ENDOGENOUS LIQUIDITY AND CAPITAL REALLOCATION*

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Abstract

This paper studies economies where firms acquire capital in primary markets, then, after idiosyncratic productivity shocks, retrade it in secondary markets that can incorporate bilateral trade with search, bargaining and liquidity frictions. We distinguish between full or partial sales (one firm gets all or some of the other's capital), and document several long- and short-run empirical patterns between these variables and the cost of liquidity measured by inflation. Quantitatively, the model can match these patterns plus the standard facts from business cycle theory. We also investigate the impact of search frictions, monetary and fiscal policy, and persistence in firm-specific shocks.

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1 Introduction

This paper studies economies where firms first acquire capital in centralized primary markets, as in standard growth theory, then, after idiosyncratic productivity shocks, retrade it in decentralized secondary markets. In the interest of realism and generality, the secondary markets can involve bilateral trade with search, matching, bargaining and liquidity frictions, although in special cases each of these can be shut down. A novel feature is that we distinguish between full sales, where the buyer gets all the seller's capital, and partial sales, where the buyer only gets some of it. Under CRS (constant returns to scale) it is efficient for firms with higher productivity to get all the capital in bilateral trade, but that may not happen in equilibrium, depending on financial considerations.

We first document some facts. Over the business cycle, the ratio of full sales to total capital expenditure, defined as new investment plus reallocation, is procyclical, while the ratio of partial sales to total capital expenditure is countercyclical. In the longer run, the ratio of full sales to total capital expenditure has increased and the ratio of partial sales to total capital expenditure has decreased. Given that 42% of full sales are facilitated by cash or cash-equivalent payments (Thomson Reuters M&A Database, 1971-2018), we examine the relationship between reallocation and the cost of liquidity, measured by inflation, as discussed below. In the longer run, full sales decrease while partial sales increase with inflation, while at cyclic frequencies the pattern is reversed.

Our theory is that high inflation raises the cost and lowers the amount of liquidity, decreasing total reallocation and full sales while increasing partial sales, consistent with the long-run evidence. Then, to get full sales increasing and partial sales decreasing with inflation at business cycle frequencies, we incorporate credit shocks. Easier credit increases total reallocation and full sales, decreases partial sales, and reduces money demand, leading to a short-term jump in inflation. With credit shocks at business cycle frequencies, total reallocation is procyclical and moves with inflation, while partial sales are countercyclical. The idea is not that credit shocks are necessarily more transitory, but that they affect the price level, implying a change in short run but not trend inflation.

While a main goal is to show the calibrated model is consistent with all these facts in the tradition of the RBC (real business cycle) literature, it can also be used to study monetary and fiscal policy. We solve for the optimal capital tax/subsidy and inflation rate, which depend on details like bargaining power not usually considered in related studies. In particular, inflation may have a nonmonotone impact on investment, output and other macro variables, and different from many models, the Friedman rule may not be optimal. We also study how search frictions and persistence in shocks matter for this.

To motivate an interest in capital reallocation, in general, economic performance requires not only getting the right amount of investment over time, but getting capital into the hands of those best able to use it at any point in time. With idiosyncratic shocks, capital should flow from lower- to higher-productivity firms (Maksimovic and Phillips 2001; Andrade et al. 2001; Schoar 2002). The ease with which capital can be retraded on secondary markets affects investment in primary markets, and vice versa, as is true for many assets (Harrison and Kreps 1978; Lagos and Zhang 2020). However, the channel is subtle: a well-functioning secondary market encourages primary investment since, if firms have more capital than they need, it is relatively easy to sell in that market, but it also discourages primary investment since, if firms want more capital than they have, it is relatively easy to buy in that market. We analyze how the net effect depends on various factors, including bargaining power and monetary policy.

Also, reallocation is sizable, with purchases of used capital reported to be between 25% and 33% of total investment (Eisfeldt and Rampini 2006; Cao and Shi 2016; Dong et al. 2016; Cui 2017; Eisfeldt and Shi 2018), which is probably an underestimate since the data ignore small firms and those that are not publicly traded, neglect mergers, and include purchases but not rentals. Studies also document several stylized facts: reallocation is procyclical while capital mismatch is countercyclical (Eisfeldt and Rampini 2006; Cao and Shi 2016); productivity dispersion is countercyclical (Kehrig 2015); the price of used capital is procyclical (Lanteri 2016); and the ratio of spending on used capital to total investment is procyclical (Cui 2017). Our goal is to match all of these facts.

As for *frictional* reallocation, many argue secondary capital markets are far from the perfectly competitive ideal (Gavazza 2010, 2011*a,b*; Kurman 2014; Ottonello 2015; Kurmann and Rabinovitz 2018; Horner 2018; Li and Whited 2015; Bierdel et al. 2021). Imperfections include financial constraints, difficulties in finding counterparties, holdup problems, and asymmetric information. Our secondary market has bilateral trade and bargaining, as in search theory.[1] It also has assets facilitating payments, as in monetary economics. While explicit modeling of this is missing from most work on capital reallocation, some studies (e.g., Buera et al. 2011; Moll 2014) argue that liquidity frictions are important, even if self financing mitigates the problem, which is just what we model.

[1] In models of capital, Ottonello (2015) finds search helps fit the facts and generates more interesting propagation. Horner (2018) shows vacancy rates for commerical real estate resemble unemployment data, and finds disperse rents on structures, similar to wage dispersion, suggesting search may be as relevant for that kind of capital as it is for labor. In aircraft markets, which have received much attention, Pulvino (1998), Gilligan (2004) and Gavazza (2011*a,b*) show used sales are thrice new sales, and Gavazza (2011*a*) shows prices vary inversely with search, while market thickness affects trading frequency, average utilization, utilization dispersion, average price and price dispersion. Also emphasized is *specificity* – capital is often customized, making it nontrivial to find the right trading partner. This all suggests search may be important.

To say more about our approach to liquidity, we use the label "money" but do not mean currency per se: it can include any asset that is widely accepted as a payment instrument, or can be converted into something that is widely accepted with little cost or delay. In reality there is a spectrum of assets with varying degrees of acceptability and return, implying a tradeoff between these attributes, and research on the foundations of monetary theory analyzes this explicitly. Kiyotaki and Wright (1989), e.g., formalizes the tradeoff, but in a way that is far too stylized for this paper, which is a study in macroeconomics.

The essence of macro is aggregation: standard models have just two uses of output, consumption or investment, and two uses of time, labor or leisure (with exceptions, like home production models with three uses for output and three uses for time). Similarly, our benchmark model has just two assets: money and capital. In reality, while cash may be the most liquid asset, there are substitutes. Hence we incorporate banking, following Berentsen et al. (2007), and define money as currency plus checkable deposits in the quantitative work. In principle other assets also provide liquidity, but as inflation lowers the return on the most liquid asset, cash, in equilibrium that can affect the return on other liquid assets and aggregate liquidity.

As Wallace (1980) says: "[inflation] is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on [inflation]... the higher the [inflation rate] the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income... Many economists seem to ignore this aspect of inflation because of their unfounded attachment to Irving Fisher's theory of nominal interest rates... [and that] accounts for why economists seem to have a hard time describing the distortions created by anticipated inflation."

Evidence for this point is presented in Fig. 1 (all figures are at the end, while Appendix A provides details on our data sources). It shows that various real rates of return, including those on transactions deposits, public and private bonds of different maturity and quality, and housing loans, are strongly negatively correlated with inflation. The message is clear: putting your money into a checking account, bonds, etc. does not completely avoid the inflation tax. Ignoring this message would be like saying "I don't care how they tax taxis – for all I care, they can drive them out of existence – because I use Uber." To the extent that taxis and Uber are substitutes for transportation, a tax on taxis gets passed through to Uber fares. To the extent that checking accounts, bonds, etc. are substitutes for cash in transactions, a tax on currency gets passed through to the cost of holding other liquid assets. Hence, inflation can capture the cost of liquidity, even in multiple-means-of-payment economies.

An explicit multiple-means-of-payment model is presented in Section 6, where liquid real assets act as a substitute for currency, and it is shown that higher inflation lowers the liquidity embodied in cash, so agents want to substitute into real assets (a kind of Mundell-Tobin effect), which increases the price and liquidity of real assets, but on net total liquidity falls. While that works well in theory, models with multiple assets having different liquidity and return are harder to take to data, so the benchmark model in Sections 3-5 has only money and capital.

The paper is organized as follows. Section 2 presents evidence. Sections 3 and 4 develop the theory and show it is tractable enough to deliver strong results on existence, uniqueness and comparative statics. Section 5 provides the main quantitative results. Section 6 discusses extensions. Section 7 concludes.[2]

2 Evidence

2.1 Macro Data

Here we use US data from 1971 to 2018. Appendix A has more detail, but capital reallocation is from COMPUSTAT, which has information on full and partial sales, measured respectively by acquisitions and sales of PPE (property, plant and equipment), plus total capital expenditures. Firm data are summed to get aggregate series. Reallocation is defined as full plus partial sales. We focus on the reallocation-to-expenditure ratio, the *R share*, and the partial-sales-to-reallocation ratio, the *P share*, capturing the importance of reallocation in investment, and the importance of partial sales in reallocation. In the early part of the sample the R share varies a lot, it but stabilizes after 1984, fluctuating around 32%. Similarly, the P share stabilizes after 1984, fluctuating around 24%.[3]

As discussed above, we entertain the possibility that liquidity plays a role, and use inflation to measure its cost, although we also tried nominal T-bill and AAA corporate

[2]When a buyer gets all a seller's capital, it resembles M&A (merger and acquisition) activity, but we are not trying to contribute to the M&A literature in finance (see Andrade et al. 2001 and Betton et al. 2008 for surveys). That involves issues of executive compensation, management strategy, tax implications etc. and ultimately connects to Coase's (1937) question: what's a firm? We are more in the macro tradition, like Jovanovic and Rousseau (2002), Jermann and Quadrini (2012), Del Negro et al. (2017) or Khan and Thomas (2013, 2017). Other work on reallocation includes Ramey and Shapiro (1998,2001), Hsieh and Klenow (2009), Asker et al. (2014), Midrigan and Xu (2014), Cooper and Schott (2016), Ai et al. (2016), David et al. (2016), Lanteri and Gavazza (2019), David and Venkateswaran (2019), and Wright et al. (2018, 2020). Related papers on money and capital include Shi (1999a,b), Shi and Wang (2006), Aruoba and Wright (2003), Aruoba et al. (2011), and of course classics like Sidrauski (1967) and Tobin (1965).

[3]Since the R and P shares are more or less stationary after 1984, for calibration we start in 1984, but here, for establishing different types of evidence, we go back to 1971. This does not affect the message.

Table 1: Business Cycle Statisticss

	Debt	R share	P share	Investment	Inflation	Output
Debt	1	0.53	-0.44	0.62	0.42	0.63
R share	-	1	-0.76	0.62	0.37	0.61
P share	-	-	1	-0.51	-0.28	-0.52
Investment	-	-	-	1	0.43	0.96
Inflation	-	-	-	-	1	0.33
Output	-	-	-	-	-	1
Relative SD.	1.41	6.45	9.46	2.56	0.89	1

Note: output is private output defined as the sum of consumption and investment. Its standard deviation is 1.73%.

bond rates, and the results are similar. Fig. 2 shows the R and P shares vs inflation, with different panels using the raw data, the trend and the cyclical component, after band-pass filtering, following Christiano and Fitzgerald (2003).

In the longer run, when inflation is high firms spend less on used capital relative to total investment, while within reallocation there are more partial sales. Given full sales are about two times partial sales, when inflation rises the fall in reallocation is mainly driven by full sales. Of course, other secular changes may affect reallocation, and the fall in inflation since 1980s may or may not have resulted from monetary policy, but in any case lower inflation is associated with more full and fewer partial sales in the longer run, while at business cycle frequencies the relationships are reversed. A plausible explanation involves credit conditions at business cycle frequencies: easier credit leads to more full and fewer partial sales, plus it reduces money demand, which raises the price level and that shows up as inflation in the short run.

We pursue this using aggregate firm debt as a proxy for credit conditions. Fig. 3 shows the cyclical components of debt, investment, the R and P shares, and output. Debt and the R share are procyclical, and the P share countercyclical. So when credit conditions ease, debt goes up, part of which funds reallocation, explaining why full sales rise, partial sales fall and total reallocation rises, and notice reallocation must be more volatile than investment to get a procyclical R share. Table 1 shows investment and reallocation positively comove, and inflation is procyclical. All of this is consistent with the intuitive discussion in the Introduction.[4]

[4] A referee points out that both full and partial sales are procyclical in Eisfeldt's data that can be found at https://sites.google.com/site/andrealeisfeldt/home/capital-reallocation-and-liquidity?authuser=0 and suggests the difference comes from the denominator of the reallocation rate, total assets or capital expenditure. Our choice follows Cui (2022), who discusses its advantages. In particular, since much of the literature focuses in terms of first moments on reallocations as a fraction of total capital expenditure, for consistency we use the same variable for cyclical facts.

2.2 Micro Data

Disaggregated COMPUSTAT data can be used to present two pieces of micro evidence. First, we show money holdings have a positive effect on full purchases and a negative effect on partial purchases. (In terms of labels, we usually use full and partial *sales*, but when the focus is on the firm getting capital it seems better to use full and partial *purchases*.) Second, we examine how inflation impacts firms' money holdings.

To begin, we regress full purchases on firms' liquidity at the end of the last period, measured by holdings of cash plus cash equivalent, including assets readily convertible into cash like certificates of deposit, banker's acceptances, T bills, and commercial paper. We examine both extensive and intensive margins of full purchases. In the first approach, the LHS is a binary variable that equals 1 if a firm engages in a full purchase this year and 0 otherwise, capturing the extensive margin. We use a linear probability model (a logistic model gives similar results). The second approach is to examine full purchase expenditure. For this we take logs and focus on firms engaging in a full purchase in a given year, capturing the intensive margin. For our purposes, it does not matter whether money holdings cause full purchases or anticipations of full purchases cause money holdings – both say that cash facilitates reallocation.

We control for factors that may affect purchases and money holdings, like earnings before interest and taxes (EBIT), total assets, the leverage ratio measured by short-term liabilities over shareholder equity (SEQ) and the relative size of a firm measured by its assets divided by the industry average. Independent variables are lagged one period to reduce simultaneity problems. We include year or year-industry fixed effects (FE) defined by the first two digits of SIC codes. Total assets are normalized by the nominal price level. Variables other than the leverage ratio and relative size are in logs and normalized by total firm assets, but results are similar if normalized by the nominal price level.

In Table 2, the first three columns give results on the probability of a full purchase. The first column includes only firm FE, the second includes firm and year FE and the third includes firm and year-industry FE. In all cases money holdings have a significant positive effect, with a 1% increase in cash raising the probability of a full purchase by 0.00018 in levels. As the average probability of a full sale is around 0.21, this means a 1% increase in cash increases the full sale probability by about 0.1%. EBIT has significant positive effects on full purchases. Total assets have a positive while leverage ratios have a negative effect.

The last three columns of Table 2 report results on full purchase spending, with a 1% increase in money leading to a 0.2% rise in spending, so a 1 dollar increase in money leads

Table 2: Full Sales and Money Holdings

	Prob	Prob	Prob	Spending	Spending	Spending
Money Holding	0.018***	0.019***	0.018***	0.190***	0.199***	0.202***
	(0.001)	(0.001)	(0.001)	(0.012)	(0.012)	(0.013)
EBIT	0.024***	0.027***	0.028***	0.270***	0.274***	0.266***
	(0.002)	(0.002)	(0.002)	(0.019)	(0.019)	(0.020)
Asset	0.087***	0.066***	0.069***	-0.322***	-0.390***	-0.381***
	(0.003)	(0.003)	(0.003)	(0.018)	(0.022)	(0.025)
Leverage	-0.001**	-0.001**	-0.001*	-0.005	-0.005	-0.005*
	(0.000)	(0.000)	(0.000)	(0.003)	(0.003)	(0.003)
Firm FE	Y	Y	Y	Y	Y	Y
Year FE		Y			Y	
Year-Industry FE			Y			Y
Adj. R^2	0.029	0.039	0.053	0.049	0.075	0.112
# observations	115822	115822	115822	33678	33678	33678

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Robust standard errors are in brackets and are clustered at firm level. All independent variables are lagged by one period. Acquisition spending, money holdings and EBIT are normalized by firms' total assets.

to an 8 cent increase, which is sizeable. Again EBIT has a positive effect on full purchase spending. We also ran dynamic panel regressions to account for the possibility that full purchases are persistent; the results are similar, with coefficients on lagged purchases that are small and insignificant. This all indicates liquidity, measured by cash or cash equivalent, encourages full purchases.[5]

Now consider partial purchases. While in COMPUSTAT we cannot identify the buyer in each purchase, and hence their cash holdings, we can aggregate data to the industry level defined by the first two digits of SIC, or to the state level and investigate how cash held in an industry or in a state affects partial sales. The former aggregation would be informative about firm-level purchases if they buy mostly from firms in the same industry, the latter would if they buy mostly from firms in the same state.

