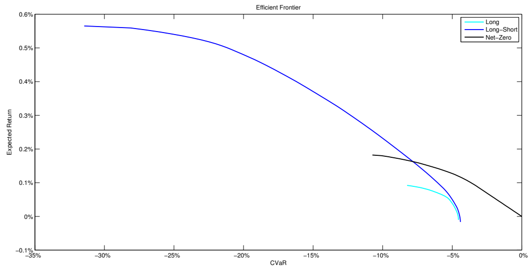



Figure 7: Comparing the efficient frontiers of the different investment strategies over the whole period October 2008 to March 2016.

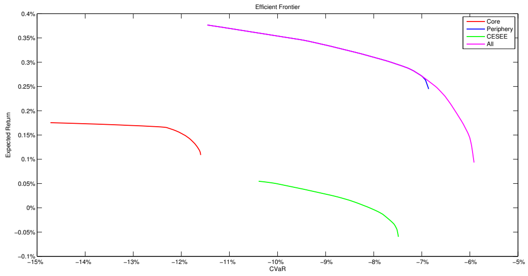
## 4.2 Diversification is regime dependent

We develop efficient frontiers for each regime separately. Figures 8–10 show how the relative position of the frontiers for each country group changes with the regime. Figure 11 summarizes the regime dependence of the frontiers for all countries. The following observations can be made from these results:

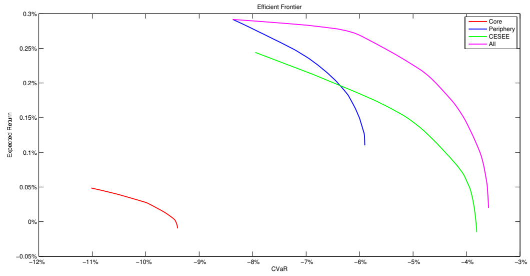
1. Diversification pays consistently in all regimes. Hence, this experiment serves as a robustness test of the "diversification pays" thesis.

2. The relative performance of the three strategies remains unaltered among regimes. The observation that `NZ` dominates at low risk levels, but cuts off the high expected return (and high risk) of the `LS` position, is robust with regime changes.

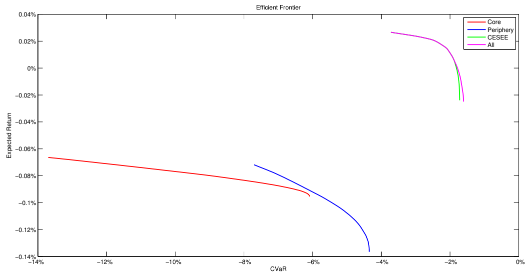
3. The relative efficiency of the different country groups changes with the regimes. For instance, before the crisis it paid to diversify from the core to CESEE and/or periphery countries for both the uncovered and covered long-short strategies. The relative advantages between CESEE and periphery were blurred during the crisis, while post-crisis the CESEE countries dominate.




(a) Regime I. Turbulent

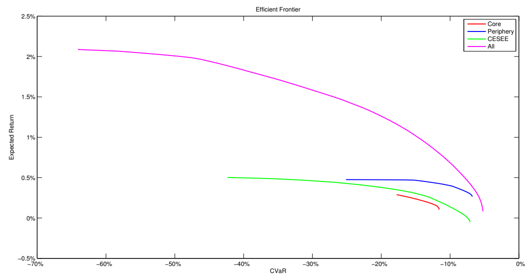



(b) Regime II. Crisis

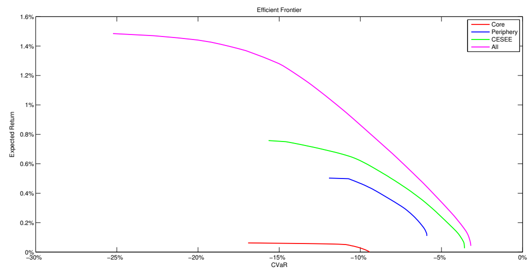



(c) Regime III. Post crisis

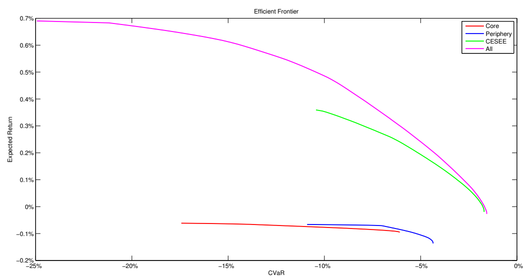
Figure 8: Regime dependent relative position of efficient frontiers for each country group for long positions.




