Public Debt Bubbles, Liquidity, and Risk:

Policy Assessments Based on the Zero-Beta Interest Rate

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Abstract

This paper studies stationary equilibria in a novel class of analytically tractable incomplete markets models with a public debt bubble (meaning that the interest rate $\bar{r}$ on riskfree government bonds is less than the growth rate). Within the models, the return to physical capital can exceed that on public debt because capital is exposed to aggregate risk and because it is less liquid than public debt. I follow di Tella et al. (2023), and define r_{zero} to be the zero-beta interest rate (on an asset with no aggregate risk and the same (il)liquidity properties as physical capital). I provide two distinct sufficient conditions in a zero-growth economy under which the government can increase welfare by following a fiscal policy which induces a higher $\bar{r}$. The first case is that $r_{zero} < 0$ (which also implies that $\bar{r} < 0$). The second case is that $\bar{r} < 0$ and $r_{zero} \approx \beta^{-1} - 1$ (which is the agents' common positive rate of time preference). I argue that the estimates of r_{zero} in di Tella et al. (2023) are in alignment with this second case.

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1 Introduction

In the advanced economies, the real return on government debt has been below the aggregate growth rate for much of the past few decades. When viewed through the lens of overlapping generations models[1] or the Bewley (1977)-Aiyagari (1994)-Huggett (1993) incomplete markets models[2], this observation suggests that governments should issue more debt so as to raise its interest rate. At the same time, though, the average real return on physical capital[3] has exceeded the aggregate growth rate. When viewed through the lens of these same models, this observation suggests that higher interest rates would be undesirable, as they would result in a socially wasteful reduction in the capital stock. Which policy conclusion about fiscal policy is correct?

To address this question, we need to provide a compelling account of the *difference* between the rates of return on private physical capital and public debt. One natural approach is to allow capital to be riskier than government bonds. But, as is well-known, the relevant return differential is large compared to what can be readily rationalized through risk. It seems that at least some part of the difference in returns is due to the superior *liquidity* of government debt relative to physical capital. These considerations mean that, to understand the welfare consequences of a fiscal policy-induced increase in the interest rate $\bar{r}$ on public debt, we need an economic framework in which:

- there is a bubble in public debt[4]: its interest rate $\bar{r}$ is smaller than the growth rate.
- physical capital can be both less liquid and more risky than public debt

In this paper, I build a novel class of analytically tractable incomplete markets models with these properties and study their stationary equilibria. Within the models, agents accumulate both physical capital and government debt so as to self-insure against transitory idiosyncratic

[1] For example, see Blanchard (2019).
[2] For example, see Kocherlakota (2023a).
[3] For example, see Gomme, Ravikumar, and Rupert (2011).
[4] I follow the terminology of Brunnermeier, Merkel, and Sannikov (2022).

shocks. This downside risk is assumed to be sufficiently large that public debt has a bubble. I also allow the adverse individual-level shocks to have an aggregate component, so that they are especially severe when the return to physical capital, net of depreciation, is low. This covariance means that physical capital is riskier than public debt. Finally, agents faced with adverse idiosyncratic shocks can only exchange at most a fraction $\theta \in (0, 1]$ of their physical capital and its returns for much-needed consumption. The remaining fraction must be re-invested into physical capital. It is in this sense that capital is less liquid than public debt.

Not surprisingly, the models imply that the equilibrium return on physical capital is higher than that on public debt for both liquidity and risk reasons. To distinguish these two factors, I follow di Tella, Hébert, Kurlat and Wang (2023) and define r_{zero} to be the *zero-beta interest rate*: the expected return on an asset which has no aggregate risk but has the same (il)liquidity properties as physical capital.[5] In the models, r_{zero} lies between the return on physical capital and the interest rate on public debt. My two main welfare results put this intermediate variable at center stage:

1. Suppose $r_{zero} < 0$ (the growth rate). Then, social welfare improves if the government adopts a fiscal policy that induces a slightly higher interest rate $\bar{r}$.

2. Suppose the interest rate $\bar{r}$ on public debt is less than zero, but r_{zero} is near the agents' common positive rate of time preference $(\beta^{-1} - 1)$. Social welfare improves if the government adopts a fiscal policy that induces a slightly higher interest rate $\bar{r}$.