Table 3 shows the results from regressing P share on cash holding at industry and state levels. P share, money held and EBIT are in logs and the latter two are normalized by total assets. Assets are in logs and normalized by the nominal price level. All the independent variables are values at the end of the last period. The first two columns report

[5]While we not trying to contribute directly to the M&A literature, the findings are consistent with empirical work in that area. Harford (2005) suggests capital liquidity drives both M/B (market to book) ratios and merger waves, finding that waves are preceded by high capital liquidity, and that including capital liquidity eliminates the power of M/B to predict waves. Harford (1999) shows firms with more cash reserves are more prone to acquire others. Andrade et al. (2008) report that in 1996-2000: 26% of M&A bids are all cash; 37% are all stock; 37% are a mix of securities; and the probability of the deal going through is higher if payment is in cash. The point is not that this is surprising, but that it is consistent with our approach.

Table 3: Money Holdings on P Share

	Industry Level		State Level	
	P Share	P Share	P Share	P Share
Cash Holding	-0.205	-0.008	-0.231**	-0.197*
	(0.125)	(0.101)	(0.095)	(0.114)
EBIT	-0.219**	-0.237**	-0.074	- 0.182*
	(0.097)	(0.073)	(0.087)	(0.087)
Asset	-0.466***	-0.054	-0.492***	-0.192
	(0.075)	(0.086)	(0.099)	(0.125)
Leverage	0.082	0.057	-0.102	-0.211
	(0.079)	(0.013)	(0.151)	(0.205)
Year FE		Y		Y
Adj. R^2	0.271	0.392	0.247	0.313
# observations	2687	2687	2237	2237

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Robust standard errors are in brackets and are clustered at industry or state level.

results at the industry level while the last two at the state level. We include industry FE in the industry-level regressions and state FE in the state-level regressions. The coefficients on cash holding are negative in all specifications and significant in the state-level regressions. These results suggest that as cash holdings increase firms tend to shift from partial to full purchases.

Next consider how money holdings depend on liquidity costs measured by CPI inflation. The results are in Table 4, with all regressions controlling for firm FE. The first column indicates that a 1% increase in inflation reduces money holdings by about 1.268%. Since inflation decreases and money holdings increase over time in the sample, negative coefficients on inflation may result from trends. To address this, we use a band-pass filter to remove the trend component of inflation, and the second column in Table 4 reports results using the cyclical component. The coefficient on inflation remains negative and highly significant, and the magnitude is much larger.

To address the concern that cyclical co-movement may drive the results, we exploit cross-sectional variation in inflation rates, by noticing that COMPUSTAT has addresses of firms. Assuming firms care about local inflation, we regress money holdings on state-level inflation from Hazell et al. (2022), including year or year-industry FE. The results in the last two columns show the coefficients on state-level inflation are again negative and highly significant, and slightly larger than the first column. Hence, like the macro data, the micro evidence is consistent with our approach, and motivates developing models with full and partial sales, taking into account credit and monetary considerations.

Table 4: Money Holdings and Liquidity Cost

Money Holding				
Inflation	-1.268***	-	-	-
	(0.170)	-	-	-
Inflation (Cycle)	-	-2.706***	-	-
	-	(0.160)	-	-
State-Level Inflation	-	-	-1.882***	-1.550***
	-	-	(0.525)	(0.535)
EBIT	0.122***	0.119***	0.107***	0.106***
	(0.005)	(0.005)	(0.007)	(0.007)
Asset	-0.091***	-0.085***	-0.087***	-0.085***
	(0.005)	(0.006)	(0.013)	(0.013)
Leverage	-0.000***	-0.000***	-0.001*	-0.001**
	(0.000)	(0.000)	(0.001)	(0.001)
Capital Exp.	-0.102***	-0.105***	-0.088***	-0.080***
	(0.005)	(0.006)	(0.008)	(0.008)
Firm FE	Y	Y	Y	Y
Year FE			Y	
Year-Industry FE				Y
Adj. R^2	0.013	0.015	0.029	0.056
# observations	142303	142303	92078	92078

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Robust standard errors are in brackets and are clustered at firm level. Money holdings, EBIT, capital expenditures are normalized by total assets.

3 Model

We build on the alternating-market structure in Lagos and Wright (2005): each period in discrete time, a continuum of infinite-lived agents interact in a frictional decentralized market, or DM, and then a frictionless centralized market, or CM. This is ideal for our purposes because in a stylized way the CM and DM correspond to primary and secondary trade. The framework also features an asynchronization of expenditures and receipts – reallocation occurs in the DM while profit accrues in the CM – crucial to any analysis of money or credit. As well, it has proved tractable in many other applications, and flexible in the sense that it allows various specifications for search, price determination, etc. (see Lagos et al. 2017 and Rocheteau and Nosal 2017 for surveys).

In the CM, households consume a numeraire good c, supply labor hours h, settle debt d and adjust their portfolios of capital k and money m, while firms produce using h and k. In the DM, (the owners of) firms meet bilaterally and potentially retrade k after observing idiosyncratic productivity shocks. Agents discount between the CM and the next DM using $\beta \in (0,1)$, but not between the DM and CM, without loss of generality. Normalizing the time endowment per period to 1, utility over consumption and leisure is $U(c, 1-h) = u(c) + \xi(1-h)$, where $\xi > 0$ is a parameter, and $u'(c) > 0 > u''(c)$.

This quasi-linearity specification enhances tractability, as described in Lemma 1 and 2 below, but Wong (2016) shows those results also hold for any $U(c, 1-h)$ with $U_{11}U_{22} = U_{12}^2$. Alternatively, Rocheteau et al. (2008) show the results hold for any $U(c, 1-h)$ if labor is indivisible, $h \in \{0, 1\}$, and agents trade employment lotteries as in Rogerson (1988). This is relevant for comparing our results on business cycles to those from a canonical RBC model, which we take to be Hansen (1985), as it jettisons many of the bells and whistles in Kydland and Prescott (1982) without sacrificing results, then improves on performance by using indivisible labor. A special case of our setup, with no idiosyncratic shocks, is exactly Hansen's indivisible-labor model.[6]

For ease of presentation, it is assumed that households own their own firms; while they could hold shares in other firms, given Lemma 1 below, they do not need to. Each has a CRS production function $f(k,h) = (A\varepsilon k)^{1-\eta} h^{\eta}$, where ε is idiosyncratic productivity and A is aggregate productivity, with the latter assumed constant until we get to the quantitative work in Section 5. The firm-specific ε has a time-invariant distribution $F(\varepsilon)$ that can be persistent: using subscript $+$ for next period, ε_+ is drawn from a conditional CDF $Q(\cdot|\varepsilon)$. As usual in growth theory, investment in k at t is productive at $t+1$, and it depreciates at rate δ.

Given a real wage w, a firm with (k, ε) in the CM chooses labor demand $\tilde{h}$ (distinct from its owner's labor supply) to maximize profit,

$$\Pi(k, \varepsilon) = \max_{\tilde{h}} \{(A\varepsilon k)^{1-\eta} \tilde{h}^{\eta} - w\tilde{h}\}.$$

The solution is $\tilde{h}(k, \varepsilon) = (\eta/w)^{\frac{1}{1-\eta}} A\varepsilon k$, and this implies $\Pi(k, \varepsilon) = B(w)\varepsilon k$ where

$$B(w) \equiv \left(\frac{\eta}{w}\right)^{\frac{\eta}{1-\eta}} (1-\eta) A. \tag{1}$$

Hence Π is linear in εk, which implies in what follows that efficient DM reallocation entails full sales: a high ε firm should get all the capital of a low ε firm.

[6] One implication is that, as in Hansen (1985) and Rogerson (1988), in the aggregate $1 - H$ can be interpreted as unemployment, or at least nonemployment, as opposed to the leisure of every agent: it is the measure of those with $h = 0$. Also note that Cooley and Hansen (1989) provide a monetary version of Hansen (1985), but that is not comparable because there households use cash to buy goods, due to a cash-in-advance constraint, while here firms use it to buy capital, and there is no such constraint, i.e., they can also use credit, but may choose to top that up with cash. In any case, with no idiosyncratic firm shocks, money is not valued, and the setup reduces to Hansen, not Cooley-Hansen.

Let ϕ be the price of money m in terms of numeraire c, the inverse nominal price level, so that real balances are $z = \phi m$ and gross inflation is $1 + \pi = \phi/\phi_+$. Let the CM and DM value functions be W and V. Then

$$W(\Omega, \varepsilon) = \max_{c,h,\hat{k},\hat{m}} \left\{ u(c) + \xi(1-h) + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} V_+(\hat{k}, \hat{z}, \hat{\varepsilon}) \right\} \tag{2}$$

$$\text{st } c = \Omega + (1-\tau_h)wh - \phi\hat{m} - \hat{k}, \tag{3}$$

where Ω is wealth, k and z are capital and real balances at the start of the CM while $\hat{k}$ and $\hat{z} = \phi_+ \hat{m}$ are capital and real balances at the end, τ_h is a labor income tax, and $\mathbb{E}_{\hat{\varepsilon}|\varepsilon}$ denotes the expectation wrt $\hat{\varepsilon}$ conditional on ε. Note the cost of real balances next period in terms of current c is inflation. Wealth is $\Omega = (1-\tau_k)B(w)\varepsilon k + (1-\delta)k + z - d - T$, where τ_k is a capital income tax, T a lump-sum tax, and d debt from the previous DM. Using the budget equation to eliminate h, (2) becomes

$$\begin{aligned} W(\Omega, \varepsilon) = {} & \xi + \frac{\xi\Omega}{(1-\tau_h)w} + \max_c \left\{ u(c) - \frac{\xi c}{(1-\tau_h)w} \right\} \\ & + \max_{\hat{k},\hat{m}} \left\{ -\frac{\xi\left(\phi\hat{m} + \hat{k}\right)}{(1-\tau_h)w} + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} V_+\left(\hat{k}, \hat{z}, \hat{\varepsilon}\right) \right\}. \end{aligned} \tag{4}$$

Because A is assumed to be constant for now, choosing $\hat{m}$ is equivalent to choosing $\hat{z}$. We then focus on the later. From (4) the following results are immediate:

Lemma 1 *$W(\Omega, \varepsilon)$ is linear in Ω with slope $\xi/[(1-\tau_h)w]$.*

Lemma 2 *An interior solution for $(\hat{k}, \hat{z})$ solves:*

$$\frac{\xi}{(1-\tau_h)w} = \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{k}, \hat{z}, \hat{\varepsilon})}{\partial \hat{k}} \tag{5}$$

$$\frac{\xi}{(1-\tau_h)w} \frac{\phi}{\phi_+} = \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{k}, \hat{z}, \hat{\varepsilon})}{\partial \hat{z}}. \tag{6}$$

This means $(\hat{k}, \hat{z})$ is the same for all agents with the same ε, although agents with different ε choose different $(\hat{k}, \hat{z})$, unless ε is i.i.d. in which case $(\hat{k}, \hat{z})$ is the same for all agents.

To complete the CM problem, let $\hat{z}(\varepsilon)$ and $\hat{k}(\varepsilon)$ solve (5)-(6), and note from (4) that c solves $u'(c) = \xi/[(1-\tau_h)w]$. Then the budget equation gives labor supply,

$$h(\Omega, \varepsilon) = \frac{c + \hat{k}(\varepsilon) + \hat{z}(\varepsilon)\phi/\phi_+ - \Omega}{(1-\tau_h)w}. \tag{7}$$

If Γ is the distribution of (k, z, ε) at the start of a period, its law of motion is

$$\Gamma_{+}(k, z, \varepsilon) = \int_{\hat{k}(x)\leq k, \hat{z}(x)\leq z} Q(\varepsilon|x)\, dF(x). \tag{8}$$

Without aggregate shocks, agents move around in the distribution but the cross section is constant. Even with aggregate shocks, tractability is preserved since $(\hat{k}, \hat{z})$ depends on ε, but not past DM trades.

Now consider the DM, where with probability α each firm (owner) is randomly matched to a potential trading partner. Similar to stories motivating frictional labor markets, $\alpha < 1$ can mean it is hard to find anyone – a pure search problem – or it is hard to find the right type – a matching problem. In a meeting, the state variables of the pair are $\mathrm{s} = (k, z, \varepsilon)$ and $\tilde{\mathrm{s}} = (\tilde{k}, \tilde{z}, \tilde{\varepsilon})$. When $\varepsilon > \tilde{\varepsilon}$, the ε firm is a *buyer* and the $\tilde{\varepsilon}$ firm is a *seller*, since the former should get some quantity $q(\mathrm{s}, \tilde{\mathrm{s}})$ of capital from the latter. Let $p(\mathrm{s}, \tilde{\mathrm{s}})$ be the cash payment by the buyer, and $d(\mathrm{s}, \tilde{\mathrm{s}})$ the value of any debt, a promise of payment in the next CM, as discussed more below. Then

$$V(k, z, \varepsilon) = W(\Omega, \varepsilon) + \alpha \int_{\varepsilon > \tilde{\varepsilon}} S^{b}(\mathrm{s}, \tilde{\mathrm{s}}) d\Gamma(\tilde{\mathrm{s}}) + \alpha \int_{\varepsilon < \tilde{\varepsilon}} S^{s}(\tilde{\mathrm{s}}, \mathrm{s}) d\Gamma(\tilde{\mathrm{s}}), \tag{9}$$

where $S^{b}(\cdot)$ and $S^{s}(\cdot)$ are buyer and seller surpluses, which by Lemma 1 are

$$S^{b}(\mathrm{s}, \tilde{\mathrm{s}}) = \frac{\xi \left\{ \left[(1-\tau_k)\, \varepsilon B(w) + 1 - \delta \right] q(\mathrm{s}, \tilde{\mathrm{s}}) - p(\mathrm{s}, \tilde{\mathrm{s}}) - d(\mathrm{s}, \tilde{\mathrm{s}}) \right\}}{w(1-\tau_h)}; \tag{10}$$

$$S^{s}(\tilde{\mathrm{s}}, \mathrm{s}) = \frac{\xi \left\{ p(\tilde{\mathrm{s}}, \mathrm{s}) + d(\tilde{\mathrm{s}}, \mathrm{s}) - \left[(1-\tau_k)\, \tilde{\varepsilon} B(w) + 1 - \delta \right] q(\tilde{\mathrm{s}}, \mathrm{s}) \right\}}{w(1-\tau_h)}. \tag{11}$$

We distinguish between two types of reallocation, a full sale $q(\mathrm{s}, \tilde{\mathrm{s}}) = \tilde{k}$, and a partial sale $q(\mathrm{s}, \tilde{\mathrm{s}}) \in (0, \tilde{k})$. While full sales are socially efficient, they may not happen due to liquidity constraints: cash payments are constrained by $p \leq z$ while credit payments are constrained by $d \leq D$ with debt limit

$$D = \chi_0 + \chi_{\Pi}\Pi + \chi_q (1-\delta)\, q + \chi_k (1-\delta)\, k. \tag{12}$$

In (12) the first term represents unsecured debt, where χ_0 can be a parameter or endogenized as in Kehoe and Levine (1993); the second term is debt secured by profit, as in Holmstrom and Tirole (1998); the third and fourth are debt secured by new and existing capital, like mortgages and home equity loans, as in Kiyotaki and Moore (1997).

