Bank Fragility when Depositors are the Asset

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Abstract

Banks' relationships with their depositors are valuable when depositors remain sticky, but this value evaporates if they leave. This tension makes banks fragile when interest rates increase and long-term asset values are depressed. In this scenario, if all of its depositors leave, the bank fails, which justifies depositors' departure in the first place. Such failures can happen even when banks only invest in liquid assets and when deposits are insured, and they are more likely for banks with the most valuable relationships. This fragility leads to sharp changes in the exposure of bank values to interest rate risk: insensitive most of the time but highly responsive when asset losses are about to catch up with them. This non-linearity complicates the evaluation of capital adequacy, with neither mark-to-market nor hold-to-maturity providing an accurate picture of bank health. We find evidence consistent with these mechanisms during the rate increase of 2022 and 2023, culminating with the failure of Silicon Valley Bank.

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1. Introduction

Banks' relationships with their depositors are valuable assets. In this paper, we argue that they are also a source of fragility when interest rates are high. The relationship of banks with their depositors has ups and downs for each party. When interest rates are low, depositors get cheap banking services as deposit rates are near market interest rates. When interest rates are high, deposit rates don't increase much, and thus the depositors pay a lot for the same banking services. Depositors typically stick through these relatively bad times because of the value of the long-term relationship – they expect better times in the future. However, if depositors expect that the bank will fail soon, they have no reason to endure the bad times, and leave immediately. Such expectations are self-fulfilling if the financial position of the bank is such that it will fail without the value of depositors. The latter is natural in high interest rate periods, since banks' hold long-term assets which lose value when interest rates rise. Such failures from a breakdown of depositor relationships can occur even if bank's assets are liquid and deposits are insured.

We propose a model to characterize the conditions leading to this fragility and its implications for bank dynamics and regulation. Banks with high leverage and high duration assets are more likely to encounter the bad equilibrium when rates increase. Furthermore, we highlight a franchise value paradox: the more valuable the banks' deposits become, the more likely depositors are to leave. In our model, depositors can leave either because they are uninsured and fear losing their money entirely, or because they fear the bank will fail and they will have to move to another bank. The latter is particularly likely if the cost of switching banks is lower, for example because online banking makes moving deposits easier.

As a bank approaches the fragile region, an on-off switch in interest rate risk is triggered. Away from the bad equilibrium, interest rate risk does not affect bank value, but as we near the fragile region the sensitivity to interest rates increases. At the point where the bank fails, all past losses on the banks' assets become realized and hence even a small increase in interest rates has a large effect near this point. We find evidence consistent with this channel during the 2022-2023 period of rising rates: large mark-to-market losses on securities during 2022 did not pass through strongly to bank equity valuations, but smaller losses from higher interest rates resulted in large declines in bank equity values.

Our model highlights the role of switching costs in understanding the behavior of depositors. Switching costs lead to depositor "stickiness" which in turn gives banks market power to exploit them. This market power leads banks to charge a spread on deposits that varies positively with the interest rate, as highlighted in Drechsler, Savov, and Schnabl (2017). In the good equilibrium, depositors accept paying higher spreads when rates are high because waiting for cheaper banking services when rates revert back dominates paying the switching cost. However, if the depositors expect the bank to fail in the near future, there is no reason to wait for better times and the optimal strategy is to pay the switching cost and leave immediately. This can lead to self-fulfilling bank failures if the remainder of the financial assets of the bank cannot ensure its survival. In other words, deposit relationships are an illiquid asset that gives the bank value but it is controlled by depositors. If the depositors leave and destroy the franchise value, the bank can not survive even if it can raise outside funding.

Importantly, the condition for such a failure occurs naturally when rates are high because banks hold long-run assets. Following Drechsler, Savov, and Schnabl (2021), such a portfolio position is the optimal way to service deposits when they remain sticky. If depositors are sticky, then the fixed costs of providing them services act like a perpetuity, and as such, the market value of the deposit liability has a long duration. To hedge this liability while providing the interest payments to the depositors, the bank invests in some short term investments and a perpetuity that exactly offsets the operating costs. So long as depositors remain at the bank, this asset position perfectly hedges the bank: a large increase in interest rates decreases the value of financial assets, but the decrease is exactly offset by an increase in the value of the deposit franchise, or equivalently a decrease in the market value of the deposit liability.

We argue that our model is a plausible explanation for some features of the events of 2022 and 2023. When rates initially rose in early 2022, bank valuations changed very little relative to what the losses on their financial assets would suggest, and bank equity valuations appeared relatively insensitive to interest rate risk. However, mark-to-market losses on bank assets continued to grow over this period and by the end of 2022 resulted in a substantial deterioration in bank's adjusted book equity of about 30%. For some banks, the losses were larger than their book equity levels. As interest rates continued to rise, bank's market equity values (stock prices) started to drop substantially and became much more sensitive to even relatively small changes in rates.

In the cross-section of banks, we show that the pass-through of losses on banks' financial assets to market equity values was far less than one for one in 2022, but appeared much larger over 2023 as rates continued to rise. In fact, the pass-through of losses on banks' assets appears much larger than one in the later period of 2023. We find that such losses pass-through and explain a large fraction of bank stock price declines even controlling for the share of uninsured deposits the bank has. Further, the declines in stock prices were larger for banks whose interest on deposits was low – exactly those for which switching would be most beneficial from the depositors perspective, and for which the deposit franchise would be the largest from the banks' perspective.

We draw inferences for debates surrounding bank regulation and accounting. Specifically, we show that in periods where the bank is far away from the possibility of the bad equilibrium, hold-to-maturity accounting makes much more sense than mark-to-market. This echoes the logic in Drechsler et al. (2021): banks' long-term financial securities are hedged by their effectively long-term liabilities, thus bank equity values are essentially immune to interest rate risk. In this region, mark-to-market losses on securities wrongly estimates a deterioration in financial health because the deposit liabilities are similarly long-term. However, as the bank nears the fragile region, where depositors might leave, mark-to-market accounting becomes much more sensible. This is because there is now a risk that depositors leave, and by leaving they destroy the franchise value of deposits. Thus, there is no longer an offsetting hedge on the liability side to offset the risk on the asset side. When enough depositors leave so that the bank fails, all current and previous mark-to-market losses on the asset side become suddenly realized.

Could the bank pursue a different risk-management policy to avoid the possibility of bad equilibria altogether? In particular, if banks might hit the bad equilibrium, where they have to redeem deposits at par, why not invest all deposits in short-term risk free securities? This issue is that these assets are a poor hedge and can lead the bank to fail. For example, in the case where interest rates fall to zero, the bank no longer earns a deposit spread, and may not be able to cover its costs of servicing depositors under this investment policy. Long-term assets – which appreciate when rates fall – protect against this scenario. The bank can overcome this issue if it starts with enough book equity.

Beyond bank-specific (that is, micro-prudential) concerns, our theory also has implications for macro-prudential regulation. At the aggregate level, what matters is whether depositors stay in the banking system and how they stay in the banking system —how much higher the rates they are earning after they move. For example, a run on a few banks entirely driven by bank-specific solvency concerns simply induces reallocation of depositors across banks and (mostly) moves the deposit franchise value from a bank to another, leaving the banking system as a whole stable. Actually, if the flow of deposits is focused on specific banks, this effectively reduces competition in the banking system, which can increase the aggregate franchise value of banks. In contrast, ”service runs” in which all depositors look for better deals are more concerning to a macro-prudential regulator. By effectively increasing competition among banks, the aggregate interest rate paid on deposits goes up. This implies that in aggregate the franchise value of the deposits go down, and the banking sector as a whole becomes more fragile.

1.1 Related Literature

Drechsler et al. (2021) study the interaction between deposit stickiness and banks' asset allocation decisions, emphasizing that deposit stickiness leads banks to take on long duration assets. Their model features zero risk in bank equity (perfect hedging) and thus can't speak directly to bank failures. Di Tella and Kurlat (2021) has a similar mechanism combined with risk-aversion by banks that generates a demand for hedging. They similarly argue that the long-duration induced by sticky deposit liabilities leads banks to take on long-term assets. Hanson, Shleifer, Stein, and Vishny (2015) show that sticky deposits gives banks an advantage to holding illiquid securities. Kashyap, Rajan, and Stein (2002) emphasize synergies between deposit taking and lending. Egan, Lewellen, and Sunderam (2022) find that deposit taking is an important source of bank value. DeAngelo and Stulz (2015) provide a model with a liquidity premium where deposits create value for banks.

Egan, Hortaçsu, and Matvos (2017) estimate a model where banks have exogenous risky assets and compete for both insured and uninsured depositors. Their model also features runs by uninsured depositors under certain conditions. We highlight the interaction of banks' risk-management and portfolio decisions with the risk of runs, particularly coming from interest rate risk and the deposit franchise.

Begenau and Stafford (2019) study whether banks portfolio decisions are well explained by passive investments in Treasury securities. In our model, this is a bad approximation during "normal" times because the value of deposits hedges long term assets, but is a much better approximation in the bad equilibrium regions where depositors run and banks have to replace with market rate funding. Similarly, Begenau, Piazzesi, and Schneider (2015) estimate a large interest rate risk exposure for banks. We help explain why this risk is only realized in certain instances where losses are large enough, but otherwise does not enter strongly into bank valuations.

Bolton, Li, Wang, and Yang (2021) provide a dynamic model where deposits provide value to banks who face costs of raising equity. They model deposits as stochastic so share the deposit flow risk with our paper. We emphasize a more sudden withdrawal when the bank is insolvent absent franchise value whereas they focus on bank behavior in low interest rate environments.

Another literature studies bank runs and bank crises. Diamond and Dybvig (1983) is the standard model of runs. Our paper shares commonality with Goldstein and Pauzner (2005) where runs or crises can be triggered by changes fundamentals, where we emphasize interest rates. Gorton and Pennacchi (1990) and Dang, Gorton, Holmström, and Ordonez (2017) both emphasize the role of banks in creating liabilities (deposits) that are "information insensitive" and thus facilitate transactions and a safe store of value. Gorton (1988) finds the banking panics are more likely after business cycle peaks, suggesting they are initially triggered by fundamentals. Our results concerning the fragility of insured deposits are driven by similar forces as the customer runs model of Hortaçsu, Matvos, Shin, Syverson, and Venkataraman (2011), who show that customer beliefs and the endogenous default decisions of firms selling durable goods can exhibit a feedback loop.[1] In a contemporaneous paper, Drechsler, Savov, Schnabl, and Wang (2023) model the impact of interest rates on the liquidity risk of banks with a focus on uninsured deposits. Jiang, Matvos, Piskorski, and Seru (2023) analyze the effect of rising interest rates on the value of U.S. bank assets. They mark-to-market losses on banks' assets due to interest rate increases and emphasize the potential for uninsured depositor runs. While our paper shares similarities to these papers for uninsured deposits, we highlight reasons why even insured depositors may withdraw funds if they believe the bank will fail.