In both cases, raising $\bar{r}$ is beneficial because the agents are able to self-insure more effectively. Also in both cases, raising $\bar{r}$ induces less physical capital ("crowding out"). But the two cases differ in terms of the welfare consequences of the crowding out of capital. In the first case ($r_{zero} < 0$), the crowd-out effect is societally beneficial, because any positivity in the return to capital is only attributable to its being risky. In the second case, the crowd-out effect

[5]IMoreira and Savov (2017, p. 2400) estimate liquidity premia in a similar fashion.

is *not* beneficial. However, physical capital is so illiquid that the two assets are essentially non-substitutable. As a consequence, while the reduction in physical capital caused by an increase in $\bar{r}$ is socially undesirable, the size of this effect is small and so it is outweighed by the improvement in self-insurance.

These two cases are not exhaustive. Are either of them empirically relevant? Di Tella et al. (2023) use US data to estimate r_{zero}. They find that it is similar to the interest rate implied by a standard representative agent model.[6] This kind of estimate corresponds to case 2 above. According to the model in this paper, such a high value for r_{zero} signals that the two assets - physical capital and public debt - are playing completely different roles in the economy. Agents accumulate public debt, not physical capital, to help finance their consumption needs in bad times. Raising $\bar{r}$ helps with this self-insurance, but has little effect on physical capital accumulation.

To the best of my knowledge, no prior work on debt bubbles allows the return to physical capital to incorporate both risk and liquidity premia. However, the class of models in this paper does unite several elements in prior work. My treatment of (il)liquidity is similar to that in di Tella, Hébert and Kurlat (2024). They in turn build on the modeling approach of Kaplan and Violante (2014). As in Kocherlakota (2022, 2023a), the adverse individual-level shocks in this paper take the form of making agents' momentary utility functions linear with slope coefficients that depend on the aggregate state of the economy. This quasilinearity greatly simplifies the history-dependence of consumption choices in equilibrium.[7]

Abel and Panageas (AP) (2022) extend Diamond (1965) by allowing for aggregate risk. I follow their lead by introducing risk into capital accumulation by assuming depreciation is stochastic.[8] Both in their paper and in mine, this approach eliminates the history-dependence

[6]Specifically, they estimate the average value of r_{zero} to be around 8%. This value is consistent with the interest rate implied by a representative agent model in which the agent has a power utility function $-C^{-2}$, a constant consumption growth rate of 2% per year, and an annual discount factor equal to 0.98. See Mulligan (2002) for a related analysis.

[7]Lagos and Wright (2005) and Challe and Ragot (2011) are examples of other studies of heterogeneous agent models that make effective use of quasilinearity.

[8]Brunnermeier, Merkel, and Sannikov (2024) study a model of public debt bubbles with a different source of aggregate risk. In their setup, physical capital accumulation is undertaken by individual entrepreneurs

of the aggregate capital stock. However, AP do not allow for any liquidity differences between capital and government bonds. They find that, regardless of the risk premium on capital, a fiscal policy-induced increase in the interest rate $\bar{r}$ on public debt improves welfare whenever $\bar{r}$ is below the aggregate growth rate. Since $\bar{r} = r_{zero}$ when capital and bonds are equally liquid, AP's characterization can be viewed as a special case of the "$r_{zero} < 0$" result derived in this paper.

2 Models

In this section, I describe the set of incomplete markets models used in the paper.

2.1 Time and Uncertainty

Time is indexed by the natural numbers. There is a unit measure of infinitely lived agents. At each date, their individual state s_t lies in the set $\{A, L\}$. In state A, the agent will find it optimal to be *accumulating* wealth. In state L, the agent will find it optimal to be *liquidating* wealth.

The individual states evolve according to stochastically independent Markov chains. The probability of transiting from state A to L is $p \in (0, 1)$. The probability of transiting from state L to state A is 1, so that the liquidation state is transitory. I assume that the initial fraction of agents in state A is given by $\mu_A = 1/(1 + p)$ and the initial fraction in state L is given by $\mu_L = p/(1 + p)$. These fractions are chosen so that they remain constant over time.

There is also an aggregate shock at each date indexed by $z_t \in [0, 1]$. This shock is independently and identically distributed over time, with pdf g and cdf G. As we shall see, the realization of the aggregate shock affects both preferences and technologies.

who bear countercyclical idiosyncratic production risk (as in Angeletos (2007)). These entrepreneurs receive higher payouts during downturns in order to compensate for the increase in risk that they bear. Hence, the dividend to a diversified claim to physical capital (akin to the S&P 500) is countercyclical, and so this claim has a higher average return than riskfree government debt. Unlike in the current paper, there is no liquidity differential between physical capital and public debt in the model of Brunnermeier et al. (2024), and so their model implies that (as in AP) $r_{zero} = \bar{r}$.