(a) Regime I. Turbulent

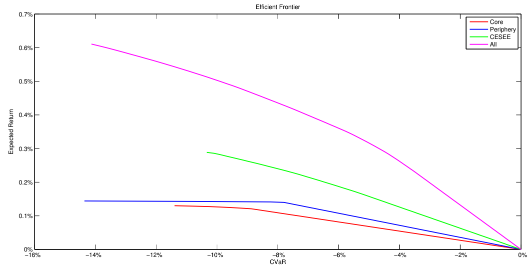



(b) Regime II. Crisis

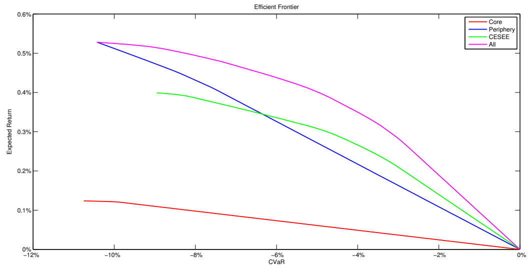



(c) Regime III. Post crisis

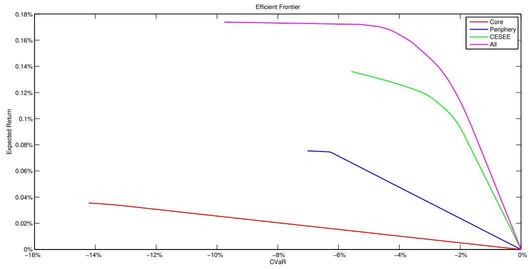
Figure 9: Regime dependent relative position of efficient frontiers for each country group for uncovered long and short positions.




(a) Regime I. Turbulent

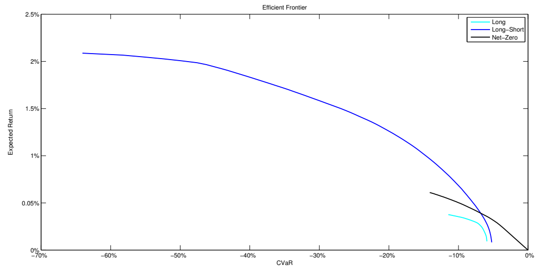



(b) Regime II. Crisis

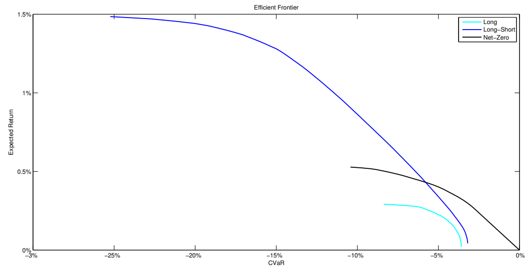



(c) Regime III. Post crisis

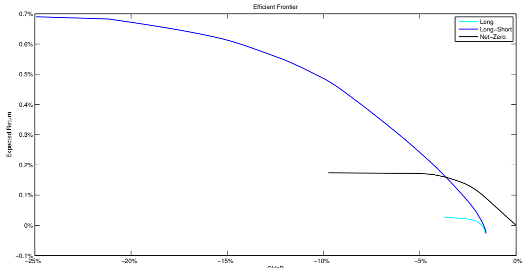
Figure 10: Regime dependent relative position of efficient frontiers for each country group for covered long and short positions.




(a) Regime I. Turbulent




(b) Regime II. Crisis




(c) Regime III. Post crisis

Figure 11: Regime dependent relative position of efficient frontiers for the different investment strategies applied to all countries in the sample.

# 5 Active portfolio management

The models are now applied for active portfolio management as follows:

Step 0. Use the first 500 days of data from 8 October 2008 to 8 September 2010 (call this date $t$) to generate the scenario return matrix $R$.

Step 1. Run the CVaR model and choose a portfolio from the efficient frontier.

Step 2. Move the clock to $t+1$, and record the portfolio return from $t$ to $t+1$.

Step 3. Update the scenario return matrix by adding the observed market returns from $t$ to $t+1$ and removing the oldest return observation, and return to Step 1.