Iyer and Puri (2012) and Iyer, Puri, and Ryan (2016) study a set of bank runs empirically using detailed depositor level data. They find uninsured depositors are more likely to run, but even

[1] See also Hortaçsu, Matvos, Syverson, and Venkataraman (2013).

insured depositors run making insurance only partially effective. The effects are long-lasting – once the relationship with depositors is broken they typically do not return.

Atkeson, d'Avernas, Eisfeldt, and Weill (2019) and Berndt, Duffie, and Zhu (2022) emphasize the role of government guarantees for bank value and argue that this value has declined over time. Atkeson et al. (2019) also argue part of the decline in banks market to book ratio comes from changes in franchise value of deposits.

Our paper speaks to the debate about whether banks should mark their assets to market. Much of the discussion centers around liquidity issues (see Allen, Carletti, et al. (2008), Heaton, Lucas, and McDonald (2010), Bai, Krishnamurthy, and Weymuller (2018) as examples that speak to this debate). In our paper the market for financial assets is fully liquid. Our paper departs from other work with liquid assets (Drechsler et al., 2021) by highlighting that mark to market may not reflect bank risk in normal times (because of offsetting effects of deposits) but makes more sense in times when interest rates increase substantially. In this sense the view is quite different from the literature on asset liquidity, where in normal times (when asset markets are liquid) marking to market makes sense but in crisis periods it does not because prices are depressed beyond fundamentals due to bank stress (see He and Krishnamurthy (2013)). The issue of how banks should account for their assets is particularly important in the recent banking crisis of 2023, since as documented by Jiang et al. (2023), monetary tightening has led to large declines in the market values of assets that are currently accounted for using hold-to-maturity values.

Cochrane (2014) argues for narrow banking to prevent runs as well as large capital buffers. In our model narrow banking – only investing in short-term risk free assets leaves the bank exposed to low interest rate environments. Large levels of capital do help prevent the run equilibrium in our model. See Admati and Hellwig (2014) for arguments that bank capital requirements should be larger.

2. Deposit Relationships and Bank Fragility

In this section, we introduce a simple model of banking focused on the value of the deposit relationship. We show that the behavior of depositors can dramatically affect the value of the bank even if the bank assets are perfectly liquid, so much so that changes in behavior can lead to bank failure. Then, we endogenize this behavior in the context of our model of switching costs and illustrate that sudden "awakenings" where all depositors exit suddenly can arise in equilibrium for a variety of reasons: fear of not being made full (as in Diamond and Dybvig (1983)), but also in presence of deposit insurance if the relationship is also valuable to depositors.

2.1 The Setting

Time is discrete and infinite. We study a bank with a measure of relationship depositors. There is an exogenous market for risk free zero coupon bonds of any maturity accessible to the bank but not depositors. There is also a competitive fringe of other banks to which depositors can move. To be clear, we will refer to the initial bank as simply "the bank."

The Bond Market. The bank can invest in risk free zero-coupon bonds of any maturity. For the sake of simplicity, we assume that a single factor, the short rate, determines the term structure of interest rates. Denote r_t the return on a one-period zero coupon bond at time t. As it will play a central role in our analysis, denote $\pi(r_t)$ the price for a standard perpetuity — a contract paying \$1 at the end of each period — at time t.

Depositors Each period, there is a measure D_0 of depositors each with \$1 to store indefinitely. If a depositor continues to store their deposit in the bank, they get a return $\tilde{r}_t$ on their dollar and some utility for the banking services provided by their current method of storage κ. For simplicity we assume that the depositor consumes this extra utility, has linear utility over consumption and banking services, and has the discount rate r_t reflecting their option to invest in cash and get no banking services. As a result, depositors' per period flow of utility is given by

$$u_t = \kappa + \frac{\tilde{r}_t}{1 + r_t}.$$

At any point in time, each depositors can withdraw their dollar and move to a new bank among the competitive fringe. The new bank provides the same level of banking services κ. Moving to the new bank requires the depositor pay a fixed switching cost ϕ. We think of ϕ as representing a search cost or the administrative cost of establishing a relationship with the new bank. For example, the depositor may need to expend effort to re-establish automatic transfers at the new bank. After moving to the new bank, the depositor can freely move among banks in the competitive fringe.

Banks and the Deposit Contract The bank takes cash from depositors and initial equity holders and invests in liquid assets, i.e., the bond market. Initial book equity E_0^b ("bankers' funds") is exogenous. The bank pays interest on deposits and provides valuable banking services. Producing these services costs the bank c paid at the end of the period. The bank offers a simple deposit contract characterized by an interest rate that is a constant fraction of the short-term risk-free rate $\tilde{r}_t = \beta r_t$ plus the right to redeem at any time. This simple contract can reflect an equilibrium in which switching costs cause banks to offer larger rate spreads when interest rates are higher because that is precisely when it is valuable to profit off of existing customers that face switching costs rather than attract new customers. We explore such a model in Appendix A.[2] The source of any potential value for bank equity holder is the deposit spread $(1-\beta)r_t$. We assume that

$$1 - \beta \geq c\pi(r_0). \tag{1}$$

This condition guarantees that the present value of the deposit spread in perpetuity exceeds the present value of operating costs in perpetuity.

The bank equity holders also have limited commitment in that they can cease operations of the bank at anytime. In the event that bank ceases to operate, all depositors receive a pro rata share of the value of the banks remaining assets up to the face value of the amount of deposits with any residual being paid to bank equity holders. We consider two cases if assets are insufficient to pay the full face value of deposits. In the first case, all deposits are uninsured and in the event the bank ceases operations, they simply get their pro rata shares of the value of the remaining assets. In the second case, all depositors are insured and they receive the full face value of their deposits if the bank ceases to operate. In either case, the each depositor must still pay the cost ϕ to establish a relationship with the new bank with their remaining deposit amount. The new bank also offers the same simple deposit contract but with a interest rate $\hat{r}_t = \hat{\beta}_t$. For simplicity we assume that

[2] In the model this occurs through a discount rate effect of higher interest rates. Banks charge higher spreads when interest rates are high because they value current profits from charging existing customers relatively more than future profits they could generate from attracting new customers.

the new bank can commit to honor this contract in perpetuity. The difference in interest rate offers could reflect a cost advantage of the new bank.

2.2 The Depositor's Problem

We first consider the problem of a single depositor fixing her beliefs about the probability the bank will continue operations and the amount of her deposit that she believes she will recover. We focus on depositor beliefs that are Markov with respect to the short rate process. Specifically, we assume that depositors believe the bank will cease operations according to a first hitting time for r_t. So for a given depositor i,

$$\tau_{i-} = \inf\{t : r_t \geq \bar{r}_{i-}\}$$

is the stopping time at which this depositor believes the bank will cease operations. We let $\theta(r_t)$ denote the depositors beliefs about the fraction of their deposits they will recover if they switch at r_t. The depositors problem is then to choose the stopping time τ_i at which to switch to the new bank that maximizes her lifetime utility. Given the nature of her beliefs, we have that τ_i must also be a first hitting time

$$\tau_i = \inf\{t : r_t \geq \bar{r}_i\}$$

for some r_i. Let $\tau = \min\{\tau_i, \tau_{i-}\}$. For a given r_i and r_{i-}, depositors i's lifetime utility is

$$U(r_i, r_{i-}) = E_0\left[\sum_{t=0}^{\tau} \delta(t)(\beta \frac{r_t}{1+r_t} + \kappa)\right] + \theta(r_\tau)E_0\left[\sum_{t=\tau+1}^{\infty} \delta(t)(\hat{\beta} \frac{r_t}{1+r_t} + \hat{\kappa})\right] - \phi E_0\left[\delta(\tau)\right]$$

where

$$\delta(t) = \prod_{s=0}^{t}\left(\frac{1}{1+r_s}\right)$$

The depositor's problem is thus to solve

$$\max_{r_i} U(r_i, r_{i-}).$$

The above formulation of the depositor's problem allows us to define what it means for the depositor to have a "relationship" with the bank in our model. The banking relationship means that even though a depositor might enjoy better terms at the new bank, she is willing to stick with her current bank as long as she can to delay paying the switching costs. For example, if a depositor forecasts that the bank will never cease operations ($\tau_{i-} \to \infty$) then she will never find it optimal to switch to the new bank ($\tau_i \to \infty$) even though the new bank offers a better bundle of deposit spread and banking services. To guarantee this is the case, we assume that $\beta, \hat{\beta}, \kappa$, and ϕ satisfy

$$E_0 \left[\sum_{t=0}^{\infty} \delta(t) (\beta \frac{r_t}{1+r_t} + \kappa) \right] > E_0 \left[\sum_{t=0}^{\infty} \delta(t) (\hat{\beta} \frac{r_t}{1+r_t} + \kappa) \right] - \phi, \quad \text{for all } r_0. \tag{2}$$

In words, switching costs are large enough that a depositor would never switch banks if she believes her current banking relationship will last forever. We will show that this condition does not imply the depositor never switches. Indeed, if she believes the bank will cease operations in some finite amount of time, she may find it optimal to leave right away. In this way the, the durability of the relationship depends on the depositors' beliefs that the bank will continue to provide services.

2.3 Would Depositors Ever Switch?

We show that when a bank is in a situation where depositors' departure would cause its failure, two equilibrium often co-exist: a good equilibrium in which depositors stay with the bank and a bad equilibrium in which depositors leave and the bank fails. To do so, we specify depositors' choice to stay with the bank. We show that the bad equilibrium can arise both when banking relationships are valuable to depositors and due to a lack of deposit insurance, as in a standard bank run. We explore the implications of depositor behavior for bank asset management in Sections 2.5 2.6.