2.2 Preferences

The agents maximize the expectation of the following utility function:

$$\sum_{s=1}^{\infty} \beta^{t-1} v(c_t; z_t, s_t)$$

where:

$$v(c_t; z_t, A) = u(c_t)$$

$$v(c_t; z_t, L) = \nu(z_t)c_t.$$

I assume that u' and $-u''$ are both strictly positive, and so the agent has "standard" preferences in state A. The utility function u also satisfies usual Inada conditions. In contrast, in state L, the agent's marginal utility is independent of consumption. The positive marginal utility parameter ν is allowed to depend continuously on the aggregate state, so that the severity of agents' liquidity needs in state L varies with aggregate economic conditions.

2.3 Production

There is a representative competitive firm (owned by the agents) with a technology that, at each date, converts any $N_t \geq 0$ units of labor and any $K_t \geq 0$ units of capital into $f(K_t, N_t)$ units of consumption. (All quantities are expressed in per-capita terms.) The function f is homogeneous of degree one and weakly concave. The partial derivatives $-f_k, -f_{kk}, f_n, -f_{nn}$ are all strictly positive. The firm rents capital from agents in a competitive market.

The production function is riskfree, but capital is nonetheless risky. In particular, consider an agent who enters period t with k_t units of capital. The agent can augment capital over time according to the technology:

$$k_{t+1} = k_t + i_t - \delta(z_t)k_t$$

Here, i_t represents the quantity of consumption goods used for physical investment. The depreciation rate $\delta(z_t)$ is a continuous function of the aggregate state with a range that is a subset of the interval $(0, 1)$.

The agents in state A are endowed with one unit of labor each in a given period, which they supply inelastically. The agents in state L are endowed with zero units of labor. It follows that, in equilibrium, the representative firm always uses μ_A units of labor.

2.4 Liquidity

At any date t, capital is partially illiquid for agents in state L. They can use at most a fraction $\theta \in (0, 1]$ of their capital and its rents for current consumption. The remaining fraction is required to be used for physical investment. In contrast, capital is fully liquid[9] for agents in state A.

We can incorporate these restrictions into the agents' problems by adding the following constraints on their physical capitalholdings:

$$k_{t+1} \geq \begin{cases} 0 & \text{in state } A \\ (1-\theta)(1-\delta(z_t)+r_{Kt})k_t & \text{in state } L. \end{cases}$$

where r_{Kt} represents the rental rate on capital in period t. Note though that the latter constraint is best viewed as financial, not physical, in nature. For example, consider the following financial arrangement at date t:

1. An agent in state L lends $(1-\delta(z_t)+r_{Kt})k_t$ units of capital and capital income to some agent in state A for a within-period IOU. The IOU is a promise to deliver $\alpha(1-\delta(z_t)+r_{Kt})k_t$ units of consumption, where $1 > \alpha > \theta$, before the end of the period.

2. The buyer in state A transforms the acquired capital into $(1-\delta(z_t)+r_{Kt})k_t$ units of

[9]The assumed liquidity of capital in state A is technically convenient but is inessential for the results. In the equilibria of interest, a liquidity constraint on capital would not be binding for agents in state A because they are always accumulating wealth.

consumption.

3. The buyer then pays off their within-period IOU and retains the remaining $(1-\alpha)(1-\delta(z_t)+r_{Kt})k_t$ units of consumption.

These kinds of Pareto-improving financial arrangements, which give agents in state L a way to access the superior decumulation technology available to agents in state A, are not allowed. Hence, it makes sense to refer to physical capital being partially *illiquid*, as opposed to referring to physical investment being partially irreversible.

2.5 Fiscal Policy

The government specifies a stochastic process that describes the quantity $B_{t+1}(z^t)$ of one-period riskfree bonds supplied at each date t after a history $z^t = (z_1, ..., z_t)$ of aggregate shocks. The bonds are sold at a market-determined interest rate $r_t(z^t)$,which represents the government's promised repayment in the following period. Then, at date $(t+1)$ and after aggregate shock history z^{t+1}, the government makes a uniform lump-sum transfer to all agents:

$$\tau_{t+1}(z^{t+1}; r_t(z^t)) = B_{t+2}(z^{t+1}) - B_{t+1}(z^t)(1 + r_t(z^t))$$

to all agents. Note that the lump-sum transfer τ_t is residually determined, so that the government's flow budget constraint is satisfied for all possible interest rate processes.