This active management strategy is repeated for 1442 daily steps from 9 September 2010 to 18 March 2016. This five-year period covers the Eurozone crisis with the three regimes identified in Section 2. Instead of reporting returns for each period we equivalently report cumulative wealth starting with an initial endowment of 100. We pick a portfolio on the efficient frontier with increasing risk appetite: (1) the portfolio that minimizes CVaR, with expected return $R_{min}$, (2) the portfolio that has a target expected return half-way between $R_{min}$ and the maximum expected return $R_{max}$, denoted by $R_{mid}$, (3) the portfolio with target expected return $\frac{1}{2}(R_{mid}+R_{max})$, (4) the portfolio with target expected return $R_{max}$.

In Figure 12 we first provide as benchmark the S&P/ISDA Eurozone Developed Nation Sovereign CDS index uding the test period, and then show in Figure 13 results with active portfolio management using the three strategies. Also shown is the cumulative growth of investment in the risk-free rate taken to be the yield of AAA rated sovereigns with three months maturity.

Several observations can be made from the results of this experiment.

1. All strategies delivered overall positive returns and significantly higher than the AAA-rated sovereign bonds, for the whole period. Investors with average risk appetite —with target expected return $R_{mid}$— achieve overall annualized return of 5.65% (long) 10.35% (covered long-short), respectively. Investors in uncovered long and short with the same level of risk appetite suffer loses -5.56% but if their risk appetite was set higher —consistently with the followed strategy— to expected target return $\frac{1}{2}(R_{mid}+R_{max})$, they achieve returns of 5.22%. These results compare favorably with the annualized return of 0% for the benchmark AAA-rated bond and -13.54% of the index.

2. The performance of the long only and uncovered long-short positions was very volatile during this period, more so for uncovered long-short than long only.

3. Increasing the risk appetite from expected target return $R_{min}$ to $\frac{1}{2}(R_{mid}+R_{max})$ improves the performance of actively managed portfolios. However, the situation deteriorates as risk appetite increases to $R_{max}$. Hence, neither too little risk $R_{min}$ nor too much $R_{max}$ serves best the investor, and this exemplifies the need for rigorous risk management models for CDS portfolios.

4. The most encouraging finding is that the covered long and short strategy performs uniformly well. It is less volatile than the other two strategies and rides smoothly

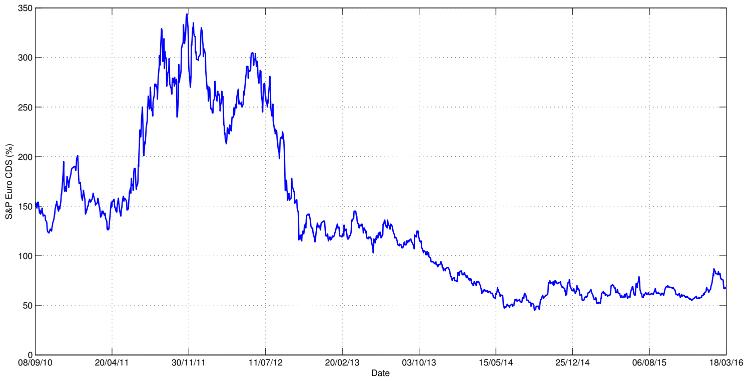



Figure 12: The S&P/ISDA Eurozone Developed Nation Sovereign CDS index.

past the big market up- and down-swings. For medium risk appetite this strategy would have delivered remarkable returns of 10.35% to 18.17%. It is only when ignoring totally the CVaR risk criterion (setting expected return to $R_{max}$) that this strategy ends up very close to zero cumulative return and finally registers loss of -6.88% at the end of the testing period.

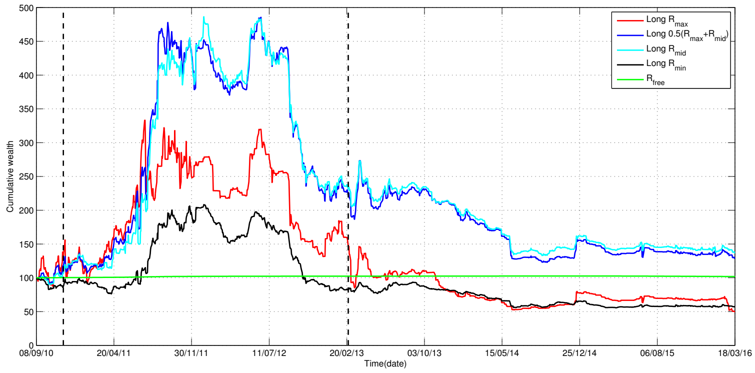
For rigorous analysis of the results we compute the Sharpe ratio of each strategy and compare it with the Sharpe ratio of the index. We compute the ratio suggested by Sharpe (1994) and the version that penalizes only downside risk suggested by Ziemba (2005). Sharpe ratios are reported on an annual basis and they are consistent with the daily ratios that we also computed but do not report. Sharpe ratios are reported in Table 4 for increasing risk appetites and they confirm the earlier observations on the relative merits of the different strategies, and the significant out-performance of the index.