2.3.1 Insured Deposits

In this section we show that strategic depositors may exit a bank with urgency even if deposits are insured. The key intuition is that banking relationships are also an asset for depositors. A depositor sticks to her bank as long as she perceives the present value of this relationship dominates looking for another bank.

Intuitively, it is not optimal to switch if the depositor can choose among all times to leave: the future value of the relationship is always expected to become good enough to justify staying today and indeed equilibrium prices should justify this. However, if the depositor is forced to choose a time within a close interval (because the bank will fail soon anyways) and the current flow value of the bank relationship is worse than the post-switching one (which is natural in high rates period), then the depositor will leave immediately upon expecting the bank to fail in the near future. As a result, a strategic complementarity across depositors can arise. To see this, assume that $\theta(r) = 1$ so that depositors expect they will never lose their deposit. Now consider a single depositor i and suppose that all other depositors set the switching threshold at $\bar{r}_{i-} < \infty$. The difference in the payoff to depositor i from setting her threshold at $\bar{r}_i = \bar{r}_{i-} - \Delta$, that is leaving before all other depositors, versus $\bar{r}_i$, that is leaving at the same time as all other depositors, is

$$U(\bar{r}_i, \bar{r}_{i-1}) - U(\bar{r}_{i-}, \bar{r}_{i-1}) = E_0 \left[\sum_{t=\tau_i}^{\tau_{i-}} \delta(t)(\hat{\beta} - \beta) \frac{r_t}{1 + r_t} \right] - \phi E_0 \left[\delta(\tau_i) - \delta(\tau_{i-}) \right]$$

If $\bar{r}_{i-}$ is large enough, then it is possible for the above quantity to be positive for some Δ for some dynamics of r_t. Thus, if depositor i forecasts that all other depositors will stay, she may never switch herself because even if r_t is currently high, eventually it will come down and the switching costs will have been a waste. However, as soon as she forecasts that other depositors will leave at some point in the future, then she may find it optimal to leave ahead of other depositors because the payoff gain from earning a higher interest rate is enough to compensate her for paying the switching early. Indeed, she will find it optimal to switch early precisely when the short term interest rate is high because that is when she is getting the worst return on her deposit relative to her outside option. Because of this strategic complementarity, there can be a bad equilibrium where all depositors leave at once. Alternatively, if depositor i forecasts that all other depositors will stay, i.e., $\tau_{i-} \to \infty$, then it is optimal for her to stay, and a good equilibrium obtains in which all depositors find it optimal to stay.

Importantly, this mechanism assumes that the deposits are completely insured. The assumption is that if all depositor leaves at once, the bank will no longer be able to provide valuable banking services so that depositors lose the long-term value of their banking relationship if the all other depositors leave. Thus, the deposit franchise induces a fragility within the bank that cannot necessarily be ameliorated by deposit insurance. Interestingly, the pressure to exit immediately is distinct from classic run models: it is not so much that being ahead of others is valuable, but instead that once failure is on the horizon, the depositor is better off breaking off the banking relationship immediately.

2.3.2 Uninsured Deposits

When deposits are uninsured, incentives similar to those in Diamond and Dybvig (1983) lead depositors to run if they expect their departure to cause bank failure. If depositors expect that others will not redeem and the bank survives forever, it is a dominant strategy to remain with the bank, the good equilibrium. However, if depositors expect that others will redeem and the bank will not be able to pay back all depositors, it becomes dominant to run ahead of others to get a chance as some partial redemption versus losing everything once the bank fails, the bad equilibrium.

While this run is reminiscent of Diamond and Dybvig (1983), a key distinguishing feature of our framework is that the assets that the bank purchases with depositors' funds are perfectly liquid. The bank failure obtains because of the additional intangible asset, the franchise value of deposit relationships. Because depositors can break the relationship unilaterally, this asset is effectively completely illiquid: as soon as depositors leave, it becomes worthless.

2.4 The Banks Problem

2.5 Stability with Sticky Depositors

We first consider the case in which depositors leave their cash in the bank in perpetuity. In this case, the bank is stable: it is able choose asset positions that make it risk-less, as in Drechsler et al. (2021).

Initial value of the bank. Under the assumption that depositors never withdraw, the bank earns the deposit spread $(1-\beta)r_t$ and pays the cost c in perpetuity for each unit of deposits. The initial value of the bank is equal to the initial book equity plus present value of this income and cost, that is,

$$E_0^m = E_0^b + \underbrace{D_0(1-\beta-c\pi(r_0))}_{\text{Franchise Value of Deposits}}. \tag{3}$$

This second term, the franchise value of deposits, is an intangible asset for the bank.[3] We can further decompose the franchise value of deposits into the face value of deposits D_0 and the market value of the deposit liability, i.e., the present value of the interest paid to depositors and the operating costs:

$$D_0^m = D_0(\beta + c\pi(r_0)). \tag{4}$$

Thus, we can express the initial market value of the bank as

$$E_0^m = E_0^b + D_0 - D_0^m. \tag{5}$$

Note that our assumption on the relation between β and c given in equation (1) guarantees that starting the bank produces positive NPV for the equity holders: depositors are the asset. Also note, that the value E_0^m given above holds regardless of the asset allocation of the bank as long as the bank never defaults given that depositors are sticky.

Asset allocation. We next turn to the asset allocation of the bank. The bank must meet its cash outflows $\beta r_t + c$ each period. It can do so by replicating the two components, using respectively short-term bonds — or equivalently variable-rate long-term assets — and long-term bonds. Then, the bank invests its remaining funds in one-period bonds. Specifically, the bank invests in a portfolio consisting of cD_0 units of a perpetuity, which costs $cD_0\pi(r_0)$, and the remainder $E_0^b + D_0(1 - c\pi(r_0))$ in the one-period bond. Thus the banks initial (book) assets can be decomposed as

$$\begin{aligned} A_0 &= E_0^b + D_0 && (6) \\ &= \underbrace{cD_0\pi(r_0)}_{\text{perpetuity}} + \underbrace{E_0^b + D_0(1 - c\pi(r_0))}_{\text{short-term bonds}}. && (7) \end{aligned}$$

Bank stability. As time progresses, the bank uses the cash flow from its assets to service the interest payments to depositors and operating costs. For simplicity, we assume that the residual is paid out as a dividend. Under this policy, and assuming depositors do not withdraw, the bank is

[3] The interest rate does not appear in the present-value of the deposit spread because the present-value of a payment r_t each period is exactly 1.

fully hedged in that it always has sufficient funds to cover its cash flow needs and the market value of its equity does not change.

Consider for example an interest rate increase in period $t = 1$; the formulas are unchanged for a rate decrease. On the one hand, the market value of long-term assets decreases:

$$A_1^m = \underbrace{E_0^b + D_0}_{A_0} + cD_0 \underbrace{[\pi(r_1) - \pi(r_0)]}_{\text{loss on perpetuity}}. \tag{8}$$

On the other hand, the franchise value of deposits increases:

$$D_0 - D_1^m = D_0(1 - \beta - c\pi(r_1)) = D_0 - D_0^m \underbrace{-cD_0[\pi(r_1) - \pi(r_0)]}_{\text{increase in franchise value}}. \tag{9}$$

With a high interest rate, operating costs decrease relative to the profits from the deposit spread. Putting together the tangible assets and intangible franchise value, we obtain the new market value of equity (ex-dividend):

$$E_1^m = A_1^m - D_1^m = E_0^b + D_0 - D_0^m = E_0^m. \tag{10}$$

In words, capital losses on the assets are exactly offset by changes in the franchise value of deposits. Induction immediately shows that $E_t^m = E_0^m$ for all t.

Moreover, equity holders indeed receive positive payouts each period. The net cash the bank generates at any given t after receiving interest on its short term bond position and the payout on its perpetuity and paying interest on deposits is

$$(E_0^b + D_0(1 - \beta - c\pi(r_0)))r_t > 0 \tag{11}$$

These payouts are positive because of assumption (1): the bank was positive NPV to begin with.

In sum, this investment policy perfectly hedges the bank against interest rate risk. The bank remains stable and its equity does not move at all over time.

2.6 Fragility when Depositors Leave

What happens if all depositors withdraw their funds for some exogenous and unexpected reason at time $t = 1$? In that case, the market value of bank equity is simply the market value of the assets after servicing the deposit withdrawal

$$E_1^m = A_1^m = E_0^b + cD_0[\pi(r_1) - \pi(r_0)]. \tag{12}$$

The market value of the bank jumps down relative to the case where depositors stay. We assumed that asset markets are perfectly liquid, so why does this seeming violation of the Modigliani-Miller theorem occur? This is because, when depositors leave, they do more than redeeming their funds. Depositors also break their relationship with the bank, effectively destroying this intangible asset. Said otherwise, the franchise value of deposits is an illiquid asset that is lost if depositors leave.

When interest rates have increased before redemptions, losses on assets that were previously compensated by the increases in the present value of the profits of the low interest rates paid to depositors catch up with the bank. If these losses are large enough, it becomes rational for any single depositor to withdraw if they expect other to do so. Note that here the value of the assets are independent of depositor behavior. Depositors themselves are the asset, so the consequences of their behavior cannot be easily fixed with a capital injection as it would be possible in Diamond and Dybvig (1983) where a deep pocked investor could buy the illiquid assets and hold to maturity. This situation happens if the market value of book equity E_1^m becomes negative, which corresponds to the condition:

$$c[\pi(r_0) - \pi(r_1)] \geq \frac{E_0^b}{D_0}. \tag{13}$$

An intuitive way to rewrite this condition is

$$\underbrace{\frac{D_0 + E_0^b}{E_0^b}}_{\text{Book Leverage}} \times \underbrace{\frac{-cD_0\pi'(r)}{D_0 + E_0^b}}_{\text{Asset Duration}} \times \Delta r \geq 1. \tag{14}$$

This result contrasts sharply with the conclusion that banks are stable when depositors are sticky. If depositors leave, traditional indicators of interest risk are the right way to think about bank failure: large increases in interest rates, long duration financial assets, and higher book leverage all make the bank more likely to succumb.

Interestingly, a "franchise value" paradox arises: banks with the most valuable deposits in high interest rate regimes are the most at risk of failing if depositors leave. In our setting, banks with high service cost c — likely because they provide more service with a lower deposit β — have a franchise value that is more sensitive to the interest rate. Therefore their franchise value is more valuable when rates are high. However these banks are also more fragile: they choose assets with higher duration, and are left with much larger asset losses if depositors leave.