2.6 Stationary Equilibrium

An equilibrium can be defined in the usual way in this economy. At each date, the rental rate on capital is equal to its marginal product and agents in state A receive a wage equal to the marginal product of labor. Agents treat the joint process of interest rates, capital rental rates and wage payments as exogenous and choose a stochastic process for their bondholdings and capital-holdings so as to maximize their expected utility. An equilibrium is then an endogenous interest rate process such that agents' bondholdings equal (the exogenously specified) bond supplies in every date and aggregate history.

In what follows, I restrict attention to *stationary* equilibria. These are equilibria in which the exogenous bond supply process and the initial cross-sectional distribution of capital are such that the endogenous interest rate on public debt and the endogenous rental rate on capital are both constants over all dates and states. The analysis in the body of the paper focuses on the properties of endogenous variables. Appendix B describes the conditions on exogenous variables that permit the existence of stationary equilibria.

3 Asset Pricing

This section describes the key asset pricing restrictions that emerge in a stationary equilibrium, in which the bond interest rate $\bar{r}$ and the return to capital r_K are both time and state invariant. For now, I simply assume that $r_K > 0$. and that $\bar{r} + \bar{\delta} > 0 > \bar{r}$, where:

$$\bar{\delta} = \int_0^1 \delta(z)g(z)dz.$$

I make use of the following notation throughout the remainder of the paper:

$$\bar{\nu} = \int_0^1 \nu(z)g(z)dz$$
$$C_{\nu\delta} = \int_0^1 (\frac{\nu(z)}{\bar{\nu}} - 1)(\delta(z) - \bar{\delta})g(z)dz.$$

In words, $\bar{\nu}$ is the mean of state L marginal utility (across aggregate states) and $\bar{\delta}$ is the mean of the depreciation rate (across aggregate states). Note that, given the assumed upper bound on the function δ, $\bar{\delta} < 1$. The parameter $C_{\nu\delta}$ is the covariance between $\nu/\bar{\nu}$ and δ across aggregate states. I restrict attention to the case in which $C_{\nu\delta} \geq 0$. This requirement means that the net return to capital is low when state L is more severe (in terms of marginal utility being high). As we shall see, it follows from this assumption that the risk premium on capital is non-negative.

I impose two additional conditions on the exogenous variables $\nu, C_{\nu\delta}, p$ and β:

$$\min_z \frac{\nu(z)}{\bar{\nu}} \geq \frac{\beta p}{1-\beta(1-p)} \tag{1}$$

$$C_{\nu\delta} \leq \frac{1-\beta}{1-\beta(1-p)}. \tag{2}$$

Appendix C shows that these two conditions are sufficient to ensure that agents in state L are indeed liquidators, in the sense that they always choose not to hold any capital or bonds in excess of their relevant lower bounds.

3.1 Optimality Restriction on the Capital Rental Rate

In this subsection, I derive the key optimality restriction that links the capital rental rate r_K and the interest rate $\bar{r}$ in a stationary equilibrium. Throughout, I assume that $\bar{r} + \bar{\delta} > 0 > \bar{r}$ and $r_K > 0$ (the latter restriction will later emerge as an equilibrium implication of firm optimality).

In period t and state A, agents choose government bonds and consumption so as to satisfy the first order conditions:

$$u'(c_{At}) = E_t\beta(1-p)(1+\bar{r})u'(c_{A,t+1}) + \beta p\bar{\nu}(1+\bar{r}), t = 1, 2, 3, ...$$

where c_{At} is state A consumption in period t and $c_{A,t+1}$ is state A consumption in period $(t+1)$. (The second term assumes that, as is demonstrated in Appendix C, agents in state L always choose to hold zero bonds.) By rolling this stochastic difference equation forwards, we can see that this first order condition implies that the consumptions of the agents in state

A are time and state invariant[10]:

$$u'(c_{At}) = u'(\bar{c}_A) = \frac{\beta p(1+\bar{r})\bar{\nu}}{1-\beta(1-p)(1+\bar{r})} \tag{3}$$

The agents in state A absorb any variations in their incomes into their assetholdings. This behavior is optimal because they are risk neutral in state L.