Often χ_Π, χ_k and χ_q are called *pledgeability* parameters. One story for $\chi_j < 1$ is that you can renege on promised payments, but then a fraction χ_j of your asset j gets seized while you abscond with the rest. Li et al. (2012) provide an alternative microfoundation, where holding more assets than you use as collateral signals quality. In any case, even at $\chi_q = 1$ credit secured using only q as collateral supports no DM trade, as that does not even cover sellers' outside option; hence buyers need other lines of credit or cash.[7]

There are many options available to determine the DM terms of trade, $q(\mathrm{s}, \tilde{\mathrm{s}})$, $p(\mathrm{s}, \tilde{\mathrm{s}})$ and $d(\mathrm{s}, \tilde{\mathrm{s}})$, including Nash bargaining (Lagos and Wright 2005), strategic bargaining (Zhu 2019) and competitive price taking or price posting (Rocheteau and Wright 2005). We use Kalai's (1977) bargaining solution, which has been popular since Aruoba et al. (2007), who argue that it has advantages over Nash when liquidity constraints are operative. Letting θ be buyers' bargaining power, to get Kalai's bargain outcome we solve

$$\max_{d,p,q} S^b(\mathrm{s}, \tilde{\mathrm{s}}) \text{ st } (1-\theta) S^b(\mathrm{s}, \tilde{\mathrm{s}}) = \theta S^s(\tilde{\mathrm{s}}, \mathrm{s}),\, q \le \tilde{k}, p \le z,\, d \le D. \tag{13}$$

We have the following result (see Appendix B for details):

Proposition 1 *Consider a DM meeting* $(\mathrm{s}, \tilde{\mathrm{s}})$ *with* $\varepsilon > \tilde{\varepsilon}$, *and define a threshold for* ε *by* $\bar{\varepsilon} = \Psi_0 - \Psi_1 \tilde{\varepsilon}$, *where*

$$\Psi_0 \equiv \frac{[z + \chi_0 + \chi_k(1-\delta)k]/\tilde{k} - (1-\delta)\left(1-\chi_q\right)}{(1-\tau_k)\left[1-\theta-\chi_\Pi\left(1+k/\tilde{k}\right)\right] B(w)} \text{ and } \Psi_1 \equiv \frac{\theta}{1-\theta-\chi_\Pi\left(1+k/\tilde{k}\right)}. \tag{14}$$

Case (i) $\chi_\Pi < \bar{\chi}_\Pi \equiv (1-\theta)/(1+k/\tilde{k})$*: If* $\varepsilon > \bar{\varepsilon}$ *there is a partial sale,* $q = Q < \tilde{k}$*, where*

$$Q \equiv \frac{z + \chi_0 + [(1-\delta)\chi_k + (1-\tau_k) B(w) \chi_\Pi \varepsilon] k}{(1-\tau_k) B(w) [(1-\theta-\chi_\Pi)\varepsilon + \theta\tilde{\varepsilon}] + (1-\delta)\left(1-\chi_q\right)}, \tag{15}$$

and payment constraints bind, $p = z$ *and* $d = D$*. If* $\varepsilon < \bar{\varepsilon}$ *there is a full sale* $q = \tilde{k}$ *and the mix between* p *and* d *is irrelevant as long as* $p + d = \{(1-\tau_k) B(w) [(1-\theta)\varepsilon + \theta\tilde{\varepsilon}] + 1 - \delta\} \tilde{k}$.
Case (ii) $\chi_\Pi > \bar{\chi}_\Pi$*: The results are the same except the regions of* $(\varepsilon, \tilde{\varepsilon})$ *space are reversed – i.e.,* $\varepsilon < \bar{\varepsilon}$ *implies a partial sale and* $\varepsilon > \bar{\varepsilon}$ *a full sale.*

While there are two cases in Proposition 1, both imply higher z raises the probability of a full sale. In particular, increasing z shifts the intercept but not the slope of the threshold $\bar{\varepsilon} = \Psi_0 - \Psi_1 \tilde{\varepsilon}$. Fig. 4 shows the case $\chi_\Pi < \bar{\chi}_\Pi$, which implies partial (full) sales occur

[7] One interpretation is that they rent capital, since in the CM it does not matter if a buyer returns $(1-\delta) q$ and pays the seller a little, or keeps it and pays a lot. Then saying that at $\chi_q = 1$ credit secured by only q cannot support DM trade is like saying you cannot rent anything if the most you promise is to return it.

above (below) the threshold; this means that higher productivity firms are more likely to be constrained. When constrained, firms use all their liquid wealth, the numerator of (15), to buy q. This occurs at low χ_Π because, while higher ε firms pay more, they also get more credit secured by Π, and the net effect depends on its pledgeability. It is also worth reiterating that CRS makes partial sales inefficient. In Fig. 4, given any $\tilde{\varepsilon}$, trade is more likely to be constrained at higher ε, so reallocation with the highest social value is most prone to inefficiency due to illiquidity.

Next consider CM clearing in m and c (by Walras' law, we can ignore h). Let the aggregate supply M grow at rate μ, with changes engineered in the CM: add seigniorage to revenue from τ_h and τ_k, subtract government spending G, and set T to balance the budget each period. Then money and goods market clearing are given by

$$\phi_+ M_+ = \int \hat{z}(\varepsilon)\, dF(\varepsilon) \text{ and } c + K_+ + G = Y + (1-\delta) K, \tag{16}$$

where $Y = B(w)\bar{K}/(1-\eta)$ is total output, $K_+ = \int \hat{k}(\varepsilon)\, dF(\varepsilon)$ is gross investment, and $\bar{K}$ is effective capital weighted by productivity after DM trade,

$$\begin{aligned} \bar{K} &= \alpha \iint\limits_{\varepsilon > \tilde{\varepsilon}} \varepsilon \left[k + q(\mathbf{s}, \tilde{\mathbf{s}})\right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma(\mathbf{s}) \\ &\quad + \alpha \iint\limits_{\varepsilon < \tilde{\varepsilon}} \varepsilon \left[k - q(\tilde{\mathbf{s}}, \mathbf{s})\right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma(\mathbf{s}) + (1-\alpha) \int \varepsilon k\, d\Gamma(\mathbf{s}). \end{aligned} \tag{17}$$

We define equilibrium as follows:

Definition 1 *Given initial conditions* (z, k) *and paths for* (μ, G, τ_h, τ_k)*, equilibrium is a list of nonnegative paths for* $(c, \hat{z}, \hat{k}, q, p, d, \phi, w, \Gamma)$*, where* $\hat{z} = \hat{z}(\varepsilon)$ *and* $\hat{k} = \hat{k}(\varepsilon)$ *for each agent while* $q = q(\mathbf{s}, \tilde{\mathbf{s}})$*,* $p = p(\mathbf{s}, \tilde{\mathbf{s}})$ *and* $d = d(\mathbf{s}, \tilde{\mathbf{s}})$ *for each pair, satisfying at all dates: (i) in the CM* $(c, \hat{z}, \hat{k})$ *solves (2); (ii) in the DM* (q, p, d) *is given by Prop. 1; (iii) markets clear as in (16); (iv) the distribution* Γ *evolves according to (8); and (v) the transversality conditions* $\beta^t u'(c_t)\, \hat{k}_t \to 0$ *and* $\beta^t u'(c_t)\, \hat{z}_t \to 0$ *hold.*

To define steady state, let (μ, G, τ_h, τ_k) be constant and note if the growth rate of M is $\mu \neq 0$ then ϕ changes over time, but $\phi M = z$ does not if $\phi/\phi_+ = 1 + \mu$.

Definition 2 *Steady state is a time-invariant list* $(c, z, k, q, p, d, w, \Gamma)$ *satisfying everything in Definition 1 except for the initial conditions.*

While monetary policy can be given by μ, for steady state it is equivalent to instead peg inflation π, or the illiquid nominal interest rate defined by $\iota = (1+\pi)/\beta - 1$. To be precise, define illiquid interest rates as follows: $1+r$ is the amount of c agents require in the next CM to give up 1 unit in this CM; and $1+\iota$ is similar except m replaces c. As usual, $\iota > 0$ imposed, but the limit $\iota \to 0$ can be considered, which is the Friedman rule, or the zero lower bound.

It is useful to have the marginal value of capital in the DM. Using (9) we derive

$$\begin{aligned} \frac{\partial V}{\partial k} &= \frac{\xi}{(1-\tau_h)w}\left\{1-\delta+(1-\tau_k)B(w)\left[\varepsilon+\alpha(1-\theta)\int_{\mathcal{S}_s(\mathbf{s})}(\tilde{\varepsilon}-\varepsilon)\,d\Gamma(\tilde{\mathbf{s}})\right.\right. \\ &\left.\left. +\alpha\theta(1-\delta)\chi_k\int_{\mathcal{S}_b(\mathbf{s})}\frac{\varepsilon-\tilde{\varepsilon}}{\Delta(\varepsilon,\tilde{\varepsilon})}d\Gamma(\tilde{\mathbf{s}})+\alpha\theta(1-\tau_k)B(w)\chi_\Pi\int_{\mathcal{S}_b(\mathbf{s})}\frac{\varepsilon(\varepsilon-\tilde{\varepsilon})}{\Delta(\varepsilon,\tilde{\varepsilon})}d\Gamma(\tilde{\mathbf{s}})\right]\right\}, \end{aligned} \tag{18}$$

where $\Delta(\varepsilon,\tilde{\varepsilon})$ denotes the denominator in (15), while

$$\mathcal{S}_s(\mathbf{s})=\{\tilde{\mathbf{s}}:\tilde{\varepsilon}>\varepsilon,\,\tilde{\varepsilon}<\overline{\varepsilon}(\tilde{\mathbf{s}},\mathbf{s})\}\text{ and }\mathcal{S}_b(\mathbf{s})=\{\tilde{\mathbf{s}}:\tilde{\varepsilon}<\varepsilon,\,\varepsilon>\overline{\varepsilon}(\mathbf{s},\tilde{\mathbf{s}})\} \tag{19}$$

are sets of meetings where sellers are constrained by k and buyers by z.[8] Similarly,

$$\frac{\partial V}{\partial z}=\frac{\xi}{(1-\tau_h)w}\left[1+(1-\tau_k)B(w)\,\alpha\theta\int_{\mathcal{S}_b(\mathbf{s})}\frac{\varepsilon-\tilde{\varepsilon}}{\Delta(\varepsilon,\tilde{\varepsilon})}d\Gamma(\tilde{\mathbf{s}})\right]. \tag{20}$$

is the analog for z.[9] Combining (18)-(20) with the FOCs we get the Euler equations

$$\begin{aligned} \frac{1}{w} &= \frac{\beta(1-\tau_k)B(w_+)}{w_+}\mathbb{E}_{\varepsilon_+|\varepsilon}\left[\varepsilon_+ + \alpha(1-\theta)I_s+\alpha\theta(1-\delta)\chi_k I_{b1}\right. \\ &\qquad \left. +\alpha\theta(1-\tau_k)B(w)\chi_\Pi I_{b2}\right]+\frac{\beta(1-\delta)}{w_+} \end{aligned} \tag{21}$$

$$\frac{Z}{w}=\frac{\beta Z_+}{w_+(1+\mu)}\mathbb{E}_{\varepsilon_+|\varepsilon}\left[1+(1-\tau_k)B(w_+)\,\alpha\theta I_{b1}\right], \tag{22}$$

[8] In words, (18) says that a marginal unit of k has several potential benefits: (i) You can get the CM resale value of $1-\delta$ per unit. (ii) You can get its contribution to CM production, the first term in square brackets, which is ε because $(1-\tau_k)B(w)$ outside the brackets converts εk into income. (iii) You can get its value from a DM sale, the second term in brackets, since you sell all of k when you meet someone with $\tilde{\mathbf{s}} \in \mathcal{S}_s(\mathbf{s})$ and enjoy a share $1-\theta$ of the surplus. (iv) You can get its DM collateral value, captured by the third term, since you hit your liquidity constraint when you buy from someone with $\tilde{\mathbf{s}} \in \mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of that surplus. (v) You can get the collateral value from more CM profit, captured by the fourth term, when you buy from someone with $\tilde{\mathbf{s}} \in \mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of that surplus. Of course you do not get all of these – you get each one with some probability.

[9] In words: (i) You can get z's CM purchasing power. (ii) In the DM you hit your liquidity constraint as a buyer when you meet $\tilde{\mathbf{s}} \in \mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of the surplus.

where Z is aggregate real balances, and to save space I_s, I_{b1} and I_{b2} denote the three integrals on the RHS of (18).

4 Analytic Results

We now study the model where ε is i.i.d across time, which allows us to obtain analytical results. For this, assume χ_Π is not too big (used to guarantee uniqueness). To begin, make a change of variable by defining

$$L \equiv \frac{(Z+\chi_0)/K - (1-\delta)\left(1-\chi_q-\chi_k\right)}{(1-\tau_k)B(w)}, \tag{23}$$

a normalized notion of liquidity determining when the constraint binds. Then write

$$\mathcal{S}_b(L) \equiv \left\{(\varepsilon,\tilde{\varepsilon}) : \varepsilon > \tilde{\varepsilon},\, \varepsilon > \frac{L-\theta\tilde{\varepsilon}}{1-\theta-2\chi_\Pi}\right\} \text{ and } \mathcal{S}_s(L) \equiv \left\{(\varepsilon,\tilde{\varepsilon}) : \varepsilon < \tilde{\varepsilon},\, \tilde{\varepsilon} < \frac{L-\theta\varepsilon}{1-\theta-2\chi_\Pi}\right\}$$

for the sets of meetings where partial and full sales occur now as functions of L. Also, write the effective capital stock, defined in (17), as $\bar{K} = J(L,w)K$ with

$$\begin{aligned} J(L,w) \;\; &\equiv \;\; \mathbb{E}\varepsilon + \alpha I_s(L) + \alpha\left[(1-\tau_k)B(w)L + \left(1-\chi_q\right)(1-\delta)\right] I_{b1}(L) \\ &\quad + \alpha\chi_\Pi (1-\tau_k) B(w) I_{b2}(L) \end{aligned}$$

where, with a little abuse of notations, I_s, I_{b1} and I_{b2} are the expected values of the integrals from (18) written as functions of L.

Then in steady state the Euler equations can be written

$$\frac{r+\delta}{B(w)(1-\tau_k)} = \mathbb{E}\varepsilon + \alpha(1-\theta) I_s(L) + (1-\delta)\chi_k\iota + \chi_\Pi B(w)(1-\tau_k)\alpha\theta I_{b2}(L) \tag{24}$$

$$\iota = \alpha\theta \iint_{\mathcal{S}_b(L)} \frac{(\varepsilon-\tilde{\varepsilon})\, dF(\tilde{\varepsilon})dF(\varepsilon)}{(1-\theta-\chi_\Pi)\varepsilon + \theta\tilde{\varepsilon} + \frac{(1-\delta)\left(1-\chi_q\right)}{(1-\tau_k)B(w)}} \tag{25}$$

where r and ι are the illiquid real and nominal rates defined above. Also, goods market clearing becomes

$$u'^{-1}\left[\frac{\xi}{(1-\tau_h)\,w}\right] + G = \left[\frac{B(w)\,J(L,w)}{1-\eta} - \delta\right] K. \tag{26}$$

Notice (24)-(26) are three equations in (K,Z,w). For what it's worth, (24) and (25) are the classical IS and LM curves: demand for Investment equals supply of Savings and demand for Liquidity equals supply of Money. While not the textbook IS-LM model, these can be used similarly by shifting curves (see fn. 10). Of course, w is endogenous, but in principle (26) can be solved for it and used to write (24)-(25) to get two equations in (K, Z). We cannot solve for w explicitly, however, except in special cases, like $\theta = 1$, or $f(k, h)$ linear in h, which are too restrictive because they rule out interesting effects.

While in principle it would be nice to work in (K, Z) space – after all, the paper is about capital and liquidity – in practice it is better to notice that (24)-(25) are two equations in (L, B). As shown in Fig. 5, their intersections in (L, B) space constitute steady states, and given (L, B), K, Z and w follow from (1), (23) and (26). Appendix B establishes the existence and uniqueness of monetary steady state under certain conditions, where some conditions are obviously needed since, as is known from related work, monetary equilibrium does not exist if θ is too small, ι is too big, or credit is too easy.

Proposition 2 *If θ is not too small, while χ_0, χ_Π and χ_k are not too big, there exists a monetary steady state iff $\iota < \bar{\iota}$, where $\bar{\iota} > 0$, and it is unique.*

Table 5 shows the effects of parameters on standard macro variables, plus the probability of a full sale in any given meeting, denoted Φ. As mentioned, here we are restricting χ_Π to be not too big, and for Table 5 we also assume χ_q is not too small. While these are not unnatural restrictions, still many entries are ambiguous, shown by $\pm$. This is not because the theory is messy; this is because it is rich enough to make some effects nonmonotone – e.g., standard macro variables can be nonmonotone in ι, as can be verified numerically. In particular, we can have $\partial K/\partial\iota > 0$ or $\partial K/\partial\iota < 0$, with the former reminiscent of the Mundell-Tobin effect reflecting the fact that K and Z are substitutes, but as payment instruments here, not arguments of utility.

Some results in Table 5 are unambiguous, including most of the effects of fiscal policy. A particularly relevant result is $\partial Z/\partial\chi_0 < 0$, which says that an increase in credit reduces money demand, pushing up the price level, and showing up as inflation in the short run even if that is pinned down by $\phi/\phi_+ = 1 + \mu$ in the long run. Similarly, most of the effects on Φ are unambiguous, including $\partial\Phi/\partial\iota < 0$, $\partial\Phi/\partial\chi_k > 0$ and $\partial\Phi/\partial\chi_q > 0$, which together with $\partial Z/\partial\chi_0 < 0$ relate directly to the discussion in the Introduction about how we intend to explain the facts.[10]

[10]As mentioned, the results in Table 5 can be illustrated by shifting curves in Fig. 5, and we use this in the proofs in Appendix B. While there we study a more general case, restricting $\chi_k = 0$ makes graphical analysis especially easy since then monetary policy in terms of of ι shifts LM but not IS. Similarly, if $\chi_q = 1$, fiscal policy in terms of τ_k shifts IS but not LM.

Table 5: Comparative Statics

	ι	χ_0	χ_k	χ_q	χ_Π	τ_k	τ_h	G	A	ι^*	$\iota^\dagger$
K	$\pm$	0	$+$	$\pm$	$\pm$	$-$	$-$	$+$	$+$	$\pm$	$+$
Z	$\pm$	$-$	$\pm$	$\pm$	$\pm$	$\pm$	$-$	$+$	$+$	$\pm$	$\pm$
w	$\pm$	0	$+$	$+$	$+$	$-$	0	0	$+$	$-$	$+$
c	$\pm$	0	$+$	$+$	$+$	$-$	$-$	0	$+$	$-$	$+$
H	$\pm$	0	$\pm$	$\pm$	$\pm$	$\pm$	$-$	$+$	$\pm$	$\pm$	$\pm$
Y	$\pm$	0	$+$	$\pm$	$\pm$	$-$	$-$	$+$	$+$	$\pm$	$+$
Φ	$-$	0	$+$	$+$	$\pm$	$-$	0	0	0	$-$	$-$

Notes: All effects assume χ_q is big and χ_π small; $*$ also assumes χ_k is small; † assumes χ_k and θ are big.