This bank fragility due to an evaporating franchise value leads to the natural question of what prompts depositors to leave in an *unexpected* way. If the bank expected specific patterns of deposit inflows and outflows, it would adjust the computation of its deposit franchise value and change its asset strategy accordingly. As long as the bank's assets can replicate the evolution of franchise value, the bank can remain stable. However, the bank can always remain vulnerable to unexpected shocks that it cannot hedge. Such a potential shock could be the unexpected entry of more efficient competitors, say a rapid adoption of online banking by a large swath of the population.

However, the previous section shows that banks' fragility to depositor behavior can sow the seeds of their failure. If a bank can fail when depositors leave surprisingly, depositors are likely to leave surprisingly.

3. Bank Dynamics Ahead of Failure

We have demonstrated that the franchise value of deposits leads to the emergence of situations in which depositors suddenly leave and the bank fails when rates are high enough. In this section, we show how this fragility manifests itself in asset dynamics ahead of failure. These results are useful because they offer a path for detecting the importance bank fragility before it is too late. We also show how they inform the debate of whether asset values of banks should be marked to market.

3.1 The on-off switch of interest rate risk

In the stable setting of Section 2.5, the bank's market value is constant. We have showed that, when rates are high, this value can suddenly jump to 0. If market participants are aware of this possibility, what are the dynamics of market value? We illustrate this behavior in a simple case with two assumptions. First, we assume that the bank sticks with the no-fragility asset allocation. Second, we assume that depositors leave as soon as the bank hits the point where its value without depositor relationships is negative — the most unstable situation possible.

To characterize the market value, we need to specify the dynamics of interest rates. For our calculations, we focus on an AR(1) model with a reflective boundary at 0. Appendix Section B shows how to easily compute the market value of equity in this setting in the limit of continuous time.

Figure 1 illustrates the behavior of the value of the bank as a function of the market value of its assets. The red dashed line corresponds to the stable case: bank value is constant, equal to its initial value. In contrast the dashed blue line represents the value of equity if depositors leave immediately. In this case, there is no franchise value of deposits to offset movement in the price of the asset. Gains and losses on assets pass through one-to-one to the value of the bank. Depositors leave the bank as soon as the bank value without them is 0. This is the point $\bar{A}^m$ at which the blue line cross 0.

The solid black line is the market value of equity. When rates are low, we are towards the right of the graph. Asset valuations are high and a departure of depositors is unlikely. Hence the bank's value is close to what it would be with no prospect of liquidation at all. The pass-through of asset risk is effectively turned off: an econometrician would be hard pressed to distinguish this situation from the full stability case in which firm value is completely unresponsive to asset values and thus unresponsive to changes in interest rates. In contrast, as interest rise, asset values drop and we move towards the liquidation threshold $\bar{A}^m$, so the bank value behaves more like its value when depositors leave. The pass-through of asset risk is turned on: the bank value responds strongly to asset value.

Actually, one can observe that the market value of equity falls more precipitously that the equity value with immediate liquidation: the slope of the black line (market value of equity) be-

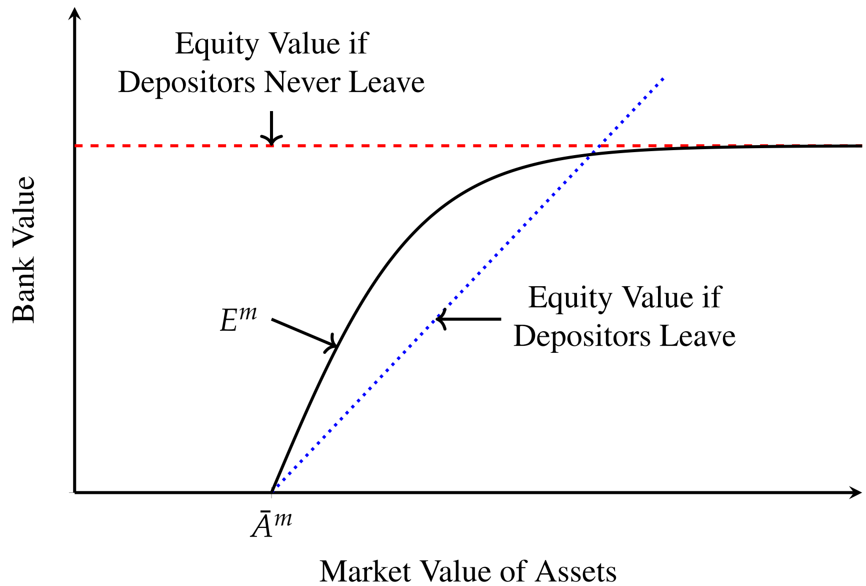

Figure 1: The Market Value of Equity when Depositors Can Leave. The red dashed line is the value the bank if depositors never leave as in the benchmark. The blue dotted line is the liquidation value of the assets less the face value of deposits. The black curve is the market value of equity when depositors can run.

comes steeper than the slope of the blue line (the value of the financial assets). In other words, the pass-through becomes larger than 1 when close to the threshold — a property that holds generically. This is because two effects combine to hurt the market value of equity as the interest rate increases. First, the liquidation value deteriorates, a pass-through of 1. Second, the likelihood of liquidation increases as well, so previous asset losses catch up with the bank.

3.2 Should Bank Assets Be Marked to Market?

An important debate is whether bank assets should be marked to market. In our model, market participants understand well all the details of how the bank functions. In practice, accounting reports are a useful tool for market participants and regulators to assess the health of banks. Across different jurisdictions, time periods, and bank sizes, the accounting of banks have followed different conventions in assessing a book measure of the equity of banks. One key distinction is that asset values are sometimes estimated as "held-to-maturity," that is, insensitive to market condition, or measured at "fair value," that is marked to market.

We can compute these two book values in our model:

$$\text{Hold-to-maturity: } E_1^b = A_0 - D_0 = E_0^b,$$
$$\text{Mark-to-market: } E_1^b = A_1^m - D_0 = E_0^b + cD_0[\pi(r_1) - \pi(r_0)].$$

In the stable case of Section 2.5, the hold-to-maturity book value coincides exactly with market equity. How can this be the case despite economically meaningful movements in asset prices? One way to understand this result is to notice that hold-to-maturity accounting ignores fluctuation in the value of assets, but also ignores fluctuations in the value of the intangible asset, the franchise value of deposit. Because the asset allocation decision is such that these two sources of fluctuations exactly offset each other, two wrongs make a right, and hold-to-maturity accounting provides an accurate picture of bank health. In contrast mark-to-market accounting is only marking the tangible assets, but not the intangible franchise value, and so wrongly estimates losses when rates increase. The only way to "fix" mark-to-market accounting is to also evaluate the fair value of deposits. Another interpretation of the success of hold-to-maturity accounting is to notice that the assets are effectively riskless because they are held to maturity in a specific sense: their payouts (perpetuity and risk-free coupons) exactly match liability payments (operating costs and interests on deposits).

When fragility is strong like in the previous section, neither accounting values coincide with the market value of equity. However, if one cares specifically about using a measure of capital to estimate distance of the bank to failure — the goal of Basel capital regulations — then mark-to-market equity is the accurate answer. Indeed, this value moves in lockstep with the value of the bank if depositors leave immediately (the dashed blue line in Figure 1). Because the bank fails when this liquidation value hits 0 keeping track of its evolution correctly measures the health of the bank.

In practice, we likely live somewhere in between these two extremes: banks become fragile when rates are high, but do not necessarily immediately fail. There is a chance of staying in the good equilibrium and a chance of going to the bad equilibrium. This dichotomy suggests that neither of the accounting estimate is a panacea. Instead an accurate measure of default risk would put some weights on these two measures. Or, analogously to the distinction between capital ratio and leverage ratio, bank regulators should require banks to report and hold a minimal level for both of these measures of equity capital.

3.3 How Much Capital Do Banks Need?

Beyond measuring the evolution bank health, a common concern is assessing whether banks have enough capital to survive the potential shocks they face. We ask how much initial capital E_0^b the bank needs to be able to survive forever. To make results clearer, we focus on the case where the initial franchise value of deposits is 0 — that is, condition 1 holds with equality.

Consider first the stable case, where bank value is constant. In this situation, the bank needs no initial capital whatsoever. It can perfectly replicate the payments it needs to make to depositors by investing in liquid assets. Actually, this bank is an example of a "narrow bank:" it is only involved in deposits and does not take engage in risky lending activities. Because it has no additional source of risk, the bank is perfectly stable and needs zero capital.

However, as long as depositors have the potential to leave, the bank needs additional equity if it wants to avoid the region with liquidation. Using static replication arguments, we can characterize some equity amounts that avoid spending any time where liquidation can happen. We study two cases: i) the bank can engage in arbitrarily complex interest rate contracts, or equivalently arbitrarily complex dynamic asset strategies, ii) the bank must choose a fixed combination of perpetuity and short-term bonds. In addition we maintain the assumption that excess interest payments are paid as dividends to shareholders each period.

In both cases, notice that the bank must at least invest in a portfolio that allows it to meet all interest payments to depositors (invest βD_0 in short-term assets) and pay its operating costs (invest in cD_0 units of perpetuity). If the bank has 0 initial equity, the value is exactly equal to the initial cash inflow from deposits because of the no-profit condition. The bank will enter the fragile region as soon as interest rate rise. When the bank engages in arbitrary contracts, the only way it can always have enough cash is by entering a contract that effectively injects just enough new cash each time interest rates rise above the previous maximum. This way, the bank is always sure to continue to survive. The value V_{inject} of such a contract is:

$$V_{inject} = \int_{r_0}^{\infty} cD_0(-\pi'(r))T(r)dr, \tag{15}$$

where $T(r)$ is the expected discount factor to the first hitting time of r. If the bank has exactly this amount of initial equity, it can avoid getting to a fragile region altogether.