The agents in state A in period t choose physical capital and consumption so as to satisfy the first order condition:

$$\begin{aligned} u'(\bar{c}_A) = u'(\bar{c}_A)\beta(1-p)\int_0^1 (1+r_K-\delta(z))g(z)dz \\ + \beta p\theta \int_0^1 ((1+r_K-\delta(z))\nu(z))g(z)dz \\ + u'(\bar{c}_A)\beta^2 p(1-\theta)(1+r_K-\bar{\delta})^2 \end{aligned} \tag{4}$$

(Note that the second term implicitly assumes that, as is justified in Appendix C, agents in state L never hold any capital in excess of its lower bound.) The last terms on the right-hand side of (4) capture the illiquidity of physical capital. If the agent is in state L in period $(t+1)$, then they can only convert a fraction θ of capital and its income into consumption. The remaining fraction $(1-\theta)$ is re-invested into capital and is carried forward to period $(t+2)$, when the agent is again in state A.

We can rewrite as:

$$\begin{aligned} u'(\bar{c}_A) = \beta(1-p)u'(\bar{c}_A)(1+r_K-\bar{\delta}) + \beta\theta p\bar{\nu}(1+r_K-\bar{\delta}) - \beta\theta p\bar{\nu}C_{\nu\delta} \\ + u'(\bar{c}_A)\beta^2 p(1-\theta)(1+r_K-\bar{\delta})^2 \end{aligned}$$

[10]See Kocherlakota (2023a, p. 503-504) for a detailed justification.

By using (3), we obtain the following key restriction:

$$\frac{\beta p(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})} = \frac{\beta\theta p(1+r_K-\bar{\delta})-\beta\theta p C_{\nu\delta}}{1-\beta(1-p)(1+r_K-\bar{\delta})-\beta^2 p(1-\theta)(1+r_K-\bar{\delta})^2} \tag{5}$$

which, as the following proposition shows, pins down r_K as a function of $\bar{r}$.

Proposition 1. *Given any $\bar{r} < 0$, there is a unique r_K that satisfies (5).*

Proof. In Appendix A. □

Since the denominator of the right hand side of (5) is strictly decreasing in r_K, Proposition 1 allows us to conclude that there exists a function $\bar{r}_K(\bar{r};\theta,C_{\nu\delta})$ that satisfies (5) and is strictly increasing in its first argument.

The restriction (5) is a generalization of the usual relationship between capital rental rates and interest rates in models without uncertainty. In particular, suppose:

$$C_{\nu\delta} = 0$$
$$\theta = 1.$$

Since the right-hand side of (5) is strictly increasing in $(r_K - \bar{\delta})$, the resulting simplified restriction implies that:

$$\bar{r}_K(\bar{r};\theta,C_{\nu\delta}) - \bar{\delta} = \bar{r}.$$

which is the absence-of-arbitrage linkage between interest rates and capital rents in a deterministic world.

3.2 The Zero-Beta Interest Rate

In this section, I define the zero-beta interest rate (as in di Tella et al. (2023)). Consider a hypothetical asset that faces the same liquidity limitations as physical capital but has a return that is *uncorrelated* with the marginal utility of the liquidators. Define the return to this asset to be the *zero-beta interest rate* r_{zero}, which is restricted to be larger than -1. Then r_{zero} satisfies a version of (5) but with $C_{\nu\delta} = 0$:

$$\frac{\beta p(1+\bar{r})\bar{\nu}}{1-\beta(1-p)(1+\bar{r})} = \frac{\beta\theta p\bar{\nu}(1+r_{zero})}{1-\beta(1-p)(1+r_{zero})-\beta^2 p(1-\theta)(1+r_{zero})^2}. \tag{6}$$

It is then straightforward to see that there is a function $\bar{r}_{zero}$ that satisfies (6):

$$\bar{r}_{zero}(\bar{r};\theta) = \bar{r}_K(\bar{r};\theta,0) - \bar{\delta}.$$

The relative illiquidity of the zero-beta asset suggests that r_{zero} should exceed $\bar{r}$. The following proposition shows that this is indeed the case if $\bar{r}$ is negative and $\theta < 1$. It also shows that r_{zero} is bounded above by the rate of time preference.