# 6 Conclusions

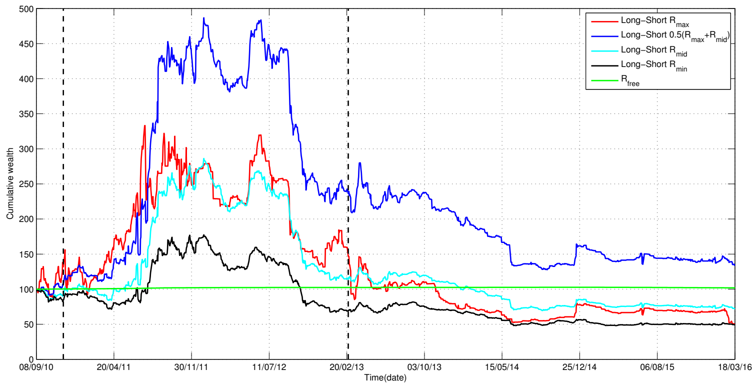
The paper has modeled several features of a CDS portfolio problem using optimization models with a risk measure (CVaR) the captures the empirically observed characteristics of CDS spread returns. Three distinct portfolio strategies were modeled and tested empirically: long positions, long and short positions, and covered long and short positions. Our empirical analysis of recent CDS spreads reveals that they are subject to regime switching and the performance of these models will depend on the regime.

The models were tested over the highly volatile period that covered the eurozone crisis and using data from core, periphery and CESEE countries. One set of tests —using a static approach of building diversified portfolios and tracing the efficient frontier— reveals that diversification pays. While the precise type of diversification is regime dependent, the observation that diversification pays is robust across regimes and for all three portfolio strategies. A second set of tests —using the models for active CDS portfolio management— establishes that, ex post, the model selected portfolios can yield results superior to the broad market index. The investor's risk appetite in selecting a portfolio has an impact on the results, but except for very conservative or extremely risky choices, the model generated portfolios consistently outperform the index.

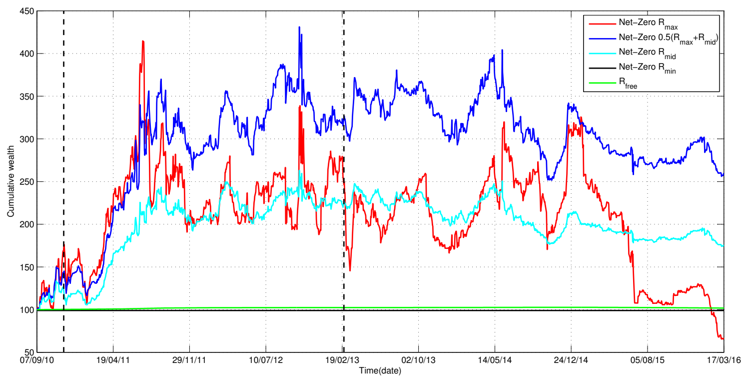
While the paper makes some novel contributions on an important problem, it also leaves several questions unanswered that deserve further research.