When the bank is restricted to potentially more realistic static strategies, it needs enough assets upfront to replicate both the liquidation value D_0 and market value $D_t^m = (\beta + c\pi(r))D_0$. The only way to hit both of these targets for all values of the interest rate (because $\pi(r) \to_\infty 0$) is to invest D_0 in short-term assets and $c\pi(r)D_0$ in perpetuities. Because of the no-profit condition, the initial value of this portfolio is exactly equal to $(1-\beta)D_0$. This is a very large amount of capital: for an empirically realistic value of $\beta = 0.3$, this implies that the initial book equity needs to be 70% of deposits. This would translate into a leverage of only 2.5, much less than typical values around 10.

These calculations suggest that even when banks are narrow and focus on deposits, stabilizing the fragility of the franchise value of deposits requires a substantial amount of capital.

4. Bank Valuations during the 2022-2023 Interest Rate Rise

We study the relation between interest rates, asset values, and bank stock prices during the between January 2022 and March 2023. During this short period, interest rates rose steadily from 0 to 4.5%. On March 10, Silicon Valley Bank, a large regional bank with about $250 billions in asset experienced a run and was taken over by the FDIC, which announced guarantees for both insured and uninsured deposits. Even after this intervention, much of the banking sectors' valuations remain depressed.

We combine data from the call reports on bank balance sheets with stock return and interest rate data. All 2022 values are cumulative for returns and use data from the fourth quarter of 2022 from the call reports. Data from 2023 uses the period up until March 14th. We focus on the top 50 banks by assets and merge with stock returns when available.

4.1 Aggregate Bank Stock Returns and Rising Interest Rate Risk

Figure 2 Panel A plots the KBWB ETF index which tracks a value-weighted basket of bank stocks. We plot the cumulative raw index value, normalized to zero at the start of 2022, as well as a cumulative abnormal return obtained by regressing daily returns on the S&P500. The yield on the

2 year Treasury, on the right axis, which we use here as a proxy for the stance of monetary policy, shows a steady increase in interest rates over this period. Yet, the bank stock index only declines substantially in March of 2023. Steeply rising interest rates in the 2022 period were not associated with declines in bank equity values, consistent with the losses on long-term securities being at least partially offset by higher deposit spreads and an increase in the value of the deposit franchise. However, the associated losses on long-term securities coming from higher interest rates meant that the banks would be closer to insolvency if depositors left and destroyed the franchise value, i.e., the franchise value only goes up with interest rates if depositors are expected to stay with the bank. We argue this risk of a depositors flight made bank stocks far more sensitive to increases in interest rates as rates continued to rise and a higher and higher share of banks values came from an illiquid asset, the deposit franchise.

Consistent with this idea, Panel B of Figure 2 plots the coefficient of bank stock returns on the negative of the change in 2 year Treasury yields using daily data and rolling 6 month regressions. We see this sensitivity increase over the course of 2022. In unreported results, we find this is also true for bank stocks more so than the rest of the stock market, which we show by comparing the interest rate sensitivity of bank stocks to that of the S&P500. We now turn to the cross-section of banks to explore these issues in more detail.

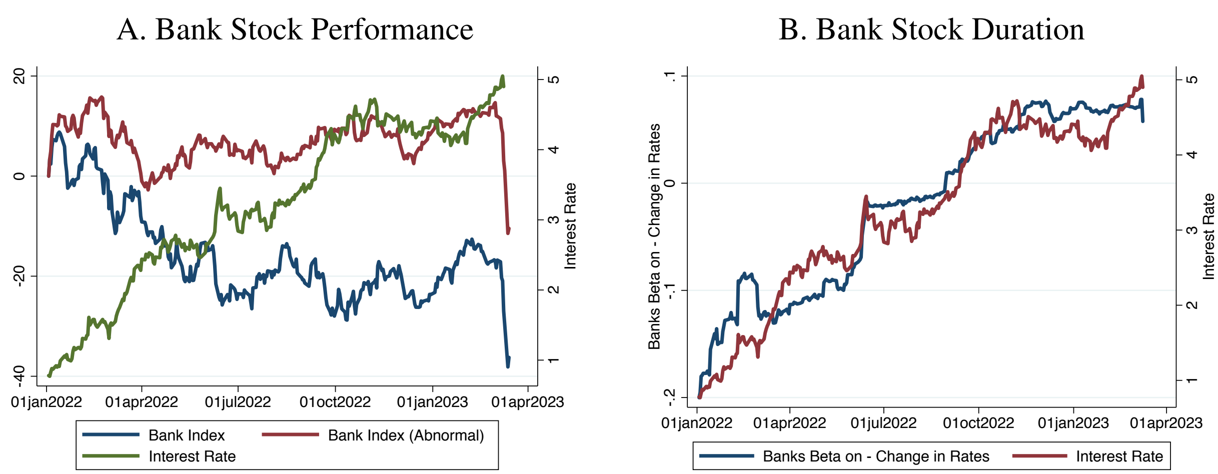

Figure 2: Aggregate Bank Stock Index and Interest Rates.
Panel A plots the cumulative return and abnormal return on the KBW bank stock index along with the 2-year Treasury yield. Panel B plots the sensitivity of the bank stock index returns to interest rates. We plot coefficients from rolling regressions of the returns on the aggregate bank index regressed on the negative of the change in 2-year Treasury yields. We use daily data and 6-month rolling windows. We plot the interest rate level (2 year Treasury yield) to show that this sensitivity rose as rates rose.

4.2 The On-Off Switch in Interest Rate Risk in the Cross-Section

We explore these issues more directly by studying the relation between bank stock returns in 2023 and mark-to-market gains or losses on banks' securities coming from the call report data. Specifically, we use hold to maturity and available for sale securities and compute losses or gains as the difference between their book value and the fair values as reported in the call report. We normalize these losses by total assets. We emphasize this is not a complete picture of losses on the banks' assets, as it does not adjust for other losses on the entire balance sheet or factor in any potentially offsetting derivatives positions.[4] However, we take it as a proxy for losses on banks' assets coming from higher interest rates. The median loss across banks in 2022 is about 2.6%, which given a median leverage of just over 10, translates to a median implied mark-to-market decline in book equity of about 30%. Losses on the banks' assets should be associated with a lower stock return, absent any offsetting hedge coming from the value of the deposit franchise.

Table 1 regresses bank stock returns on these asset losses. Column 1 shows that for 2022 stock returns, asset losses have a coefficient of around -3, but are not statistically significant. In a perfect pass-through model, with no offsetting hedge from the deposit franchise, this number would be equal to the average leverage of the banks in our sample (around 10). We see stronger coefficients in columns 2 and 3 when using stock returns in 2023. In particular, column 3 shows that the 2022 losses strongly pass-through to the stock returns in 2023 with a significant coefficient of -7. This is the sense in which unrealized losses become realized in the share price. In the fragile region of our model, all past and current losses catch up with the bank, which can explain why even past losses can show up in returns following bad shocks.

While we only have losses on securities for 2022 from the call reports, in column 3 we proxy for losses in 2023 by using the 2022 losses and the continued increase in rates in early 2023. Specifically, we use the 2022 losses along with the increase in interest rates over the course of 2022 to construct implied duration of the securities a bank holds. We then use the change in interest rates through March 14th of 2023 and the same duration number to proxy for mark to market asset losses over the 2023 period. We see substantially a substantially larger coefficient of -55, a value far too large to be explained by a mechanical pass-through channel. This channel would

[4] Our total numbers for losses on securities roughly matches the numbers in Jiang et al. (2023) when excluding losses on loans.

Table 1: The pass-through of Asset Losses to Bank Stock Returns

This table presents stock return regressions in the cross-section of banks. Column 1 uses 2022 cumulative returns for each bank in our sample, columns 2-3 use returns from 2023, and column 4 uses the cumulative return from the start of 2022 through mid-March of 2023. Losses are the difference between book value and fair value of securities taken from 2022Q4 call reports. Losses 2023 computes implied losses over the 2023 period using implied duration from the 2022 losses and changes in interest rates over 2023. Standard errors in parentheses. See text for details.

	(1)	(2)	(3)	(4)
VARIABLES	Stock Ret '22	Stock Ret '23	Stock Ret '23	Cumulative Ret
$Losses_{2022}$	-3.09		-6.91***	
	(2.08)		(2.26)	
$Losses_{2023}$		-55.71***		
		(18.19)		
$Losses_{Cumulative}$				-5.92***
				(1.78)
Observations	44	44	44	44
R-squared	0.13	0.31	0.31	0.27

predict an upper bound on the interaction term coefficient of around 10 given the typical banks' leverage ratio, while the actual coefficient is over 55 in magnitude. Both column 2 and column 3 suggest a pass-through of losses much that is much larger in the 2023 data. This is consistent with an increased sensitivity to losses and pass-through greater than one-to-one as banks approach the region where depositors leave.

Column 4 uses the cumulative stock return from 2022 to 2023 regressed on the cumulative asset losses. We find a large significant negative coefficient of 6. This suggests that a large part of the losses have been passed through to returns. Recall again that conditional on the bank failing in our model, all cumulative losses will be passed through to the share price as the value of the deposit franchise erodes, not just current losses. Note as well this is a purely cross-sectional regression, so it omits the average decline in bank stocks that might come from other broader aggregate economic channels. Further, because of substitution across banks, one would expect the overall banking system to be less sensitive to aggregate losses as we see through 2022: stronger banks–even if facing losses– might well benefit from an abnormal increase in charter value as deposits flow out of weaker banks towards stronger banks.

Figure 3 shows these regressions in a scatter plot, with a far steeper slope in the 2023 period. In our model, the sensitivity of losses is low or zero when the bank is far from the multiple equilibrium region, but increases substantially as banks' approach this region. In contrast a view where losses are simply passed through to market values would predict similar slopes.

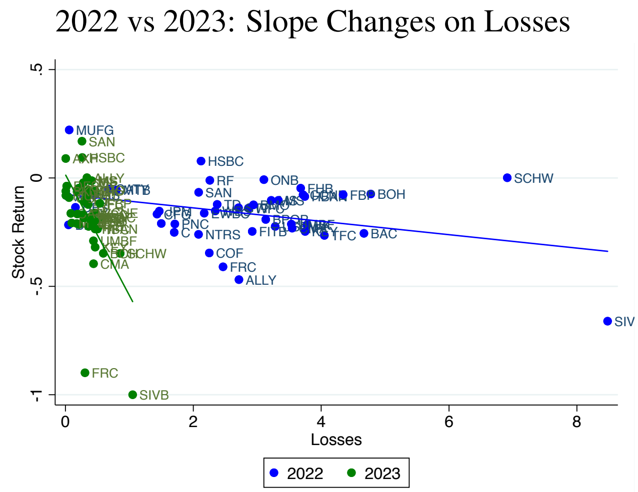

Figure 3: Bank Stock Returns and Asset Losses on Securities.