In addition to existence, uniqueness and comparative statics, welfare can be studied analytically. Appendix B compares the equilibrium outcome with no tax distortions, $\tau_k = \tau_h = 0$, to the solution to a planner problem. For q to be efficient in equilibrium we need full sales in all meetings, which for an arbitrary ε distribution requires $\iota \to 0$. For K to be efficient we need sellers in the DM to reap the full benefit of their investments, meaning $\theta = 0$, but there is no monetary equilibrium at $\theta = 0$. More generally, there is a two-sided holdup problem: given $\iota > 0$, high θ is needed to get money demand right; low θ is needed to get capital demand right; and we cannot have both.[11]

However, for any $\theta > 0$, efficiency obtains when $\iota \to 0$ if we set $\tau_h = 0$ and implement a corrective subsidy on capital formation financed by the lump sum tax T.

Proposition 3 *Efficiency is not possible at $\iota > 0$. When $\iota \to 0$, monetary steady state is efficient if $\tau_h = 0$ and $\tau_k = \tau_k^*$, where $\tau_k^* \leq 0$ with strict inequality unless $\theta = 0$, is given by*

$$\tau_k^* = 1 - \frac{\int_0^\infty \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon})\, dF(\tilde{\varepsilon})\, dF(\hat{\varepsilon})}{\int_0^\infty \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha (1-\theta) \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon})\, dF(\tilde{\varepsilon})\, dF(\hat{\varepsilon})}. \tag{27}$$

Note that this does not imply $\iota = 0$ is always optimal. If $\tau_h \neq 0$ or $\tau_k \neq \tau_k^*$, welfare can be increasing in ι over some range, as can be verified numerically.

Before moving to numerical work, consider a pure credit setup with $\phi M = 0$ (no money) and $\chi_0 > 0$, $\chi_j = 0$ for $j \neq 0$ (no collateral), which does not mean perfect credit, since $d \leq \chi_0$ may still bind. This version has one nice feature: it immediately gives two equations in (K, w). It also facilitates comparison with other papers on capital reallocation, which do not have money, and we compare pure credit and money quantitatively in Section 5.5.

[11] Past work in different contexts suggests that efficiency may emerge, given there are no policy distortions, if there is posting instead of bargaining and directed instead of random search, a solution concept called competitive search equilibrium (see Wright et al. 2021 for a survey). That is left to future reseearch.

Proposition 4 *With pure credit steady state exists. It is unique if α is not too big. It is efficient if χ_0 is not too small, $\tau_h = 0$ and $\tau_k = \tau_k^*$ as given in (27).*

5 Quantitative Results

5.1 Preliminaries

Before describing our calibration, it is useful to extend the benchmark model on two dimensions. First, in reality firms hold cash substitutes such as interest-bearing bank deposits that can partially reduce the cost of inflation, and based on experience we want to allow for that because it can help quantitative analysis. Hence we add banks as in Berentsen et al. (2007). After the CM closes, and before the DM opens, suppose some information is revealed that affects agents' desired $\hat{z}$. For simplicity, assume this information is whether each agent will have a meeting in the next DM. Those that will not have a meeting have excess cash; those that will could use more. This liquidity mismatch creates a role for banks, like Diamond and Dybvig (1982), except they deal in money not goods. What makes them essential is that agents cannot easily trade liquidity among themselves using promised CM repayment, for the same reason they cannot trade capital in the DM using promised CM repayment: lack of commitment and lack of concern for reputation.

Assume bankers have reputational concerns, so their promises are credible, and a comparative advantage in collecting debt, giving them a role intermediating the reallocation of liquidity (see Gu et al. 2023 for a survey of banking models with these features). While they are not needed for the theory, banks aid in the calibration because they prop up money demand. The insight in Berentsen et al. (2007) is that the ability to retrade makes investing in liquidity less costly, since if you find yourself with more than you need you can put it in the bank, at interest financed by loans to those who want more – another instance the ease with which assets can be retraded on secondary markets affects demand in primary markets. Conveniently, the only impact this has on the equilibrium conditions is that 1 replaces α in (22).[12]

[12]One can check, e.g., He et al. (2015) for details, but the idea is simple: 1 replaces α in (22) since when you deposit you effectively lend to someone with a DM meeting, so you get the same marginal benefit, which props up money demand (simply setting $\alpha = 1$ without banks props it up, too, but is not equivalent , as it has other implications, and it turns out that banks work better). A key point is that with banking not all agents get 0 returns: the ones who do not need liquidity keep it in the bank at interest; the ones who withdraw and actually top it up by borrowing get the liquidity value in DM trade, but earn no interest on deposits and pay interest on loans. While this is realistic, at calibrated parameters the interest rate on deposits is a bit high, around 3.8% in real terms, compared to checking accounts, if not CD's or related investments. Of course there is the old idea that bank accounts provide more that interest, they provide services like record

Second, we add endogenous DM entry, as in many classic search models (e.g., Diamond 1982; Pissarides 2000) and monetary papers featuring a cost of participating in certain markets (e.g., Chatterjee and Corbae 1992; Chiu 2014). As in Khan and Thomas (2007, 2013), the entry cost γ is random across agents, and for simplicity let us assume entry happens before seeing the ε shocks, so only those realizing γ below a common threshold γ^* enter. With a CRS meeting technology, entry affects the measure of agents in the DM but not individual arrival rates.

Like adding banks, making γ^* endogenous is not crucial for the theory, but it helps in the quantitative work. In particular, the model without entry implies a counterfactual positive correlation between inflation and credit conditions, while with entry it implies a negative correlation, as in the data. Moreover, endogenous entry is natural since it means the number of firms trading in the DM, and not just the total quantity traded, varies over time. Calculating the fraction of firms trading in secondary capital markets in COMPUSTAT, we find a strong positive correlation with output over the cycle.

5.2 Calibration

We assume for now that ε is i.i.d., but that is relaxed in Section 6. Also we now consider the sample period 1984 to 2018, where the P and R shares are relatively stable. This leads to the parameter values in Table 6, and we now describe how they are set.

Many parameters are standard in RBC research, which we follow where possible (i.e., we do not take parameter values from the literature, but use methods in literature for setting parameters to hit empirical targets). For fiscal policy, we set $G/Y = 0.161$, $\tau_k = 0.25$ and $\tau_h = 0.22$, consistent with the methods discussed in Gomme and Rupert (2007). For monetary policy, measuring π by annual CPI inflation, we get 2.65% on average over the sample. For the illiquid nominal rate ι, we use the AAA corporate bond yield of 6.72%, so the illiquid real rate solves $1 + r = (1 + i)/(1 + \pi) = 1.0396$, and $\beta = 0.962$.[13]

Then we use $u(c) = \log(c)$ and set the coefficient on leisure to get hours worked as a fraction of discretionary time 33%, a standard target from time-use surveys (Gomme et

keeping, overdraft protection, the ability to stop checks if necessary, etc. Getting every last detail of banking right require adding detail, probably at least some limit on how much agents can borrow, and so is relegated to future work.

[13] Although it would not matter a lot, AAA corporate bonds are used here rather than T-bills. One reason it that the model is about firms. Another is that while corporate bonds may have more risk than T-bills, our agents are effectively risk neutral wrt yield by Lemma 1, making these bonds correspond well to our definition of ι (the dollars agents require in the next CM to give up one in this CM ignoring risk premia). Also, it is generally agreed in finance that corporate bonds are less liquid than T-bills, or at least less "convenient" (Krishnamurthy and Vissing-Jorgensen 2012), which we interpret as less liquid, and that is what ι is supposed to capture.

Table 6: Calibrated Parameter Values

Parameter	Value	Explanation	Parameter	Value	Explanation
ι	0.067	nominal AAA yield	μ_γ	-1.7670	R share
β	0.9619	real AAA yield	χ_q	0.8869	P share
ξ	2.2498	labor hours	χ_Π	0.0989	cash/output
η	0.6110	investment/output	τ_k	0.25	capital tax rate
δ	0.1000	depreciation rate	τ_h	0.22	labor tax rate
σ_ε	1.2853	COMPUSTAT	G	0.1479	gov't share
σ_γ	0.2216	acquisition elasticity	θ	0.50	symmetry

al. 2004). The depreciation rate is set to $\delta = 0.10$, which combined with a labor's share of $\eta = 0.611$ matches an investment-output ratio of 20%, which we get using private plus public investment excluding national defence. While there is not universal agreement on labor's share – e.g., Christiano (1988) argues one can reasonably say it is anywhere from 0.57 to 0.75, depending on interpretation, mainly how one treats proprietors income – our η is in the range typically used.

For less standard parameters, to begin with credit, we set $\chi_0 = \chi_k = 0$ for this reason: The general framework allows several kinds of credit that are interesting in principle, but it would be difficult to handle them all in practice. Most papers use just one; we use two, χ_q and χ_Π. As regards χ_q, it is reasonable to think new capital secures purchases of new capital the way houses secure mortgage loans (Kiyotaki and Moore 1997). As regards χ_Π, the pledgeability of profit is regarded as relevant and standard in finance (Holmstrom and Tirole 1998; Li 2022). Hence we focus on χ_q and χ_Π, but before calibrating them we need to discuss some other parameters that are all set jointly.

Assume ε is log-normal, with normalized $\mu_\varepsilon = 0$ and $\sigma_\varepsilon = 1.2853$, in line with previous studies (e.g., Imrohoroglu and Tuzel 2014). To explain, it is standard to fit an AR(1) for the log of productivity, which gives a persistence coefficient 0.70 and standard deviation 0.357 in COMPUSTAT after 1984. The unconditional variance and standard deviation are $0.357^2/(1-0.7^2)$ and 0.50. That is for total productivity, while ε in the model is capital productivity, so $\sigma_\varepsilon = 0.50/(1-\eta) = 1.2853$. Assume the DM entry cost γ is also log-normal with mean μ_γ and standard deviation σ_γ. Then calibrate μ_γ and σ_γ jointly with χ_q and χ_Π to hit four sample averages: R share of 0.24; P share of 0.32; firm money holdings over output of 4.3%; and the elasticity of acquisition spending wrt inflation of -0.64.

In these calculations, firm money holdings are checkable deposits plus currency held by nonfinancial corporate and noncorporate businesses, from FRED, divided by output to make the series stationary. For the elasticity of acquisition spending wrt inflation, it is estimated using COMPUSTAT with acquisitions defined as full sales. We tried a few specifications, with estimates ranging between around -0.3 and -1, and settled on the number in the middle. This method is meant to emulate the one used for household money demand, where cash over output and its elasticity are key targets. While there is existing work on firm money demand (see Gao 2021 for a recent example), and in principle one could appeal to that, it seemed prudent to get numbers from the same data used for reallocation variables.

Regarding the DM entry cost, as usual one must decide how to do the accounting – it is in terms of utility, labor or numeraire? Our convention is to say entry requires labor services from a financial sector, which is not modeled explicitly, but the idea is simple enough: participating in the DM uses hours employed in this service multiplied by w and a wage premium parameter, and this is added to CM output to get total output. We set wage premium parameter to 1.17 to match the share of output in financial services relevant for capital investment, which is 1.2% in BEA data. This seems to us a reasonable approach, and we are consistent in the way we do the accounting in the model and data.

Finally there is bargaining power. We set $\theta = 1/2$ as a benchmark, which is natural with ex ante identical agents. However, rather than focus exclusively on one number below we check how this matters by presenting results for different values of θ.

5.3 The Long Run

The first experiment concerns the impact of inflation on steady state. Fig. 6 shows the results for three bargaining powers, $\theta = 0.50$, 0.45 and 0.55, recalibrating other parameters in each case. On the horizontal axis is ι, but this conveys the same information as π in steady state. The top row shows standard macro variables, Y, K, C and H; the middle row shows reallocation variables, the R and P shares, plus the probability Φ of a full sale and the average DM price; the bottom row shows productivity, welfare, money and credit. A vertical line indicates a threshold beyond which monetary equilibria vanish, the $\bar{\iota}$ from Prop. 2 (although one does not need take this too seriously, as presumably monetary equilibria would survive beyond $\bar{\iota}$ if, say, cash were also demanded by households, or by firms for reasons other than reallocating k).

First notice that, although in general output is nonmonotone, the top row shows that numerically it decreases with ι, except for low θ and a small range of ι. Second notice that there is a Phillips curve: H increases with ι, with the same qualification, except for low θ and a small range of ι, and recall (from fn. 6) that $1 - H$ can be interpreted as unemployment, not just leisure. The nonmonotonicity of K is more pronounced, but it increases with ι over a big range and the effect is sizable. The bottom row shows welfare is at maximized at $\iota = 0$ for these parameters, if not in general, where the welfare cost is defined in a standard way as the amount of consumption, in percent, agents would give up to go from a benchmark inflation to 0. At $\theta = 0.5$, eliminating 10% inflation is worth around 1.4% of consumption, although $\pi = 0$ inflation is not the best we can do, and going all the way to $\iota = 0$ is worth around 5.4%.[14]

The middle row of Fig. 6 concerns reallocation. As in the data, in the long run, inflation decreases the R share, increases the P share, and decreases the probability of full sales. It also raises the DM price $(p + d)/q$, consistent with the facts. The graph shows the average price, of course since q can vary across bilateral meetings – i.e., the law of one price does not hold, also consistent with facts on secondary capital trade (recall fn. 1). The bottom row show inflation lowers average productivity by hindering reallocation.

While we interpret Fig. 6 as providing numerical comparative statics, at the suggestion of a referee Fig. 7 shows how the results compare with long-run data. There is not a unique way to do this, but here the data is filtered to leave very smooth paths for investment and consumption over output, as well as the R and P shares. These are compared to model predictions when inflation is the only driving process. Clearly this is a counterfactual exercise, as many other factors changed over the period; it should be interpreted as showing how these variables would have moved abstracting from innovations in fiscal policy, regulation, demographics, etc. So one should not expect, or desire, a perfect fit, but interestingly enough the model accounts for the data fairly well.

5.4 The Medium Run

In between looking at trends as we did above, and studying business cycles as we do below, one can ask about the medium run. As suggested by a (different) referee, one might worry that the long-run findings are basically driven by two observations: once there was high inflation, a low R share and a high P share; then there was low inflation, a high R share and a low P share.

To address this, Fig. 8 plots the empirical R and P shares starting in 1971 along with the predictions of the model when we input actual inflation, focusing on the medium run

[14] For comparison, in models of household money demand, without capital, like Lagos and Wright (2005), going from 10% to 0 is worth 4.6% of consumption and going to $\iota = 0$ is worth 6.8%. In similar models with capital, like Aruoba et al. (2011), the numbers are lower, while in reduced-form monetary models with or with capital, like Cooley and Hansen (1989) or Lucas (2000), they are much lower. We initially expected bigger numbers here since: (i) macro public finance tells us taxing capital is generally a bad idea; and (ii) micro public finance tells us big distortions come from taxing things with close substitutes, and CM k is a substitute for DM k. These effects are present, but are attenuated by inflation stimulating investment, which tends to be too low due to holdup problems coming from bargaining.

by averaging over 5-year subperiods, and looking at the model's steady state in each one, using both fixed-window and rolling averages. The model tracks the data well, with changes in inflation accounting for much of the pattern in the R and P shares not only over two broad episodes, one with high and one with low inflation, but across all subperiods. As with Fig. 7, one should not expect a perfect fit, since the exercise abstracts from technical progress, financial innovation, etc. but it does suggest that inflation may be relevant.[15]

5.5 The Short Run

Next we ask how the model accounts for business cycles. Motivated by the discussion of economic intuition and empirical findings, we allow shocks to aggregate productivity A and to credit conditions, as captured by χ_q, although for comparison results with only A shocks are also presented. The specification is

$$\ln A_t = \rho_A \ln A_{t-1} + \varsigma_{A,t} \text{ and } \ln \chi_{q,t} - \ln \chi_q = \rho_\chi \left(\ln \chi_{q,t-1} - \ln \chi_q \right) + \varsigma_{\chi,t},$$

where $\varsigma_{A,t} \sim N(0, \sigma_A^2)$ and $\varsigma_{\chi,t} \sim N(0, \sigma_\chi^2)$ are i.i.d. and orthogonal. We use $\rho_A = 0.83$, as is standard in annual models, corresponding to 0.95 in quarterly models; with less precedent, for the sake of illustration, we also use $\rho_\chi = 0.83$. With only A shocks, $\sigma_A = 3.85\%$ to match output volatility; with both shocks, $\sigma_A = 2.75\%$ and $\sigma_\chi = 8.21\%$ to match the volatility of output and the R share; then we ask how the model captures other variables' volatility and output correlation.