(a) Long exposures




(b) Uncovered long and short exposures




(c) Covered long and short exposures

Figure 13: Wealth accumulation with active portfolio management of sovereign CDS over the period September 2010 to March 2016.

| **Strategy** | **Sharpe ratio (Sharpe, 1994)** | **Sharpe ratio (Ziemba, 2005)** |
|---|---|---|
| Long exposures | | |
| Target expected return $R_{min}$ | -0.055 | -0.063 |
| Target expected return $R_{mid}$ | 0.275 | 0.922 |
| Target expected return $\frac{1}{2}(R_{mid} + R_{max})$ | 0.275 | 0.992 |
| Target expected return $R_{max}$ | 0.091 | 0.140 |
| Uncovered Long-Short exposures | | |
| Target expected return $R_{min}$ | -0.186 | -0.180 |
| Target expected return $R_{mid}$ | 0.106 | 0.173 |
| Target expected return $\frac{1}{2}(R_{mid} + R_{max})$ | 0.276 | 0.930 |
| Target expected return $R_{max}$ | 0.091 | 0.140 |
| Covered Long-Short exposures | | |
| Target expected return $R_{min}$ | -0.503 | -0.317 |
| Target expected return $R_{mid}$ | 0.306 | 1.565 |
| Target expected return $\frac{1}{2}(R_{mid} + R_{max})$ | 0.355 | 2.636 |
| Target expected return $R_{max}$ | 0.112 | 0.190 |
| Index | -0.304 | -0.251 |

Table 4: Sharpe ratios for the active management strategies and the index.

First, we model CDS-only portfolios, but it is an interesting extension with great practical significance to incorporate the underlying sovereign bond(s) in the models. Second, the models do not account for collected premia or payments in the case of default. They can be extended to overcome this simplifying assumption by using multi-period stochastic programming and explicitly modeling defaults. This is a topic for future research for which we have precedence in the multi-period stochastic programming model for corporate bonds including default, Jobst et al. (2006), and in modeling defaults on credit derivatives, Schönbucher (2003). Third, given that regime switching has an effect on the diversification strategy it is natural to introduce regime switching in the model. Diversification with regime switching requires multi-period models, and a stochastic process to model regime switching.

# References


P. Augustin, M.G. Subrahmanyam, D. Y. Tang, and S. Q. Wang. Credit default swaps: a survey. *Foundations and Trends in Finance*, 9(1–2):1–196, 2014.

P. Augustin, M.G. Subrahmanyam, D. Y. Tang, and S. Q. Wang. Credit default swaps: Past, present, and future. *Annual Review of Financial Economics*, (in print), 2016.

J. Bai and P. Perron. Evaluating and testing linear models with multiple structural changes. *Eonometrica*, 66(1):47–78, 1998.

S. Criado, L. Degabriel, M. Lewandowska, S. Linden, and P. Ritter. Sovereign CDS report. Official report DG COMP, DG ECFIN and DG MARKT, European Commission, Brussels, 2010.

European Union. Regulation (EU) of the European Parliament and the Council on short selling and certain aspects of credit default swaps. Regulation EU 236/2012, Offical Journal of the European Union, 2012.

F. Fabozzi, R. Giacometti, and N. Tsuchida. Factor decomposition of the eurozone sovereign CDS spreads. *Journal of International Money and Finance*, (in print), 2016.

IMF. *A New Look at the Role of Sovereign Credit Default Swaps*, volume Global Financial Stability Report, chapter 2, pages 57–92. International Monetary Fund, Washington, DC., April 2013.

N. Jobst, G. Mitra, and S.A. Zenios. Integrating market and credit risk: A simulation and optimization perspective. *Journal of Banking and Finance*, 30(2):645–667, 2006.

F. Packer and C. Suthiphongchai. Sovereign credit default swaps. *BIS Quarterly Review*, pages 79–88, December 2003.

Z. Qu and P. Perron. Estimating and Testing Structural Changes in Multivariate Regressions. *Econometrica*, 75(March):459–502, March 2007.

R. Rockafellar and S. Uryasev. Conditional Value-at-Risk for general loss distributions. *Journal of Banking and Finance*, 26:1443–1471, 2002.

R.T. Rockafellar and S. Uryasev. Optimization of conditional Value–at–Risk. *Journal of Risk*, 2(3):21–41, 2000.

P. Schönbucher. *Credit Derivatives Pricing Models: Models, Pricing and Implementation.* Wiley Finance, New York, NY, 2003.

W.F. Sharpe. The Sharpe ratio. *The Journal of Portfolio Management*, pages 49–58, Fall 1994.

R. Stulz. Credit default swaps and the credit crisis. *Journal of Economic Perspectives*, 24(1):73–92, 2010.

S.A. Zenios. *Practical Financial Optimization. Decision making for financial engineers.* Blackwell, Malden, MA, 2007.

W. T. Ziemba. The symmetric downside risk Sharpe ratio. *Journal of Portfolio Management*, 32(1):108–122, 2005.