Overall, these results are strongly consistent with the on-off switch of interest rate risk in our model, where all the cumulative losses start showing up quickly once the bank becomes likely to reach the multiple equilibrium region. This naturally explains why pass-through is normally low but becomes much larger than 1 as losses mount.

4.3 What Determines Whether Asset Losses are Passed Through?

We've argued that the on switch triggers losses on banks' balance sheets to be passed through to stock returns when interest rates rise enough to make deposit relationships fragile. In our model this can trigger depositors to leave the bank for several reasons. The first is that they have a run incentive if they are uninsured. We show that uninsured deposits across banks on their own do not explain the declines in stock returns. However, consistent with uninsured deposits being more run-prone, we find that the interaction of uninsured deposits, leverage, and losses is strongly associated with stock returns in the cross-section.

Next, we show that low deposit rate banks were hit the most, regardless of whether they had uninsured deposits. This is consistent with the fact that (1) low deposit rate banks have the most to lose if depositors leave in terms of franchise value, and (2) low deposit rate banks suffered the worst losses on their asset side because they took on the most duration risk.

4.3.1 The Role of Uninsured Deposits

We show that uninsured deposits alone do not correlate with declines in stock returns, but that uninsured deposits are an important source of amplification for the pass-through of losses on securities to stock returns. Specifically, Table 2 shows that both the 2022 and 2023 drop in stock returns were not strongly related to the uninsured deposit share across banks. We compute this share as the fraction of uninsured deposits to total deposits. A regression of stock returns on uninsured deposits alone produces insignificant results. Similarly, in a regression including the uninsured share, losses, and leverage, the uninsured share is also insignificant. Asset losses show up negatively, particularly for 2023, as before. Of course the share of uninsured deposits can play a role in exacerbating a run when losses are present. But many banks with large losses can still be at risk of depositor withdrawals even if they are insured. This could come from it being easier to switch to higher interest rate banks. High interest rates heighten the risk of depositor withdrawal on bank values, regardless of the reason for withdrawal.

While the uninsured share of deposits alone does not explain which bank stocks fell the most, the interaction of uninsured deposits with losses is associated with stock returns in the cross-section. Specifically, leverage interacted with losses and uninsured deposits has a strongly negative coefficient, and provides somewhat more explanatory power than losses and leverage alone. This is intuitive: the largest drops in stock returns come from banks with a combination of large asset losses, high leverage, and a higher share of uninsured deposits. Once again, the magnitude of this relationship is much stronger in 2023 than it was in 2022.

Figure 4 plots leveraged losses against stock returns, and shows that this relation is somewhat stronger when the share of uninsured deposits is high. The relationship is also significantly stronger in 2023 than in 2022.

4.3.2 The Franchise Value Paradox: Low Deposit Rate Banks Are More Fragile

We have shown the pass-through of asset losses is larger in 2023, and the pass-through is somewhat stronger for banks with larger share of uninsured depositors, despite the uninsured share on its own providing little information. We now show that the interest banks pay on deposits is a strong indicator of both whether they had losses on securities and whether these losses are reflected in their

Table 2: Stock Returns, Leverage, and Losses

This table presents stock return regressions in the cross-section of banks. Columns 1-3 use 2022 cumulative returns for each bank in our sample, columns 4-8 use returns from 2023. Leverage, losses, and uninsured are book leverage, mark to market losses, and the fraction of uninsured deposits taken from 2022Q4 call reports. Losses 2023 computes implied losses over the 2023 period using implied duration from the 2022 losses and changes in interest rates over 2023. Standard errors in parentheses. See text for details.

VARIABLES	(1) 2022	(2) 2022	(3) 2022	(4) 2023	(5) 2023	(6) 2023	(7) 2023	(8) 2023	(9) Cum	(10) Cum
Uninsured (Un)	-0.18	-0.09		-0.34	-0.22		-0.22		-0.19	
	(0.15)	(0.13)		(0.23)	(0.16)		(0.16)		(0.16)	
$Loss_{22}$		-3.81*			-6.24**					
		(1.98)			(2.57)					
Leverage (Lev)		0.50			-0.04		-0.04		0.04	
		(0.40)			(0.37)		(0.37)		(0.43)	
$Lev \times Loss_{22} \times Un$			-0.29*			-0.68***				
			(0.16)			(0.14)				
$Loss_{23}$							-50.30**			
							(20.71)			
$Lev \times Loss_{23} \times Un$								-5.47***		
								(1.13)		
$Loss_{Cum}$									-5.55***	
									(1.92)	
$Lev \times Loss_{Cum} \times Un$										-0.54***
										(0.12)
Observations	44	44	44	44	44	44	44	44	44	44
R-squared	0.06	0.19	0.16	0.10	0.35	0.42	0.35	0.42	0.30	0.32

stock prices. Intuitively, banks that pay the lowest deposit rates should have the largest franchise value of deposits when interest rates are high. They are thus most affected by entering the fragile region.

Table 3 relates these results more closely to our model by looking at bank deposit rates. We proxy for the deposit rate of a bank by computing total interest expense on deposits divided by total deposits. Column 1 shows that banks with lower deposit rates had larger losses on their mark-to-market securities and banks with the highest deposit rates had low losses. Banks that pay high deposit rates should take less interest rate risk on the asset side because they need to match with a short-term interest rate on deposits that is closer to the market interest rate, and vice versa. Low deposit rate banks are those with high fixed operation costs, so some long duration assets are important to finance this fixed liability. These banks also experience the highest increases in franchise value of deposits as rates rise, offsetting losses on long-duration assets. Column 2

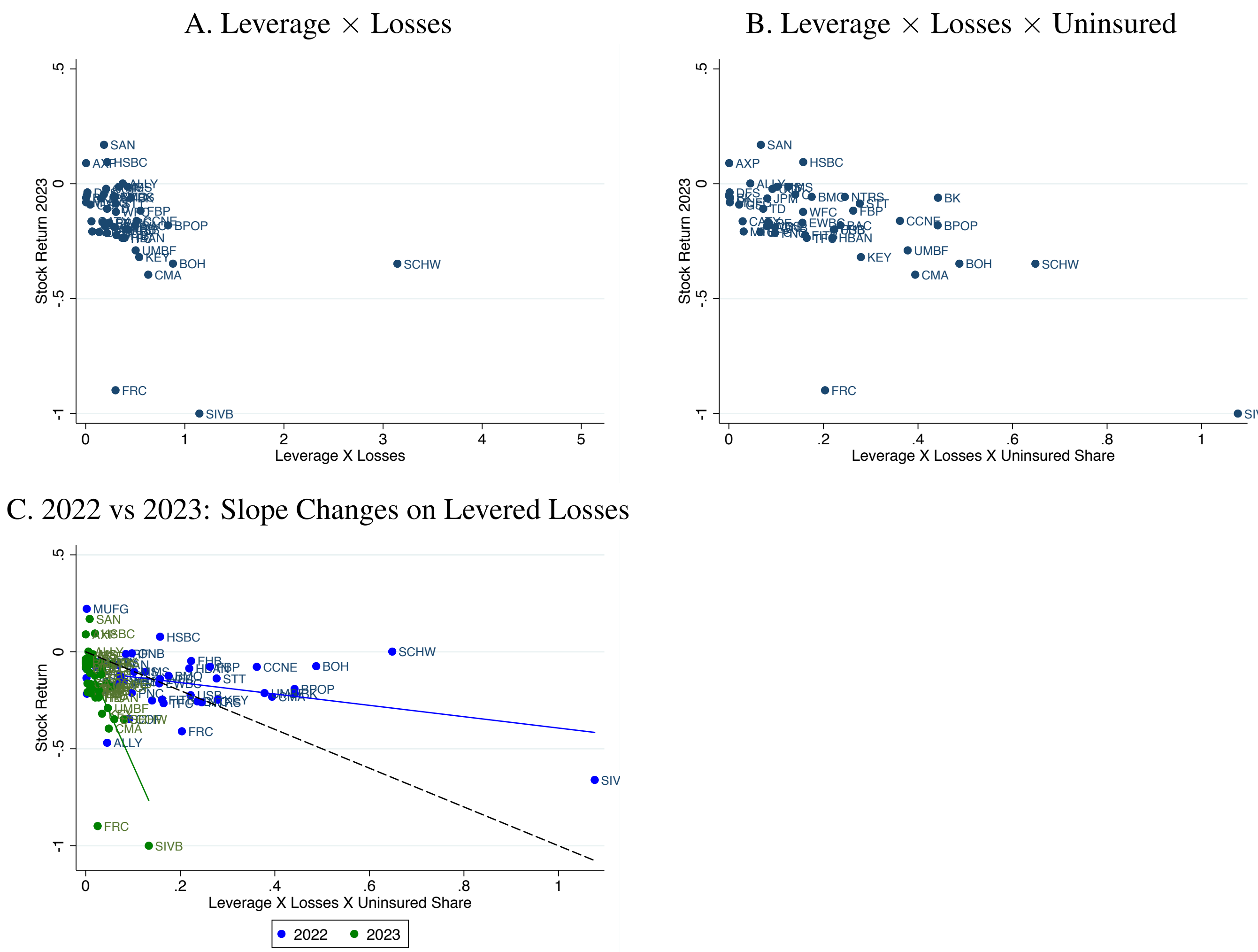

Figure 4: Bank Stock Returns, Asset Losses, and Leverage.
We plot stock returns against measures of risk at the bank level. Panel A plots book leverage from Q4 2022 multiplied by implied losses on assets from the same quarter on the x-axis. The y-axis is the cumulative stock return in 2023. Panel B interacts leverage times losses with the fraction of deposits which are uninsured. Panel C plots leverage times losses times fraction uninsured against the 2022 and 2023 stock returns for each bank. Losses in Panel C use implied duration and changes in interest rates for the 2023 returns. The steeper slope for 2023 returns indicates that stock returns are more sensitive to losses. See text for more details.

confirms that high deposit rate banks have a lower fraction of hold to maturity securities, and low deposit rates have a high fraction. Column 4 shows that the deposit rate has no association with bank stock returns in 2022, despite being correlated with losses.