The first three columns of Table 7 show standard deviations from the data, the model with A shocks, and the model with both shocks; other columns show correlations with Y. Model as well as data statistics are computed after taking logs and filtering, as usual. For Y, C, I and H, with only A shocks or with both shocks, the model does well accounting for volatility and correlation by the standards of the literature. This is similar to the textbook RBC model, and perhaps not too surprising, but it is good to know our new features do not impair performance of standard theory in capturing the basic facts.

How about reallocation dynamics? On that dimension, the model with only A shocks is way off. For the R share, the standard deviation is too small, and the correlation with

[15] We do not show firm money demand in Fig. 8, since it is hard to fit. One reason is that firms value liquidity for purposes other than capital reallocation. Other reasons include the factors mentioned above, especially financial innovation and regulatory changes that make theoretical and empirical notions of money moving targets. Lucas and Niolini (2015) show household money demand in the data is a stationary function of ι over long periods if, and only if, we agument M1 to account for regulatory changes, and an analogous procedure has not been done for firms. So we are more comfortable looking at long-run averages of firm money holdings than at how they change over time, but future work could explore this.

Table 7: Business Cycle Statistics

	Standard deviation			Correlation with output		
	Data	A only	A & χ_q	Data	A only	A & χ_q
Output	1.73	1.73	1.73	1.00	1.00	1.00
Consumption	0.67	0.44	0.58	0.96	0.97	0.89
Investment	2.56	2.89	2.73	0.96	0.99	0.95
Employment	0.88	0.44	0.76	0.85	0.97	0.92
TFP	0.69	0.62	0.48	0.76	0.99	0.91
R share	6.45	1.78	6.45	0.61	-0.99	0.48
P share	9.46	1.17	7.45	-0.52	0.98	-0.56
Inflation	0.89	0.39	1.18	0.33	-0.76	0.42

Note: Standard deviation of other variables are relative to output.

Y is -0.99 instead of $+0.61$. For the P share, the standard deviation is far too small, only 1.17 compared to 9.46 in the data, and the correlation with Y again takes the wrong sign, $+0.98$ instead of -0.52. The conclusion is that with only aggregate productivity shocks, the theory is very poor at capturing empirical reallocation dynamics

How about the case with shocks to both χ_q and A? That does much better – by the standards of the literature, it accounts for the data very well indeed. Of course it matches the volatility of the R share, since σ_χ is calibrated to that, but we did not target the correlation between the R share and Y, which now has a similar magnitude and the correct sign. For the P share, the standard deviation and correlation with Y look reasonably good, and certainly better than with only A shocks.

One might say this is the major finding, but there are many subsidiary results. First, with only A shocks inflation is counterfactually countercyclical and not volatile enough, but with both shocks its correlation with Y has the correct sign and its volatility is not too far off. Also, although not shown in the Table, with both shocks the correlation between inflation and the R share is 0.53 and the correlation between inflation and the P share is -0.55, which have the right signs if somewhat higher magnitudes than the data, 0.37 and -0.28, while with only A shocks the model does much worse. We do not push this too hard, however, since we have abstracted from features that may have a big influence of inflation dynamics, such as a more detailed model of monetary policy. We prefer to emphasize that the average DM price is procyclical, its correlation with Y being 0.67, and productivity dispersion measured by the coefficient of variation is countercyclical, its correlation with Y being -0.65. Hence, the model can match all the stylized facts on capital reallocation summarized in the Introduction.

It is also interesting to see how the two shocks affect different variables, e.g., the results on the DM price are driven mainly by A shocks, while the results on productivity dispersion are driven by χ_q shocks. Moreover, two-shock models have another, well-understood, advantage (see,Christiano and Eichenbaum 1992): they break the tight relationship between A and H in one-shock models. Although the labor market is not our main focus, the correlation between A and H is 0.64, which is a big step in the right direction compared to the result with one shock, which is 0.98, and the standard deviation of H is not too bad. So the model does well at capturing some standard business cycle facts.

Table 8: Business Cycle Statistics without Money

	SD			Corr with output		
	Data	A only	A and χ_q	Data	A only	A and χ_q
Output	1.73	1.73	1.73	1.00	1.00	1.00
Consumption	0.67	0.43	0.51	0.96	0.96	0.92
Investment	2.56	2.89	2.78	0.96	0.99	0.97
Employment	0.88	0.43	0.66	0.85	0.97	0.90
TFP	0.69	0.63	0.54	0.76	0.99	0.94
R share	6.45	2.45	6.45	0.61	-0.97	0.24
P share	9.46	1.17	4.93	-0.52	0.96	-0.36
Inflation	0.89	-	-	0.33	-	-

Note: SD is standard deviation relative to output, except for output itself.

The final exercise in this Section is to ask if the results are due mainly to credit shocks, or if money also matters. To that end, consider the nonmonetary model from Prop. 4. This version resembles Kahn and Thomas (2013), who do not examine capital reallocation directly, but find that credit shocks help capture empirical features of other variables. The results are in Table 8, with parameters re-calibrated, and in particular σ_χ still set to match the volatility of the R share. On standard macro variables and the volatility of reallocation variables, the nonmonetary model does fairly well, but the P share volatility is only 52% of the data; it does not deliver a sufficiently positive correlation between the R share and Y. Having money helps get these right.[16] Moreover, obviously a monetary model is useful for understanding the results on inflation and reallocation, the welfare cost of inflation, etc. So, in general, the answer is yes, money matters.

[16]Heuristically, volatility in A does not generate enough volatility in reallocation and it leads to a big negative correlation between the R share and Y, and a big positive correlation between the P share and Y, because when A is high DM buyers are more constrained. Credit shocks help by generating variation in reallocation due to variation in credit; but credit shocks are not correlated enough with output. Adding money helps with the correlation because firms hold more z when A is high, relaxing the liquidity constraint. We thank a referee for helping us articulate this intuition.

6 Extensions and Ideas for Future Work

6.1 Multiple Liquid Assets

Suppose that, in addition to money, there is a long-lived real asset a in fixed supply with CM price ψ and dividend ρ (a Lucas tree, although things are basically the same with bonds). In the DM both z and a can be used for some payments: with probability α_1 only z is accepted; with probability α_2 only a is accepted, and with probability α_3 both are accepted. We also allow general χ_z and χ_a, the fraction of money and real asset that can be used in transactions, but fix $\chi_q = 1$ and $\chi_k = \chi_\pi = 0$, since we are more interested in asset liquidity than credit for this exercise.

Now wealth Ω includes $(\psi + \rho)\, a$, and the CM problem becomes

$$\begin{aligned} W(\Omega, \varepsilon) &= \max_{c,h,\hat{a},\hat{k},\hat{z}} \left\{ u(c) - \xi h + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} V_+(\hat{a}, \hat{k}, \hat{z}, \hat{\varepsilon}) \right\} \\ \text{st } c &= \Omega + (1-\tau_h) w h - \hat{z}\phi/\phi_+ - \hat{k} - \psi \hat{a}. \end{aligned}$$

It is routine to derive the Euler equations emulating what was done above. Normalizing the supply of the real asset to 1 and letting $Z_a = (\rho + \psi)\, a$, we get

$$\begin{aligned} \frac{r+\delta}{B(w)(1-\tau_k)} &= \mathbb{E}\varepsilon + (1-\theta)\left[\alpha_1 I_s(L_1) + \alpha_2 I_s(L_2) + \alpha_3 I_s(L_1+L_2)\right], \\ \iota &= \alpha_1 \chi_z \lambda(L_1) + \alpha_3 \chi_z \lambda(L_1+L_2), \\ r Z_a &= (1+r)\chi_a \rho + \beta Z_a \chi_a \left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1+L_2)\right] \end{aligned}$$

after imposing steady state, where

$$\lambda(L) \equiv \iint_{\mathcal{S}_b(L)} \frac{\alpha\theta(\varepsilon - \tilde{\varepsilon})\, dF(\tilde{\varepsilon}) dF(\varepsilon)}{(1-\theta)\varepsilon + \theta\tilde{\varepsilon}},\ L_1 \equiv \frac{\chi_z Z}{(1-\tau_k)B(w)K} \text{ and } L_2 \equiv \frac{\chi_a Z_a}{(1-\tau_k)B(w)K}.$$

Notice L_1 and L_2 represent the liquidity embodied in money and in real assets. If $\theta = 1$ the system reduces to two equations in (L_1, L_2),

$$\begin{aligned} \iota &= \chi_z\left[\alpha_1 \lambda(L_1) + \alpha_3 \lambda(L_1+L_2)\right], \\ r &= \Upsilon + \chi_a\left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1+L_2)\right], \end{aligned}$$

where $\Upsilon \equiv (1+r)\chi_a \rho/(1-\tau_k)B(w) K L_2$. Suppose ρ is small, so that $\partial\Upsilon/\partial L_1$ and $\partial\Upsilon/\partial L_2$

are also small. Then

$$\frac{\partial L_1}{\partial \iota} < 0,\ \frac{\partial L_2}{\partial \iota} > 0 \text{ and } \frac{\partial (L_1 + L_2)}{\partial \iota} < 0.$$

Intuitively, as ι rises the liquidity embodied in cash falls, so agents try to substitute into real assets (another Mundell-Tobin effect). This increases the price and hence the liquidity embodied in real assets, but total liquidity, $L_1 + L_2$, falls. By continuity, if $\theta < 1$ is not too small and $\rho > 0$ not too big, the results still hold. Wallace (1980) did not derive these effects, as his framework at the time used Walrasian markets, where liquidity has no role, but they are consistent in spirit with his words as quoted in the Introduction. The point is that inflation can reduce overall liquidity even if it directly taxes only cash. While it would be interesting to quantify this version, it is more complicated, and therefore that is left to future research.

6.2 Effects of Search Frictions

Motivated by claims (recall fn. 1) for the relevance of search in secondary capital markets, we now ask how much the DM arrival rate α matters.[17] One issue is how the business cycle results in Table 7 depend on α. For standard macro variables, α does not matter much, which is not too surprising given both $\alpha = 0$ (basically a textbook RBC model) and our calibrated α does well on that dimension. Of course, α matters for reallocation dynamics, since at low α the DM shuts down – no one is willing to pay the entry cost – but we are more interested in less obvious results.

To focus on the long run, Fig. 9 shows steady state as α varies, from a low point where the DM is closed to a high point of 1, for three specifications: perfect credit; constrained credit without money; and constrained credit with money. These can all generate the same outcome, since having big χ_k in the second case and having $\iota = 0$ in the third case make them equivalent to the first case. To keep outcomes distinct, consider $\chi_k = 0.20$ and $\iota = 0.02$, with other parameters at calibrated values, making the threshold for the DM to open around $\alpha = 1/3$. Perhaps not surprisingly, higher α raises output and welfare, and the effects are sizable. To put it in perspective, with perfect credit the impact on welfare of raising α to 1 is similar to the impact of lowering inflation from 10% to 0 as discussed in Section 5.3. Qualitatively, raising α works by reducing primary investment, leading to

[17]We did other similar experiments, like asking what happens if low ε firms always meet high ε firms, to focus on matching rather than search problems, or asking what happens if the DM has Walrasian pricing rather than bargaining, to focus on holdup problems. We obviously cannot report them all, but these are straightforward enough to leave as exercises.

more consumption and leisure, as firms become more confident in the secondary market. Given that these frictions matter, it would be useful to investigate alternative specifications, as mentioned in fn. 11, especially since this is known to matter quantitatively in models of household liquidity. That is left to future work.

6.3 Effects of Fiscal Policy

Of the many experiments that could be run on taxation, Fig. 10 plots steady state against τ_k, ranging from a high of the calibrated $\tau_k = 0.25$ down to large negative values, as suggested by the optimal capital subsidy discussed in Section 4. Clearly τ_k has big effects on standard macro variables here, but that is true in other RBC-style models (e.g., McGrattan et al. 1997; McGrattan 2012). However, τ_k does not matter too much for the R share. In fact, the absolute amount of reallocation moves a lot, but the R share is approximately constant because investment also moves a lot. We do see an impact on money demand when τ_k rises, since that makes reallocating capital less lucrative.

A caveat for these results is that the revenue change after adjusting τ_k is made up by adjusting the lump sum T, which is not very realistic. Still, we report that steady state welfare is maximized, given the calibration, including τ_h and ι, at $\tau_k = -0.575$. Alternatively, the optimum when $\tau_h = 0$ and $\iota \to 0$ is $\tau_k^* = -0.86$. Further investigating fiscal policy would be useful. Aruoba and Chugh (2010) solve a Ramsey policy problem in a related model of household liquidity, and find that search-and-bargaining frictions matter a lot. Again this seems to be an interesting problem, but it must be left to future research.

6.4 Decreasing Returns

A referee urged us to consider a DRS, instead of a CRS, production function, so we did. DRS is more complicated, since we lose the linearity of profit in k, but still feasible. However, when we tried to calibrate returns to scale, the data pushed us to 1. So we instead forced returns to scale to 0.9, and recalibrated other parameters taking that as given. Some results changed just a little – e.g., for business cycle statistics other than the correlation between the R share and output, the DRS version performs well if there are both productivity and credit shocks, and not otherwise, like the CRS version, although the results are perhaps slightly better with CRS.

On other dimensions, forcing returns to scale to 0.9 does significantly worse, e.g., the calibration could not match well empirical investment-output and consumption-output ratios. Moreover, the DRS specification implied all reallocation involved partial sales: DM trade always left the seller with some k because the marginal product is huge near $k = 0$. Lastly, although the DRS specification still generates a positive correlation between the R share and output (0.19 under DRS), the CRS specification matches data better on this statistic (0.48 under CRS versus 0.61 in the data). So while a DRS version is feasible, and does well on some dimensions, we do not think it is an improvement because: DRS does not allow us to study the P share; it is no better, and if anything slightly worse, than CRS on business cycle statistics; calibration wanted returns to scale to be 1; in general DRS make the analysis less tractable; and it is harder to compare with standard RBC models.

6.5 Persistent Shocks

The firm-specific shocks used above are i.i.d. Now suppose ε can be decomposed into a persistent component ε_1 and a transient component ε_2,

$$\log \varepsilon = \log \varepsilon_1 + \log \varepsilon_2.$$

Assume $\log \varepsilon_1 \in \{1 - x, 1 + x\}$, with $x \in [0, 1)$, so ε_1 is a two-state Markov process, with $\log \varepsilon_2$ i.i.d. normal. For simplicity, assume the two-state Markov process to be symmetric with a switching probability $1 - \omega$. Firms' $(\hat{k}, \hat{z})$ choices in the CM now depend on their persistent component ε_1, so we lose the degeneracy of the (k, z) distribution at the end of the CM.[18]

For now, let us concentrate on the impact of x on steady state, with $2x$ being the gap between the persistent component of high and low productivity firms. We keep other parameters the same and set $\omega = 0.75$, then vary x but keeping average productivity the same. The results are shown in Figure 11, where the horizontal axis is the gap and we report variables for both high and low ε_1 firms.[19]

If x is larger, high ε_1 firms invest in more k in the CM, and low ε_1 firms less, reflecting differences in expected future productivity, but cash holdings can go up or down depending on details including bargaining power θ. Perhaps surprisingly, for these parameters high ε_1 firms hold less cash. This is because they know their productivity is likely to be higher, and find it optimal to shift investment from the DM to the CM for two reasons: they

[18] We do maintain history independence – i.e., Lemma 2 holds conditional on ε_1 – but still this version is computationally harder. Hence, here we shut down entry and fix the number of agents in the DM. This seems fine because the intent is to compare versions of this model with different productivity gaps, not this model with the benchmark specification.

[19] Notice that when ε_1 is a 2-state process there is a two-point distribution of (k, m) after the CM, but a much more interesting distribution after the DM. Obviously it may be even more interesting to use an N-state process, although that is computationally more intense. In principle, the framework can be matched to the empirical firm-size distribution.

may not be able to get enough k in the DM; and for them the liquidity value of k is big due to χ_{Π}. Similarly, low ε_1 firms substitute out of capital and into cash for two reasons: after the CM closes it is easier to trade cash, especially the way we incorporate banking specification, than capital; and for them the liquidity value of k is small.

Liquidity in terms of cash plus credit can increase or decrease with the gap. Even if high ε_1 firms have lower liquidity, they need not be more constrained: high ε_1 buyers are less constrained than low ε_1 buyers when trading with low ε_1 sellers, since the former can leverage their size advantage. Also, notice the R share falls slightly with the gap because the DM is used to partially insure the i.i.d. component of idiosyncratic shocks. As the gap increases, the i.i.d. component contributes less volatility and investment shifts to the primary market, so the R share drops. The P share, however, increases, as some firms get bigger and hold less cash.