We interpret the deposit rate here as a proxy for the value of the depositor franchise of the bank– the lower the rate, the higher the earned spread. The regressions say that the franchise value acts as a hedge to bank profitability: while banks with low deposit rates (high franchise value) see large mark-to-market losses as rates rose, the low rate on deposits boosts profitability, leading

Table 3: Deposit Rates, Duration, and Losses

This table shows that banks with lower deposit rates chose longer duration assets and suffered larger mark-to-market losses in 2022. Lower deposit rates were not correlated with stock returns in 2022. However, in 2023, lower deposit rates were associated with lower stock returns. Standard errors in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Losses	HTM Sec	Stock Ret '23	Stock Ret '23	Stock Ret '22	Stock Ret '22
DepositRate	-2.11***	-9.31***	20.59***	17.65***	-7.16	-9.41
	(0.53)	(2.73)	(4.06)	(5.33)	(6.70)	(7.08)
UninsInsured				-0.28		-0.21
				(0.23)		(0.14)
Observations	44	44	44	44	44	44
R-squared	0.21	0.11	0.13	0.20	0.03	0.12

to effectively no change in market equity values. Effectively, as rates rose, the franchise value of deposits rose most for banks with low deposit rates. This picture changes in 2023 (column 3). In 2023, low deposit rate banks had low returns. A 1% lower deposit rate is associated with a 20% lower stock return in 2023. This is directly consistent with the channel in our model, and suggests that the deposit franchise value began to erode in 2023 from losses accumulating. This provides indirect evidence that the "on" switch in interest rate risk came from a deterioration in the franchise value of deposits, which leads the losses on securities to pass through to equity values directly.

4.4 Assessing the Potential Deterioration in Bank Capital

While it should be clear that losses from interest rate rises in 2022 were large, we formally show the effect they had relative to banks equity capital ratios. See also Jiang et al. (2023) who similarly find these losses are large relative to banks' capital ratios.

The left panel of Figure 5 shows the ratio of book equity to total assets for a cross-section of banks taken from the 2022 Q4 call reports. The median value is around 9.5% (equivalently, this implies book leverage of around 10). The second bar shows book equity adjusted for mark-to-market losses or gains on securities. We compute this from the call reports by taking hold to maturity and available for sale securities. We compute losses or gains as the difference between their book value and the fair values reported in the call report. Book equity adjusted for these changes in security values is substantially lower, with a median of just under 7%. Because the denominator is the same, we can compute the implied return on book equity adjusting for these losses at around -30% for the median bank, a substantial decline. There is also clear heterogeneity across banks, though virtually every bank shows a decline in equity adjusting for gains or losses. To take a more extreme example, the losses on Silicon Valley Bank (SIVB) imply negative equity. We emphasize this is not a complete picture, as it doesn't adjust for other losses on the entire balance sheet or factor in any offsetting derivatives positions.[5]

In our model, the larger the decline in book equity coming from mark to market losses, and the closer this adjusted number is to 0, the larger the sensitivity of stock market values to interest rate shocks. As these adjusted losses push adjusted book equity closer to 0, they trigger the "on" switch in interest sensitivity. This comes from the fact that as adjusted book equity approaches zero the bank becomes exposed to self-fulfilling runs as deposits become increasingly backed by future profits of the deposit business. When adjusted book equity is above zero the safety of each depositors funds is independent of the action of the other depositors, but once equity is below zero, your money is only safe if a sufficient number of depositors stick with the bank. This creates fragility. Depositors may suddenly leave and destroy the valuable hedge coming from the deposit franchise, making the bank indeed insolvent. This risk increases as losses on securities grow and the reliance on future profits on deposits grow.

Panel B shows the sensitivity of bank stock returns to the change in the risk-free interest rate r_f, proxied by the 2 year Treasury yield. Specifically, we take each banks stock return over 2022 and 2023, respectively, and divide by the change in interest rates over the same period as a crude proxy. The median implied duration for 2022 is around 3.5, while the median for 2023 is an order of magnitude larger at around 35. Thus, increases in interest rates and the banks realized losses by themselves cannot explain what happened. The typical bank stock experiences a very large decline in value over the 2023 period compared to the (mild) increase in interest rates, while 2022 features relatively small sensitivity. This pattern of a slow drip and then a flood is in line with our model mechanism. As adjusted book equity approaches zero, our model implies the duration of the banks' stock return becomes increasing sensitive to triggers that drive depositors to expect that other deposits will leave. This makes the duration dramatically increase as the hedge from the

[5]Our total numbers for losses on securities roughly matches the numbers in Jiang et al. (2023) when excluding losses on loans which we don't observe.

deposit franchise disappears.

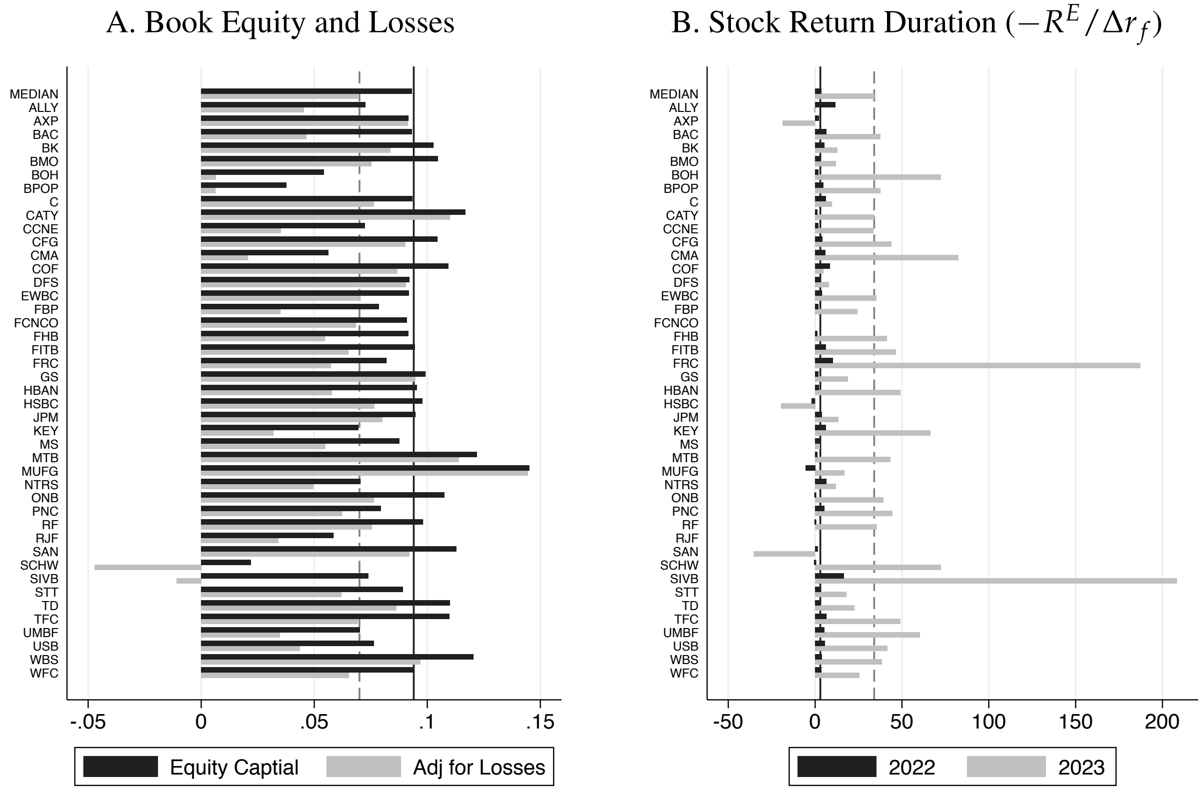

Figure 5: Book Equity Losses and Bank Stock Duration

This figure plots adjustments to banks capital ratios (left panel) that include mark to market losses on assets. The right panel plots duration implied by bank stock prices for 2022 and 2023 by dividing the stock return by the change in the 1 month Treasury bill for the period. The returns for 2023 go through March 14th. Bank tickers are given on the y-axis and the top row gives the median, which corresponds to the respective vertical lines.

5. Conclusion

Banks' relationships with their depositors are valuable assets. We argue they are also a source of fragility when interest rates are high. When depositors stick with their bank and are effectively sleepy, the value of the deposit relationship naturally hedges the bank's long-term assets. When rates increase, the profits from deposits increase while the value of long-term assets decreases and the bank is left unaffected. This apparent stability hides a fundamental imbalance: if depositors decide to leave and redeem their deposits, the value of deposit relationships is gone and the bank, left with its asset losses, might fail. Depositors' awakening, or becoming more mobile, and the subsequent bank failure can be an equilibrium in many situations, even if the bank's assets are liquid and deposits are insured.

We provide a model with these features. Beyond characterizing banks' fragility, our theory highlights that banks are not sensitive to interest rates in normal times, but experience a substantially increased exposure when rates move high enough. In this region both current and past losses catch up with bank valuations. We find empirical support for this prediction. In 2022, as interest rates rose, mark-to-market losses on banks' assets increased but did not pass through strongly to their equity prices. However, this changed as the losses grew and pushed the bank toward the risk of failure. The pass-through of losses to equity values strongly increased over the first portion of 2023, despite relatively smaller increases in interest rates.

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Appendix

A. A Simple Switching Costs Model of Bank Deposit Pricing

In this section we introduce a simple switching cost model of bank deposit pricing that follows Beggs and Klemperer (1992). In this model, depositors are sticky because they face switching costs. As a result, banks can earn profits off of existing customers. Banks face a tradeoff between offering high deposit rates to attract new depositors and offering low deposit rates to exploit their existing depositors. When rates are high, the bank values current profits relatively more than future profits due to higher discount rates and thus chooses to exploit old customers rather than attract new ones. It achieves this by setting a deposit rate that is low relative to the prevailing short term risk free rate. Thus, the deposit spread, i.e., the difference between the prevailing short term interest rate and deposit rates, is increasing in the level of the short term interest rate consistent with evidence presented in Drechsler et al. (2017).

A.1 The Setting

Time is discrete and infinite. We study a market consisting of two banks, Bank A and Bank B, competing to serve a measure of depositors.

Depositors Each period, a measure of new depositors $(1-\rho)D$ enters the market. Each new depositor has one dollar to store and holds a reservation value of κ per period for a unit of banking services. Additionally, each new depositor is positioned at a location $y \in [0,1]$ on a line segment with Bank A located at 0 and Bank B located at 1. In the first period upon entering the market, a depositor incurs a transport cost of ty if they choose to deposit their dollar at Bank A, and a transport cost of $t(1-y)$ if they opt for Bank B.[6] Transport costs are paid at the end of the period. Upon choosing a bank, the depositor faces a switching cost ϕ should they decide to move to the other bank in subsequent periods. For simplicity, we assume that this switching cost is independent of the depositor's location. Apart from storing their dollar at a bank, depositors can store their dollars in cash, in which case they will get no banking services. Cash earns the risk free rate r. We reflect this option by assuming that depositors discount future payoffs at the rate r.[7] We assume that depositors consume the interest they earn on their deposits. Thus, a depositor has the per period utility in dollar terms after any switching or transportation costs are sunk of

$$u(\tilde{r}) = \kappa + \frac{\tilde{r}}{1+r}$$

given that she earns the rate $\tilde{r}$ At the end of a period, a fraction $1-\rho$ of existing depositors exits the market. We will assume parameters are such that all depositors that enter the market deposit their dollars in one of the two banks. As a result, the measure of depositors in any given period is D.

[6] These transport costs represent preferences that create a competitive dynamic among banks for acquiring new depositors.

[7] Note that since borrowers can save at the risk-free, any profits that banks earn will be due to their provision of banking services and not because depositors cannot access other savings technologies besides the banks.

Banks At the start of each period, bank $i \in \{A, B\}$ posts an interest rate $\tilde{r}_i$ that it pays on deposits. In then invests the deposits it has in that period at the risk free rate. Providing banking services costs the bank c at the end of the period. The bank can also raise financing at the rate r without providing any services, and thus the discounts future cash flows at the rate r. Thus, the profit per period per dollar of deposits the bank earns if it pays the interest rate on deposits $\tilde{r}$ is given by

$$\pi(\tilde{r}) = \frac{r - \tilde{r} - c}{1 + r}$$

Equilibrium Let the stock of existing customers for bank i at the start of a period be denoted x_i. Since we assumed that parameters are such that all new depositors choose one of the two banks upon entering the market, we have $x_A + x_B = \rho D$. To define equilibrium, we focus on depositor choice bank and bank rate offer strategies that are Markovian in banks stock of existing customers at the start of each period. We let $\tilde{r}_i(x)$ denote the rate offer of bank i given it's stock of customers x. First consider the choice of bank of an existing depositor in bank i. Such a depositor will remain at her current bank if

$$u(\tilde{r}_i) + \frac{1}{1+r}\rho W_i(x_i) \geq u(\tilde{r}_j) + \frac{1}{1+r}\rho W_j(x_j) - s \tag{16}$$

where $W_i(x_i)$ represents the continuation utility that a depositor has if she is a current customer of bank i and that bank has a current stock of existing customers x_i. Now consider the choice of bank for a new customer at distance y from bank i. Such a customer will choose bank i if

$$u(\tilde{r}_i) + \frac{1}{1+r}\rho W_i(x_i) - \frac{ty}{1+r} \geq u(\tilde{r}_j) + \frac{1}{1+r}\rho W_j(x_j) - \frac{t(1-y)}{1+r}. \tag{17}$$

Given the choices of bank depositors in response to rate offers, let $m_i(\tilde{r}_i, \tilde{r}_j, x_i)$ be the market share of bank i given it's stock of existing customers at the start of the period x_i. Bank i's problem is then to solve

$$V(x_i) = \max_{\tilde{r}_i} \left\{ m_i(\tilde{r}_i, \tilde{r}_j, x_i) D\pi(\tilde{r}_i) + \frac{1}{1+r} V(\rho m_i(\tilde{r}_i, \tilde{r}_j, x_i) D) \right\}. \tag{18}$$

Definition 1. *A Markov perfect equilibrium is given by market share function $m_i(\tilde{r}_i, \tilde{r}_j, x_i)$ and a rate offer function $\tilde{r}_i(x)$ for $i \in \{A, B\}$ such that for all (x_A, x_B)*

- *The market share functions are consistent with depositor decisions given by* (16) *and* (17) *given the rate offer functions.*
- *The rate offer functions solve the banks' problem in* (18) *given the market share functions.*

Deriving an equilibrium in this market follows steps given in Beggs and Klemperer (1992). They show, up to some restrictions on parameters, there exists a unique equilibrium in which all agents follow strategies that are affine in x_i and existing consumers never switch. Since the banks in our model have the same cost of producing banking services, the steady state of this equilibrium will feature equal market shares for each bank. Moreover, the equilibrium rate spread at that market share is common across banks and above what would obtain in the absence of switching costs. We summarize this in the following proposition.

Proposition 1. *In the steady state of the unique affine equilibrium bank A and B split the market for new depositors and post the same rate offer leading to the following deposit spread*

$$r - \tilde{r} = c + t\left(\frac{1}{1-\rho} - \frac{2\rho}{3(1+r)}\right). \tag{19}$$

Moreover, the rate offer $\tilde{r}$ is lower than would obtain without switching costs.

An immediate consequence of this equilibrium is that rate spread is an increasing function of the risk free rate r. Specifically we have

$$\frac{\partial(r - \tilde{r}_i)}{\partial r} = \frac{2t\rho}{3(1+r)^2} > 0 \tag{20}$$

The intuition for this result follows from the tradeoff faced by banks in setting their rate offers. On the one hand, banks set high rate offers to compete for new customers so that they can profit off of those customers in the future. On the other hand, banks set low rate offers to profit of existing customer that are sticky due to switching costs. As the rate increases, future profits are less valuable and the latter effects dominates. In contrast, if there were no switching costs, there is no way to earn profits off existing customers, and the only force present is competition for new customers. Note that the effect of the level of the risk free rate on the deposit spread is also increasing in the survival rate ρ of existing customers. The more likely existing depositors are to survive, the less the banks needs to attract new customers to maintain market share, and the stronger the incentive for the banks to profit off of existing customers.

The result that depositor switching costs lead banks to increase spreads as the risk free rate increases is in line with a key finding of Drechsler et al. (2017). They document that banks increase spreads in response to increases in the federal funds rate. They provide a rational for this fact based on concentration in the market for deposits. We show that a similar effect can arise in a model with depositor switching costs even if the market for deposits are other wise fully competitive. Indeed, without switching costs, our model would predict the deposit spread is pinned down by the cost of providing banking services and invariant to the underlying risk-free rate.

Admittedly the above model omits some important aspect of the market for deposits. In reality, interest rates are stochastic and both sides of the market chooses actions based not only on current rates, but also on expectations of future changes in rates. Moreover, banks take on interest rate risk by investing in long term assets even though deposits are demandable. Rather then fully specify a model of switching costs that takes these features into account, we take the main message of this simple model, that deposit spreads are increasing in the short-term rate due to switching costs, and embed it in a model of banking that allows for risk.

B. A continuous-time version of the model for easy computation

We present a continuous-time version of the model of Section 2 that lends itself to simple computation.

B.1 Setting

At each instant, the bank distribute dividends $d\delta_t$ to its shareholders:

$$d\delta_t = D(x * 1/\pi_0 - c + ((1+e) - x\pi_0 - \beta)r_t), \tag{IA.1}$$

where e is the initial book equity as a fraction of deposits (that is, $D + eD = A$ at date 0), and x is the dollar position in perpetuities as a share of deposits (dividing by π_0 yields the quantity perpetuities).

The market value of equity is the present value of these dividend payments, where we assume risk-neutral pricing:

$$E_t^m = E\left[\int_t^\tau e^{-r_s(s-t)} d\delta_s\right], \tag{IA.2}$$

where τ denotes the time at which depositors leave the bank.

B.2 Sticky depositors

If the depositors never leave and the bank never closes, equity can be written as

$$E_t^m = D(x * \pi_t/\pi_0 - c\pi_t + ((1+e) - x - \beta)). \tag{IA.3}$$

It is clear from above this formula that if the bank chooses an asset allocation $x = c\pi_0$, then the value of equity is constant and equal to $D((1+e) - c\pi_0 - \beta)$. Market equity trades above book equity as long $c\pi_0 + \beta < 1$, i.e., the present value of providing deposit services is less then the dollar value of the deposits.

B.3 When depositors leave

Assume that the banks follows the previously optimal asset management decisions, and that depositors leave as soon as the market value of assets is greater that the value of deposits in the bank. The point at which depositors leave is therefore the first time at which

$$x\frac{\pi_\tau}{\pi_0} + (1 + e - x) = 1 \tag{IA.4}$$

If interest rates are driven by a single factor, $\pi_t = \pi(r_t)$, the default boundary can be directly stated in terms of an upper limit for the interest rate $\bar{r}$, which is implicitly defined by the moment that the market value of bank assets equal the deposit liabilities:

$$\frac{\pi(\bar{r})}{\pi(r_0)} = 1 - e/x. \tag{IA.5}$$

Assume the interest rate process follows a Vasicek process $dr = -b(r^* - r)dt + \sigma_r dB_t$, where r^* is the unconditional average of interest rates and b is the speed of mean-reversion towards this long-term average. We add to this process a reflective boundary at 0 to effectively enforce the zero lower-bound.

Then, the market value of bank equity satisfies the Hamilton-Jacobi-Bellman equation

$$V(r_t) = d(1 + e - x - \beta)r_t dt + e^{-rdt} E[V(r_{t+dt})], \tag{IA.6}$$

which can be rewritten as

$$rV(r) = d(1 + e - x - \beta)r_t - V_r(r) * b(r^* - r) + \frac{1}{2} V_{rr}(r)\sigma_r^2. \tag{IA.7}$$

With this second-order ODE come two boundary conditions.

First, when the bank hits the liquidation point, the value of the bank is zero since shareholders get nothing in liquidation: $V(\pi_0 - \frac{e}{c}, \bar{r}) = 0$. Second, because of the reflective zero-lower-bound, we have the condition $V'(0) = 0$ when rates are 0.

The ODE together with these two conditions has a unique solution that can easily be solved calculated numerically.