The last row of Figure 11 shows the composition of full sales in terms of trading parties. The first panel is the fraction of full sales where both the buyer and the seller are big, i.e., high ε_1 firms; the second is the fraction where the buyer is big and the seller small; and so on. As the gap increases, it is easier for big firms to fully purchase small firms, and the reverse is harder. A full sale is most likely to occur when a big firm meets a small firm. Interestingly, there are also a fair number of full sales where small firms buy small firms. Small firms hold a lot of cash, which allows them to fully purchase other small firms if their productivity turns out to be high.[20] On aggregate, one can show analytically that big firms are more likely to conduct full purchases than small firms. The intuition is the following. Because of the BCW bank, big and small firms have the same marginal benefit from holding liquidity, which is equal to the marginal cost of liquidity, i.e. ι. But big firms are more likely to have high productivity and would benefit more from purchasing capital. Consequently, big firms would value liquidity more than small firms if they are more likely to be constrained than small firms, which implies big firms must be less likely to be constrained and more likely to conduct full purchases.

We have also done some preliminary exploration on the short-run dynamics with persistent firm-specific productivity shocks. The main message is similar to the baseline model. However, when the gap gets bigger the model fit is not as good, suggesting more

[20]The editor, Harald Uhlig, motivated this discussion by emphasizing that it is usually big firms that swallow small ones. This stylized fact arise here as an implication of heterogeneity induced by persistent shocks, although there may be other explanatioins. We briefly mention other idea suggested by referees. One is to notice that reallocation is currently high despite an uptick in expected inflation. While complications, like a pandemic are perhaps relevant, trying to explain this situation via the model may be interesting. Another idea is to ask if decisions between partial and full sales are driven mainly by liquidity costs or agency considerations; we abstract fromt the latter, but future work could pursue this. One can also introduce vintages, with partial sales due to buyers wanting only some types of capital.

work needs to be done on models with persistent firm heterogeneity, which is left for future research because it is computationally much more intense.

7 Conclusion

This paper developed a model consistent with empirical relationships related to different types of capital reallocation and inflation. Theory predicts higher inflation lowers liquidity, which decreases (increases) full (partial) sales. This captures long- and medium-run patterns in the data. Then we added credit shocks. Better credit conditions reduce the demand for money, increasing short- but not long-run inflation, as well as increasing (decreasing) full (partial) sales. This captures business-cycle patterns in the data. Importantly, the model can also account for business cycle patterns in standard macro variables.

For some observations a nonmonetary version of the model does a decent job; for several other observations (especially reallocation) money matters. The framework also provides qualitative and quantitative insights into how bargaining, search and fiscal policy affect reallocation and standard macro variables. Additionally, it allows us to study how persistence in idiosyncratic shocks affects capital and liquidity positions. The model yields analytic results on existence, uniqueness and comparative statics, and is amenable to calibration. This suggests there may be other applications for the framework, some of which were sketched above. There is much left for additional research.

Appendix A: More on Data

Financial data are from the Flow of Funds Accounts (Z1 Report of the FRB). We use the Coded Table released in 2018; new editions may use different coding. Corporate plus noncorporate nominal debt is the sum of Debt Securities (Table F.102, item 30) and Loans (Table F.102, item 34), with the GDP implicit price deflator (Table 1.1.9 in NIPA) putting these in 2012 dollars. T-Bill and corporate bond yields, as well as PPI and CPI inflation are from FRED. Interest rates on transaction deposits are from Call Reports.

Real quantities are from NIPA in 2012 dollars. Consumption is measured by private consumption. Investment includes private investment, excluding changes in inventories, plus government investment, excluding military spending. Government spending is government consumption in NIPA. Capital reallocation sometimes involves capital previously purchased by government, but their investment is only around 1% or 2% of GDP, so that does matter much for our results. Because of cyclical features of government spending not in the model, output is measured by private consumption plus investment.

BEA also publishes industry value-added tables, and we use value-added of “Securities, commodity contracts, and investments” (available from 1997) for financial services that facilitate capital reallocation and investment in the model. For total factor productivity (TFP), we use the annual table Fernald (2014) for the growth rate, then normalize the starting year observation to 1. From this we construct TFP for each year until 2018. For labor supply, we use BEA’s hours worked by full-time and part-time employees.

For capital reallocation, COMPUSTAT (North America) has information on ownership changes of productive assets starting in 1971. Capital reallocation is measured by sales of property, plant and equipment (SPPE, data item 107 with combined data code entries excluded), plus full purchases (AQC, data item 129 with combined data code entries excluded) from 1971 to 2018. We also use capital spending (CAPX, data item 128). Since capital spending in COMPUSTAT excludes full sales, capital expenditures for each firm is the sum of AQC and CAPX. For the micro data in Section 2, industries are excluded with standard industry classification (SIC) codes below 1000 (agriculture, forestry and fishing), above 9000 (public and non-classified), and between 6000 and 6500 (financial).

Appendix B: Proofs

Prop 1: Either the constraints on p and d are both binding or both slack. Suppose they bind, and solve (13) ignoring the constraint on q. The Kalai condition $(1-\theta)\,S^b\,(\mathrm{s},\tilde{\mathrm{s}}) = \theta S^s(\tilde{\mathrm{s}},\mathrm{s})$ yields $q = Q$. If $Q < \tilde{k}$ the actual solution is $q = Q$ and the constraints on p and d bind. If $Q > \tilde{k}$ the actual solution is $q = \tilde{k}$ and the Kalai condition gives the payment. Finally, the threshold comes from rearranging $Q < \tilde{k}$. ■

Prop 2: Set $\chi_\Pi = 0$ and consider IS. If $L \le \underline{L} \equiv \varepsilon_L$, the integral in (24) is 0, and $B = \overline{B} \equiv (r+\delta) / [(1-\tau_k)\mathbb{E}\varepsilon + (1-\delta)\chi_k \iota]$. IS is decreasing and $B \to \widetilde{B}$ as $L \to \infty$, where

$$\widetilde{B} \equiv \frac{r+\delta}{(1-\tau_k)\left[\mathbb{E}\varepsilon + \alpha(1-\theta)\iint\limits_{\varepsilon<\tilde{\varepsilon}} (\tilde{\varepsilon}-\varepsilon)\, dF(\tilde{\varepsilon}) dF(\varepsilon)\right] + (1-\delta)\chi_k \iota}.$$

Thus, if L is larger the liquidity constraint is looser and opportunities for resale are better, so firms invest in more k even if the benefit from production B is low.

Now consider LM. If $L \le \underline{L}$, buyers are always constrained and $B = \underline{B}$ where

$$\iota = \iint\limits_{\varepsilon>\tilde{\varepsilon}} \frac{\alpha B\theta(1-\tau_k)(\varepsilon-\tilde{\varepsilon})}{\Delta(\varepsilon,\tilde{\varepsilon})} dF(\tilde{\varepsilon}) dF(\varepsilon).$$

Notice $\underline{B}$ increases with ι, and $\underline{B} = 0$ at $\iota = 0$. As L increases, buyers are less constrained. To make them willing to hold cash it must be that the benefit B from reallocation is higher. Notice $B \to \infty$ as $L \to \tilde{L}$, where $\tilde{L}$ solves

$$\iota = \iint\limits_{\mathcal{S}_1(\tilde{L})} \frac{\alpha\theta(\varepsilon-\tilde{\varepsilon})}{(1-\theta-\chi_\Pi)\varepsilon + \theta\tilde{\varepsilon}} dF(\tilde{\varepsilon}) dF(\varepsilon).$$

If a monetary steady state exists, it uniquely pins down B and L, and they uniquely determine w and Z/K. It remains to show K is unique. By the definition of $J(L,w)$ and (24), $J(L,w)B(w) \ge (r+\delta)/(1-\tau_k) > r$. So there is a unique $K > 0$ solving (26), and steady state is unique. Existence is standard, so details are omitted. ■

Prop 3: First, efficiency dictates that labor for any firm solves $h^*(k,\varepsilon) = [\eta u'(c)/\xi]^{\frac{1}{1-\eta}} A\varepsilon k$. Aggregating across firms gives total hours, and $h \le 1$ is assumed slack. Given this, consider a planner problem

$$\begin{aligned} W^*(k_0) &= \max_{k_{t+1}} \sum_{t=0}^{\infty} \beta^t [u(c_t) + \xi(1-h_t)] \qquad (28) \\ \text{st } c_t &= y_t + (1-\delta)k_t - G_t - k_{t+1} \\ y_t &= (1-\alpha)\int_0^\infty (A\hat{\varepsilon}k_t)^{1-\eta} h^*(k_t,\hat{\varepsilon})^\eta dF(\hat{\varepsilon}) \\ &\quad + \alpha \int_{\hat{\varepsilon}>\tilde{\varepsilon}} (A\hat{\varepsilon}2k_t)^{1-\eta} h^*(2k_t,\hat{\varepsilon})^\eta dF(\hat{\varepsilon})\, dF(\tilde{\varepsilon}) \end{aligned}$$

where output y_t includes production by the $1-\alpha$ measure of firms that did not have a DM meeting, the α measure that had a meeting and increased k, plus the α measure that had a meeting and decreased k. It is already imposed that in DM meetings the high ε firm gets all the capital, and assuming ε is i.i.d. they all leave the CM with the same k. Also, the initial condition k_0 is taken as given, as is G_t and DM frictions.

Routine methods yield the Euler equation

$$r_t + \delta = (1-\eta) A \left[\frac{\eta u'(c_{t+1})}{\xi} \right]^{\frac{\eta}{1-\eta}} \left[\int_0^\infty \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) \, dF(\tilde{\varepsilon}) \, dF(\hat{\varepsilon}) \right]. \tag{29}$$

where r_t satisfies $1 + r_t = u'(c_t) / \beta u'(c_{t+1})$. Recall that in equilibrium with $\iota \to 0$, transactions are efficient in the DM and the Euler equation for k_t is

$$r_t + \delta = (1-\tau_k) B(w_{t+1}) \left[\int_0^\infty \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha (1-\theta) \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) \, dF(\tilde{\varepsilon}) \, dF(\hat{\varepsilon}) \right],$$

where $1 + r_t = u'(c_t) / \beta u'(c_{t+1})$, $B(w) = (\eta / w)^{\frac{\eta}{1-\eta}} (1-\eta) A$ and $u'(c) = \xi / [(1-\tau_h) w]$. Comparing this with (29), one can see that $\theta > 0$ implies agents do not fully internalizes the benefits of investment, so there is under accumulation of capital under under at $\tau_k = \tau_h = 0$. But if $\tau_h = 0$ and τ_k is given by (27), the first best is achieved. ■

Prop 4: The equilibrium is the same as the monetary equilibrium with $\iota = 0$. Then, (24) defines a unique k for any $w \in (\underline{w}, \bar{w})$, where

$$B(\underline{w}) = \frac{r+\delta}{\mathbb{E}\varepsilon(1-\tau_k)}, \; B(\bar{w}) = \frac{r+\delta}{[\mathbb{E}\varepsilon + \alpha(1-\theta) I_s(\infty)] (1-\tau_k)}.$$

Suppose ε is bounded away from 0 and ∞. If $w = \underline{w}$, any sufficiently large K solves (24). If $w = \bar{w}$, any sufficiently small K solves (24). Also, (26) implies w is increasing in k. Moreover $w = 0$ if $k = 0$ and k is finite if $w = \bar{w}$. By continuity, there is a steady state. If α is not too big, the curve defined by (24) is decreasing in w, implying uniqueness. ■

Comparative statics: In what follows we set $\chi_\Pi = 0$. Then (24) defines a downward sloping IS curve and an upward sloping LM curve.

Comparative statics wrt ι: If ι increases, the IS curve shifts down while the LM curve shifts up, resulting in a decrease in L and Φ. Other variables may go up or down. If $\chi_k = 0$, the IS curve does not change, B goes up, hence c and w go down. By continuity, this holds if χ_k are small. If θ is big and χ_k is not too small, we can use (25) to eliminate ι in (24). This defines B as an increasing function in L, which we referred to as IS curve. The unique intersection of IS' and LM corresponds to the equilibrium. An increase in ι does not change the IS' curve but shifts the LM curve up. Because the IS' curve is upward sloping, both B and L increase. Therefore, w increases and Φ decreases. Now notice

$$J(L,w) \equiv \int \varepsilon dF(\varepsilon) + \alpha \iint_{\varepsilon > \tilde{\varepsilon}} (\varepsilon - \tilde{\varepsilon}) \min\left\{1, \frac{L + \chi_\Pi \varepsilon + \frac{(1-\chi_q)(1-\delta)}{B(1-\tau_k)}}{(1-\theta-\chi_\Pi)\varepsilon + \theta\tilde{\varepsilon} + \frac{(1-\chi_q)(1-\delta)}{B(1-\tau_k)}}\right\} dF(\varepsilon) dF(\tilde{\varepsilon}).$$

Therefore, both B and J decreases. Then (26) implies K and Y increase.

Comparative statics wrt χ_0: As χ_0 does not affect (24)-(26), w, K, Y and L stay the same. Therefore, $(Z + \chi_0)/K$ is constant. If χ_0 increases, Z decreases.

Comparative statics wrt χ_k: Higher χ_k shifts the IS curve down and does not affect the LM curve. Hence B and L decrease, so w increases. If $\chi_q = 1$ then L stays constant and B decreases. Thus w and K increase. Additionally, Y increases because both c and K increase. By continuity, the same is true if χ_q is not too small.

Comparative statics wrt χ_q: If χ_q increases LM shifts down and IS stays the same. Hence L increases and B decreases, w and c go up and Φ increases. If θ is close to 1, the change in B is close to 0. As a result, w and c are almost unchanged. $B(w)J(w,L)/(1-\eta)$ increases because L increases, so K and Y decrease. If θ is not close to 1, the effects on K and Y are ambiguous. Φ increases because L increases.

Comparative statics wrt χ_Π: If χ_Π increases both LM and IS shift down. Hence B decreases, w and c go up and L may go up or down.

Comparative statics wrt τ_k: This shifts up both LM and IS, so $B(w)$ increases, L increases if χ_q close to 1, and w decreases. So Φ increases, and since χ_q is close to 1, $B(w)J(w,L)/(1-\eta)$ increases, so K, c and Y decrease.

Comparative statics wrt τ_h: This does not change B or L, so w and Φ stay the same, while c decreases. Then K decreases, which implies Y decreases. Also, Z decreases because L is unchanged and K decreases. Lastly, $H = (\eta/w)^{\frac{1}{1-\eta}} AJ(L,w)K$ decreases.

Comparative statics wrt A: This does not change B or L, so Φ stays unchanged. Because $B = (\eta/w)^{\frac{\eta}{1-\eta}} A$, w increases, which implies c increases. Then both K and Y increase by the goods market clearing condition. Also, Z increases because L is unchanged. The effect on H is unknown.

Comparative statics wrt G: The argument for G is similar to A.

References

H. Ai, K. Li and F. Yang (2016) "Financial Intermediation and Capital Reallocation," *mimeo*.

G. Andrade, M. Michell and E. Stafford (2001) "New Evidence and Perspectives on Mergers," *J Econ Perspectives*, 103-120.

S. Aruoba and S. Chugh (2010) "Optimal Fiscal and Monetary Policy When Money Is Essential," *JET* 145, 1618-47.

S. Aruoba, G. Rocheteau and C. Waller (2007) "Bargaining and the Value of Money," *JME* 54, 2636-55.

S. Aruoba, C. Waller and R. Wright (2011) "Money and Capital," *JME* 58, 98-116.

S. Aruoba and R. Wright (2003) "Search, Money and Capital," *JMCB* 35, 1085-105.

J. Asker, A. Collard-Wexler and J. Loecker (2014) "Dynamic Inputs and Resource (Mis)allocation," *JPE* 112, 1013-63.

A. Berentsen, G. Camera and C. Waller (2007) "Money, Credit and Banking," *JET* 135, 171-95.

S. Betton, B. Eckbo and K.Thorburn (2008) "Corporate Takeovers," *Handbook of Empirical Corp Fin* 2, 291-429.

A. Bierdel, A. Drenik, J. Herreno, and P. Ottonello (2021) "Asymmetric Information and Capital Accumulation," *mimeo*.

F. Buera, J. Kaboski and Y. Shin (2011) "Finance and Development: A Tale of Two Sectors," *AER* 101, 1964-2002.

M. Cao and S. Shi (2016) "Endogenously Procyclical Liquidity, Capital Reallocation and q," *mimeo*.

L. Christiano (1988) "Why Does Inventory Investment Fluctuate So Much?" *JME* 21, 247-80.

L. Christiano and T. Fitzgerald (2003) "The Band Pass Filter," *IER* 44, 435-65.

L. Christiano and M. Eichenbaum (1992) "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations," *AER* 82, 430-50.

R. Coase (1937) "The Nature of the Firm," *Economica* 4, 386-405.

T. Cooley and G. Hansen (1989) "The Inflation Tax in a Real Business Cycle Model," *AER* 79, 733-48.

R. Cooper and I. Schott (2016) “Capital Reallocation and Aggregate Productivity,” *mimeo*.

S. Chatterjee and D. Corbae (1992) “Endogenous Market Participation and the General Equilibrium Value of Money,” *JPE* 100, 615-46.

J. Chiu (2014) “Endogenously Segmented Asset Markets in an Inventory-Theoretic Model of Money Demand,” *MD* 18, 438-72.

W. Cui (2022) “Macroeconomic Effects of Delayed Capital Liquidation,” *JEEA*, in press.

J. David, H. Hopenhayn and V. Venkateswaran (2016) “Information, Misallocation, and Aggregate Productivity,” *QJE* 131, 943-1005.

J. David and V. Venkateswaran (2019) “The Sources of Capital Misallocation,” *AER* 109, 2531-67.

P. Diamond (1982) “Aggregate Demand Management in Search Equilibrium,” *JPE* 90, 881-94.

D. Diamond and P. Dybvig (1983) “Bank Runs, Deposit Insurance, and Liquidity,” *JPE* 91, 401-19.

M. Del Negro, G. Eggertsson, A. Ferrero and N. Kiyotaki (2017) “The Great Escape? A Quantitative Evaluation of the Fed’s Liquidity Facilities,” *AER* 107, 824-57.

F. Dong, P. Wang and Y. Wen (2016) “A Searched-Based Framework of Capital Reallocation,” *mimeo*.

A. Eisfeldt and A. Rampini (2006) “Capital Reallocation and Liquidity,” *JME* 53, 369-99.

A. Eisfeldt and A. Rampini (2008) “Managerial Incentives, Capital Reallocation, and the Business Cycle,” *JFE* 87, 177-99.

A. Eisfeldt and Y. Shi (2018) “Capital Reallocation,” *Annual Rev Fin Econ* 10, 361-38.

J. Fernald (2014) “A Quarterly, Utilization-Adjusted Series on Total Factor Productivity,” FRB San Francisco WP2012-19.

X. Gao, T. Whited and N. Zhang (2021) “Corporate Money Demand,” *RFS* 34, 1834-66.

A. Gavazza (2010) “Asset Liquidity and Financial Contracts: Evidence From Aircraft Leases,” *JFE* 95, 62-84.

A. Gavazza (2011a) “The Role of Trading Frictions in Real Asset Markets,” *AER* 101, 1106-43.

A. Gavazza (2011b) “Leasing and Secondary Markets: Theory and Evidence From Commercial Aircraft,” *JPE* 119, 325-77.

T. Gilligan (2004) “Lemons and Leases in the Used Business Aircraft Market,” *JPE* 112, 1157-86.

P. Gomme and P. Rupert (2007) “Theory, Measurement and Calibration of Macroeconomic Models,” *JME* 54, 460-97.

P. Gomme, P. Rupert, R. Rogerson and R. Wright (2004) “The Business Cycle and the Life Cycle,” *NBER Macro Annual*.

C. Gu, C. Monnet, E. Nosal and R. Wright (2023) “Diamond-Dybvig and Beyond: On the Instability of Banking,” *EER*, in press.

C. Gu, F. Mattesini and R. Wright (2016) “Money and Credit Redux,” *Econometrica* 84, 1-32.

C. Gu and R. Wright (2016) “Monetary Mechanisms,” *JET* 163, 644-57.

G. Hansen (1985) “Indivisible Labor and the Business Cycle,” *JME* 16, 309-37.

J. Harford (1999) “Corporate Cash Reserves and Acquisitions,” *JF* 54, 1969-97.

J. Harford (2005) “What Drives Merger Waves?” *JFE* 77, 529-60.

J. Harrison and D. Kreps (1978) “Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations.” *QJE* 92, 323-36.

J. Hazell, J. Herreno, E. Nakamura and J. Steinsson (2022) “The Slope of the Phillips Curve: Evidence from U.S. States, ” *mimeo*.

C. He, R. Wright and Y. Zhu (2015) “Housing and Liquidity,” *RED* 18, 435-55.

B. Holmstrom and J. Tirole (1998) “Private and Public Supply of Liquidity,” *JPE* 106, 1-40.

K. Horner (2018) “Capital Unemployment with Search-in-Use,” *mimeo*.

C. Hsieh and P. Klenow (2009) “Misallocation and Manufacturing TFP in China and India,” *QJE* 124, 1403-48.

A. Imrohoroglu and S. Tuzel (2014) “Firm-level Productivity, Risk, and Return,” *Management Sc* 60, 2073-90

U. Jermann and V. Quadrini. (2012): “Macroeconomic Effects of Financial Shocks,” *AER* 102, 238-71.

B. Jovanovic and P. Rousseau (2002) “The Q-Theory of Mergers,” *AER* 92, 198-204.

E. Kalai (1977) “Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons,” *Econometrica* 45, 1623-30.

T. Kehoe and D. Levine (1993) “Debt-Constrained Asset Markets,” *RES* 60, 865-88.

M. Kehrig (2015) “The Cyclical Nature of the Productivity Distribution,” *mimeo*.

A. Khan and J. Thomas (2007) “Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics,” *Econometrica*.

A. Khan and J. Thomas (2013) “Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity,” *JPE* 121, 1055-107.

N. Kiyotaki and J. Moore (1997) “Credit Cycles,” *JPE* 105, 211-48.

N. Kiyotaki and R. Wright (1989) “On Money as a Medium of Exchange,” *JPE* 97, 927-54.

A. Krishnamurthy and A. Vissing-Jorgensen (2012) “The Aggregate Demand for Treasury Debt,” *JPE* 120, 233-67.

A. Kurmann (2014) “Holdups and Overinvestment in Capital Markets,” *JET* 151, 88-113.

A. Kurmann and S. Rabinovich (2018) “Dynamic Inefficiency in Decentralized Capital Markets,” *JET* 173, 231-56.

F. Kydland and E. Prescott (1982) “Time to Build and Aggregate Fluctuations,” *Econometrica* 50, 1345-70.

R. Lagos, G. Rocheteau and R. Wright (2017) “Liquidity: A New Monetarist Perspective,” *JEL* 55, 371-440.

R. Lagos and R. Wright (2005) “A Unified Framework for Monetary Theory and Policy Analysis,” *JPE* 113, 463-84.

R. Lagos and S. Zhang (2020) “Turnover Liquidity and the Transmission of Monetary Policy,” *AER* 110, 1635-1672.

A. Lanteri (2016) “The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle,” *mimeo*.

A. Lanteri and A. Gavazza (2019) “Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets,” *mimeo*.

S. Li and T. Whited (2015) “Capital Reallocation and Adverse Selection,” mimeo.

H. Li (2022) “Leverage and Productivity,” *J Develop Econ* 154, 102752

Y. Li, G. Rocheteau and P. Weill (2012) “Liquidity and the Threat of Fraudulent Assets,” *JPE* 120, 815-46.

R. Lucas (2000) “Inflation and Welfare.” *Econometrica* 68, 247-74.

V. Maksimovic and G. Phillips (2001) “The Market for Corporate Assets: Who Engages in Mergers and Asset Sales and Are There Efficiency Gains?” *JF* 56, 2019-65.

E. McGrattan (2012) "Capital Taxation During the U.S. Great Depression," *QJE* 127, 1515-50.

E. McGrattan, R. Rogerson and R. Wright (1997) "An Equilibrium Model of the Business Cycle, with Home Production and Fiscal Policy," *IER* 38, 267-90.

V. Midrigan and D. Xu (2014) "Finance and Misallocation: Evidence from Plant-Level Data," *AER* 104, 422-58.

B. Moll (2014) "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *AER* 104, 3186-221.

P. Ottonello (2015) "Capital Unemployment," *mimeo*.

T. Pulvino (1998) "Do Asset Fire Sales Exist? An Empirical Investigation of Commercial Aircraft Transactions," *JF* 53, 939-78.

V. Ramey and M. Shapiro (1998) "Costly Capital Reallocation and the Effects of Government Spending," *Carnegie-Rochester Conf Series* 48, 145-94.

V. Ramey and M. Shapiro (2001) "Displaced Capital: A Study of Aerospace Plant Closings," *JPE* 109, 958-92.

V. Rios-Rull (1996) "Life-Cycle Economies and Aggregate Fluctuations," *RES* 63, 465-90.

G. Rocheteau and E. Nosal (2017) *Money, Payments and Liquidity*.

G. Rocheteau, P. Rupert, K. Shell and R. Wright (2008) "General Equilibrium withNonconvexities and Money," *JET* 142, 294-317.

G. Rocheteau and R. Wright (2013) "Liquidity and Asset Market Dynamics," *JME* 60, 275-94.

G. Rocheteau, R. Wright and S. Xiao (2018) "Open Market Operations," *JME*.

R. Rogerson (1988) "Indivisible Labor, Lotteries and Equilibrium," *JME* 21, 3-16.

A. Schoar (2002) "Effects of Corporate Diversification on Productivity," *JF* 67, 2379-403.

S. Shi (1999a) "Search, Inflation and Capital Accumulation," *JME* 44, 81-103.

S. Shi (1999b) "Money, Capital, and Redistributive Effects of Monetary Policies," *JEDC* 23, 565-90.

S. Shi and W. Wang (2006) "The Variability of the Velocity of Money in a Search Model," *JME* 53, 537-71.

M. Sidrauski (1967) "Inflation and Economic Growth," *JPE* 75, 796-810.

J. Tobin (1965) "Money and Economic Growth," *Econometrica* 33, 671-84.

N. Wallace (1980) “The Overlapping Generations Model of Fiat Money.” In *Models of Monetary Economies*, eds. J. Kareken and N. Wallace.

T. Wong (2016) “A Tractable Monetary Model under General Preferences,” *RES* 83, 402-20.

R. Wright, P. Kircher, B. Julien and V. Guerrieri (2021) “Directed Search and Competitive Search Equilibrium: A Guided Tour,” *JEL* 59, 90-148.

R. Wright, S. Xiao and Y. Zhu (2018) “Frictional Capital Reallocation I: Ex Ante Heterogeneity,” *JEDC* 89, 100-16.

R. Wright, S. Xiao and Y. Zhu (2020) “Frictional Capital Reallocation with Ex Post Heterogeneity,” *RED* 37, S227-53.

Y. Zhu (2019) “A Note on Simple Strategic Bargaining for Models of Money or Credit,” *JMCB* 51, 739-54.

Figure 1: Inflation and Real Rates on Different Assets

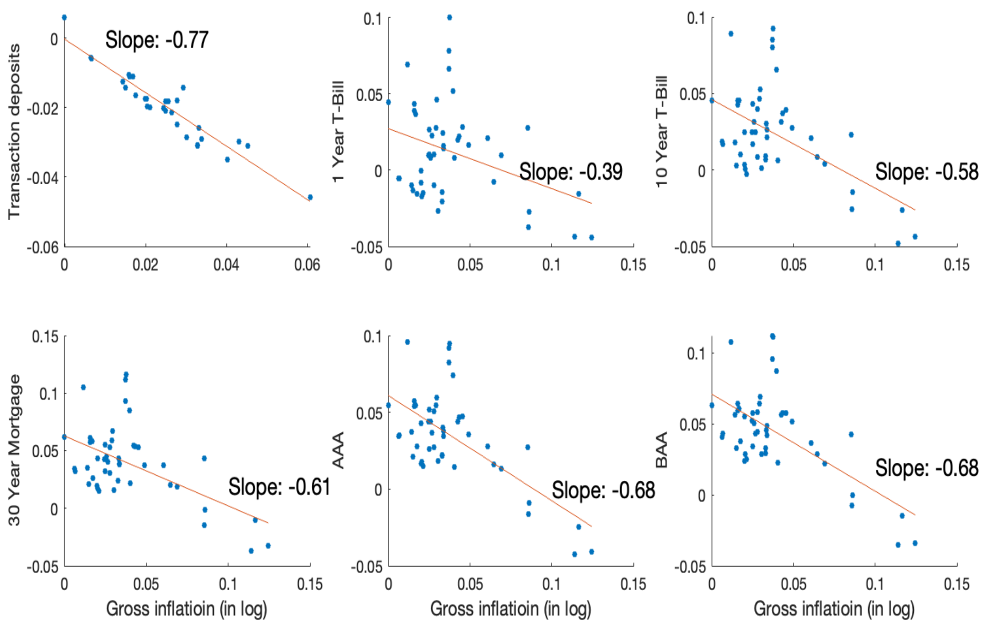
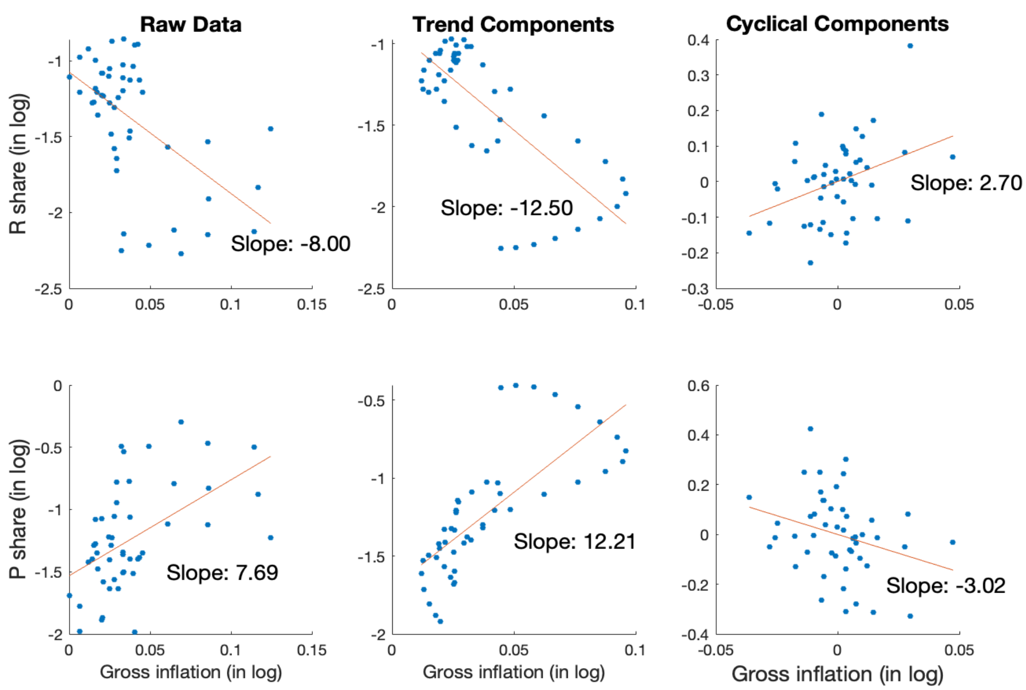
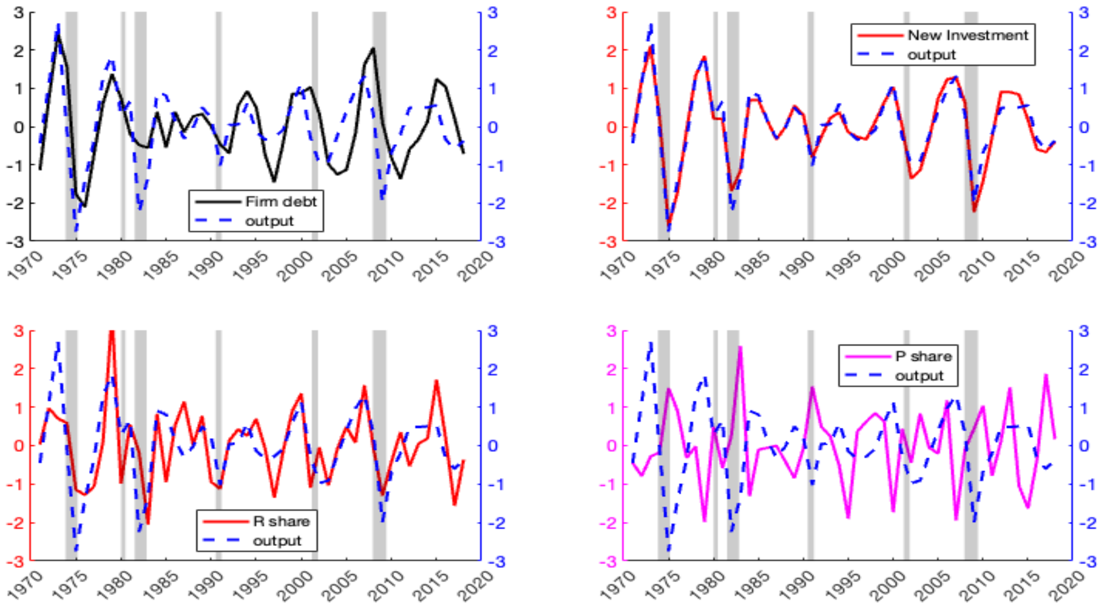


Figure 2: Reallocation and the Cost of Liquidity



Note: The R^2 values are (0.31, 0.56, 0.14) for graphs in the first row and (0.27, 0.56, 0.08) in the second row. The t-statistics for the slopes are (-4.50, -7.63, 2.72) in the first row and (4.18, 7.71, -2.01) in second row. Therefore, we reject that inflation has no effects on reallocation at least at 5% level.

Figure 3: Debt, Investment, and Reallocation



Note: shaded areas denote NBER recession dates.

Figure 4: DM Trade: Full Sale or Partial Sale

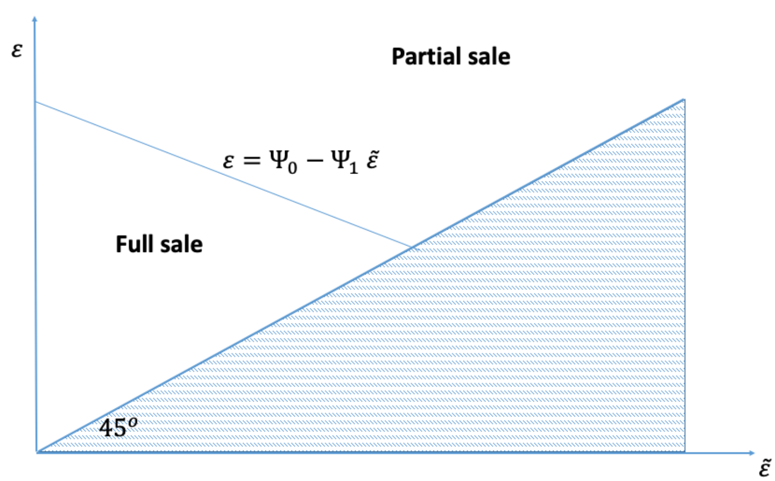


Figure 5: Steady State

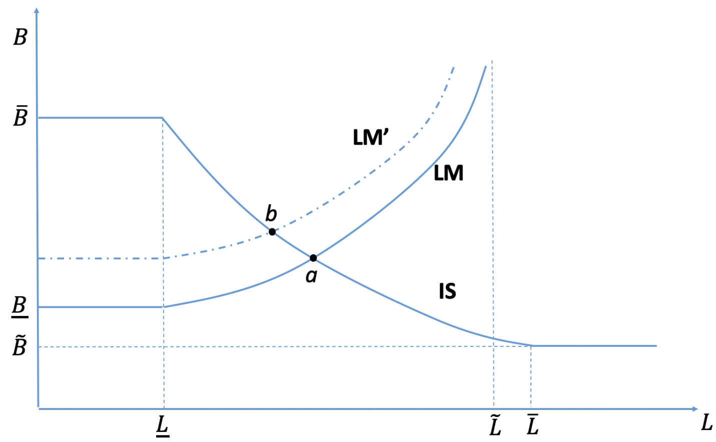
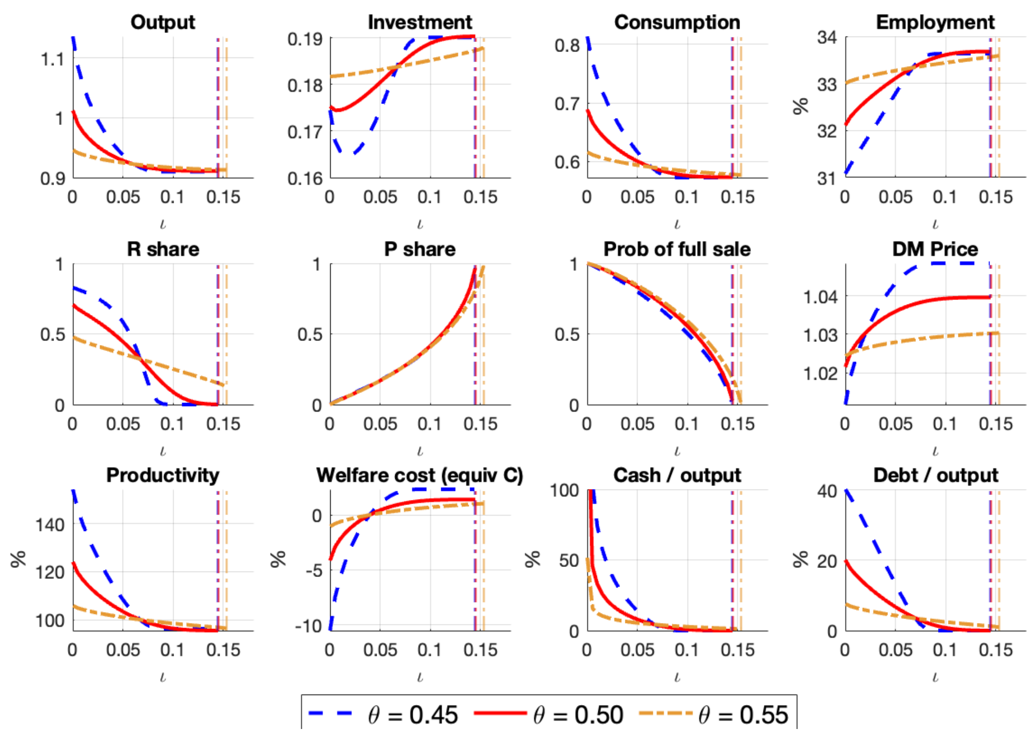
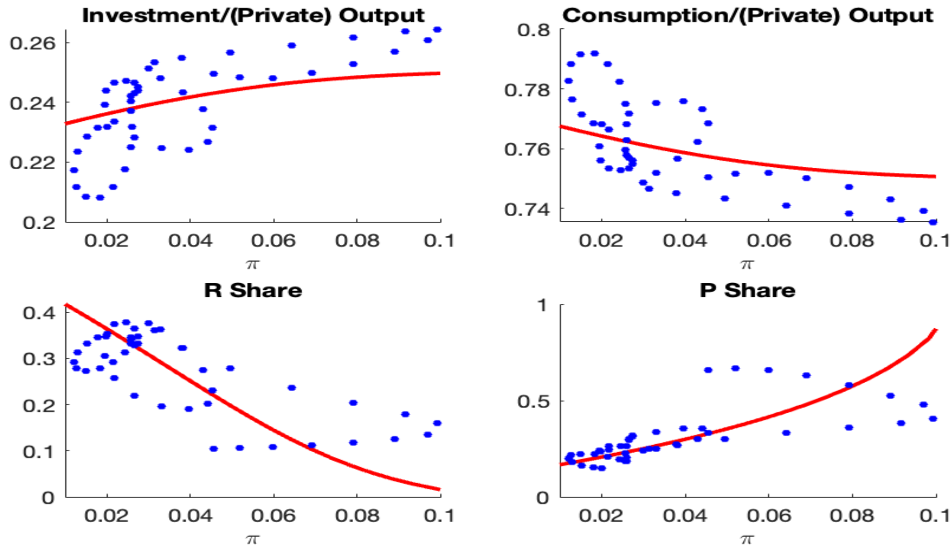


Figure 6: The Long-Run Effects of Inflation



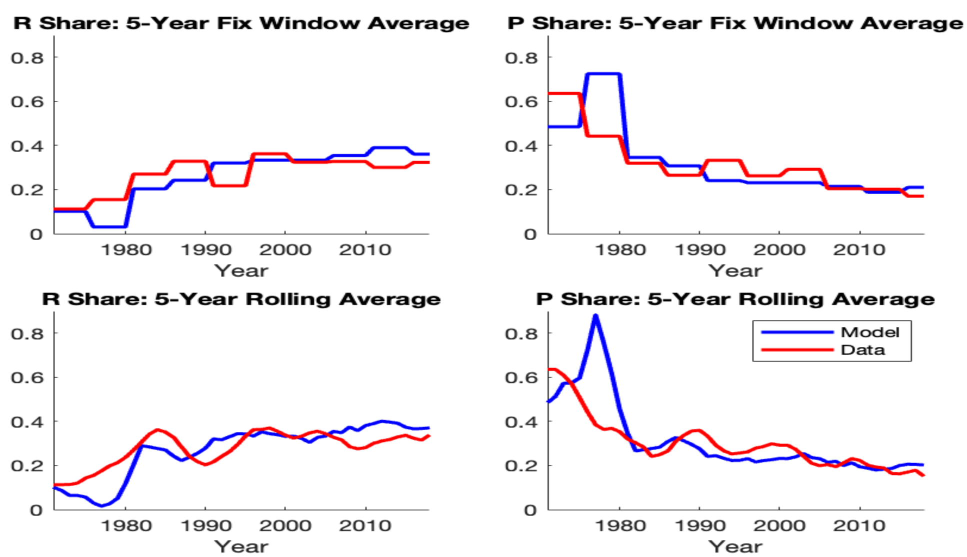
Note: The vertical lines divide non-monetary (left) and monetary (right) regions. For productivity, the level in the benchmark calibration is used as the normalization. For welfare, the level at zero inflation is used as the normalization.

Figure 7: Long-Run Effects: Data vs Model



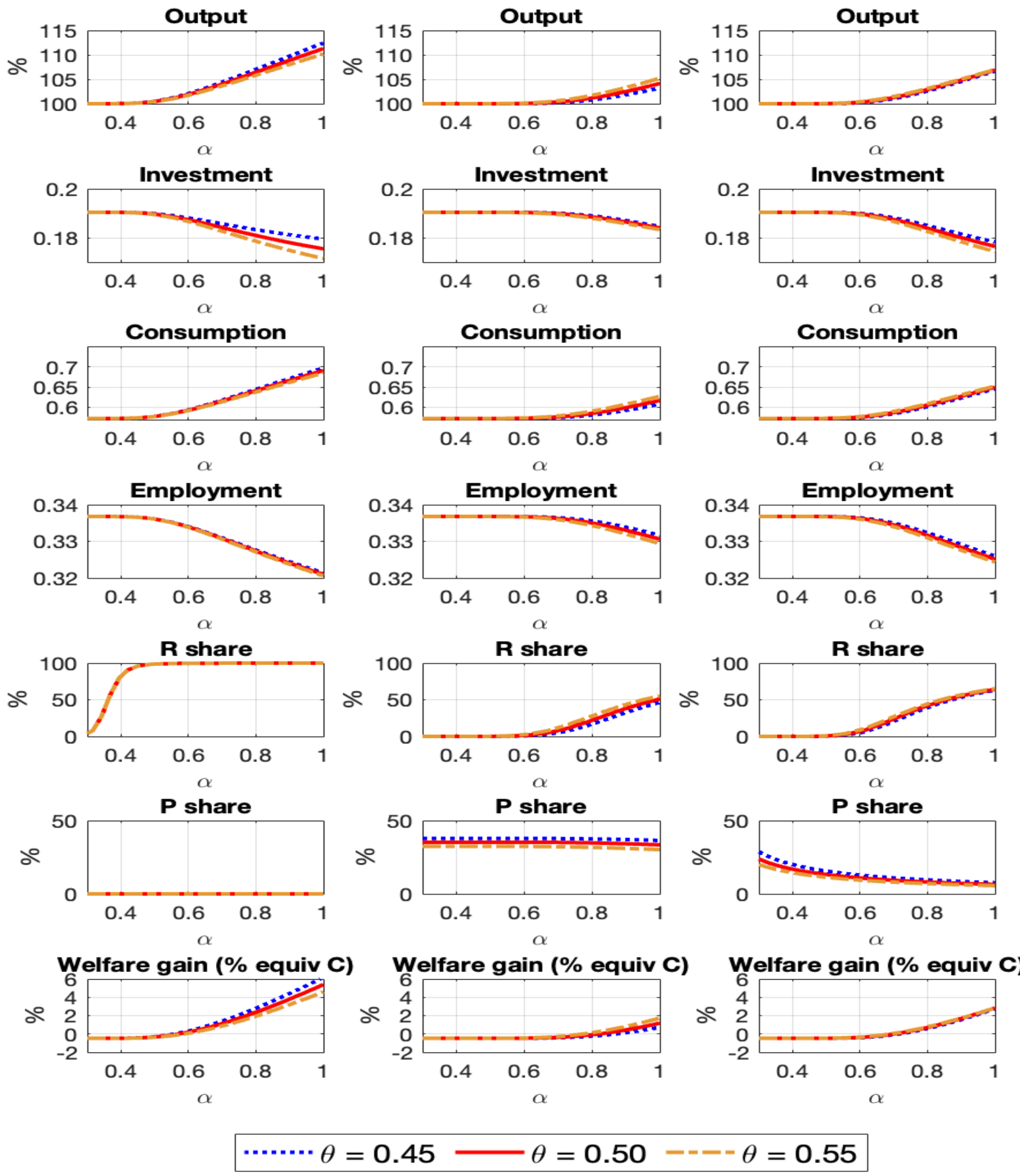
Note: Blue circles are scatter plots of data. To captures the long-run, we use the previous trend components (frequencies above 9 years) from the B-Pass filter. The results are robust with different specification. The red curves are model predictions.

Figure 8: Medium-Run Effects of Inflation: Data vs Model



Note: The figure shows R share and P share from model (blue) and data(red). Graphs in the first row are obtained by taking average in fixed 5 year windows. Therefore, the time series are constant within each 5-year window. Graphs in the second row are calculated by taking 5-year rolling average.

Figure 9: Long-run Effects of Search Frictions



Note: The first column is for the case of perfect credit; the second is for imperfect credit and no money; the third is for imperfect credit and money.

Figure 10: Long Run Effects of Capital Taxation

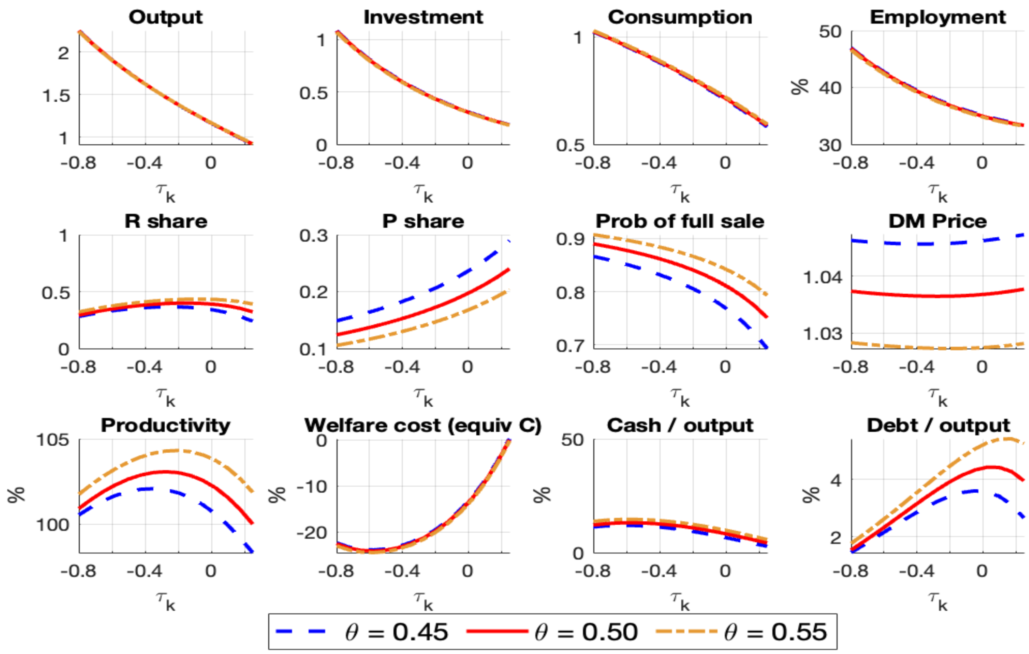


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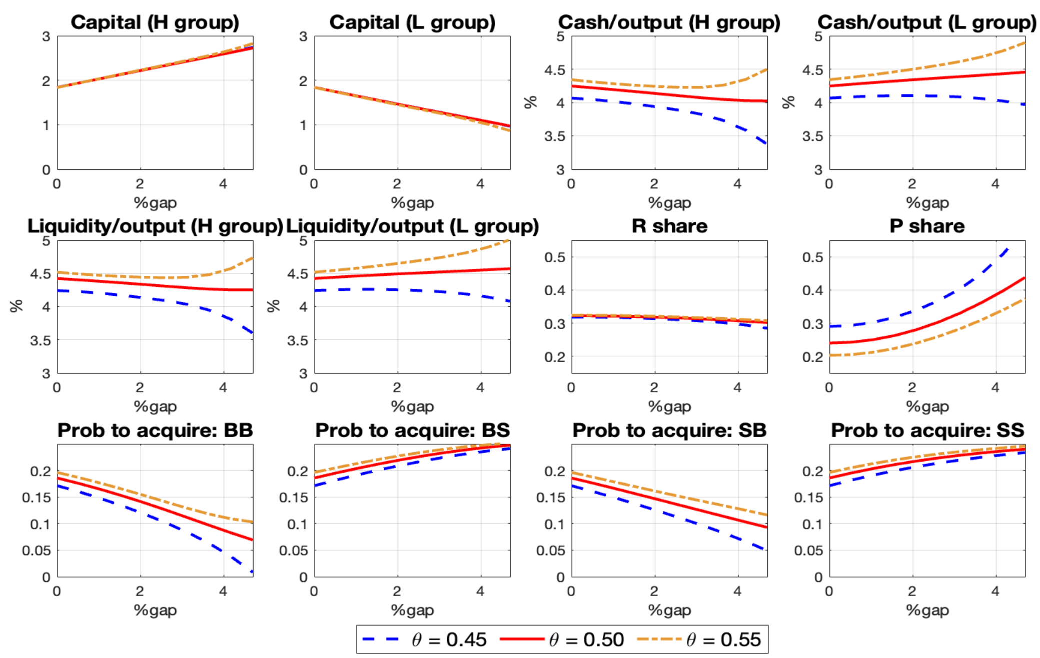
Figure 11: Effects of Differences in the Persistent Productivity Component



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