Proposition 2. *Suppose that* $\bar{r} < 0$ *and* $0 < \theta < 1$. *Then:*

$$(\beta^{-1} - 1) > \bar{r}_{zero}(\bar{r},\theta) > \bar{r}$$

Proof. The definition (6) implies:

$$\frac{\beta p(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})} = \frac{\beta\theta p(1+\bar{r}_{zero}(\bar{r},\theta))}{1-\beta(1-p)(1+\bar{r}_{zero}(\bar{r},\theta))-\beta^2 p(1-\theta)(1+\bar{r}_{zero}(\bar{r},\theta))^2}$$

Suppose $\bar{r}_{zero}(\bar{r};\theta) = \beta^{-1} - 1$. Then, the RHS equals 1, which is larger than the LHS (since $\bar{r} < 0$). The RHS is strictly increasing in $\bar{r}_{zero}$ and so it must be true that:

$$\beta^{-1} - 1 > \bar{r}_{zero}(\bar{r},\theta).$$

In terms of the lower bound, take reciprocals of (6). We obtain:

$$\frac{1}{\beta p(1+\bar{r})} - \frac{1-p}{p} = \frac{1}{\beta\theta p(1+\bar{r}_{zero}(\bar{r},\theta))} - \frac{1-p}{p\theta} - \beta(1+\bar{r}_{zero}(\bar{r},\theta))\frac{(1-\theta)}{\theta}. \tag{7}$$

Suppose $(1+\bar{r}_{zero}(\bar{r},\theta)) = (1+\bar{r})$. Then, the RHS of (7) becomes:

$$\frac{1}{\beta\theta p(1+\bar{r})} - \frac{1-p}{p\theta} - \beta(1+\bar{r})\frac{(1-\theta)}{\theta}.$$

If we subtract this from the LHS of (7), we get:

$$\begin{aligned}
[\beta(1+\bar{r}) - \frac{1}{\beta p(1+\bar{r})} + \frac{1-p}{p}](\frac{1}{\theta}-1) \\
< (1 - \frac{1}{p} + \frac{1}{p} - 1)(\frac{1}{\theta}-1) \\
= 0
\end{aligned}$$

where the inequality follows from the restriction that $\beta(1+\bar{r}) < \beta < 1$. Since the RHS of (7) is strictly decreasing in r_{zero}, the proposition follows. □

The next proposition derives a relationship between the capital rental rate r_K and the zero-beta interest rate r_{zero}. The key is that a positive covariance between ν and δ makes capital a less useful hedge against the risk in state L than a zero-beta asset.

Proposition 3. *If the covariance $C_{\nu\delta} > 0$, then $\bar{r}_K(\bar{r};\theta,C_{\nu\delta}) - \bar{\delta} > \bar{r}_{zero}(\bar{r};\theta)$.*

Proof. Recall that $\bar{r}_K$ is defined to solve (5). The RHS is strictly decreasing in $C_{\nu\delta}$ and strictly increasing in $\bar{r}_K$. Hence, the function $\bar{r}_K$ is strictly increasing in $C_{\nu\delta}$. But this proves the proposition, as $\bar{r}_{zero}(\bar{r};\theta) = \bar{r}_K(\bar{r};\theta,0)$. □

3.3 Summary

As we saw in Section 3.1, if capital is liquid ($\theta = 1$) and has no aggregate risk ($C_{\nu\delta} = 0$), then:

$$r_K - \bar{\delta} = \bar{r}.$$

However, suppose $\theta < 1$ and/or $C_{\nu\delta} > 0$, so that capital is illiquid and/or risky. Then, capital's average return exceeds that on (riskfree and liquid) public debt:

$$\begin{aligned} & r_K - \bar{\delta} - \bar{r} \\ = & (r_K - \bar{\delta} - r_{zero}) + (r_{zero} - \bar{r}) \\ & > 0 \end{aligned}$$

The latter term is a *liquidity premium* which is positive if $\theta < 1$. The former term is a *risk premium* which is positive if the covariance $C_{\nu\delta} > 0$.

3.4 Highly Illiquid Capital

In this section, we study the properties of r_{zero} and r_K when capital is highly illiquid, so that θ is near zero. In this case, the relevant Euler equation is well-approximated by:

$$u'(\bar{c}_A) = \beta(1-p)u'(\bar{c}_A)(1 + r_K - \bar{\delta}) + u'(\bar{c}_A)\beta^2 p(1 + r_K - \bar{\delta})^2$$

Agents in state L cannot use capital or its returns. Hence, agents in A accumulate capital only to fund consumption in future periods in which they are in state A. This means that capital accumulation is not affected by borrowing constraints or aggregate risk. In other words, capital becomes a zero-beta (highly) illiquid asset, suggesting that it should have no risk premium but will have a large liquidity premium.

The following proposition verifies these intuitions.

Proposition 4. *If* $\bar{r} < 0$*, then:*

$$\lim_{\theta \to 0} \bar{r}_{zero}(\bar{r}; \theta) = \lim_{\theta \to 0}(\bar{r}_K(\bar{r}; \theta, C_{\nu\delta}) - \bar{\delta}) = \beta^{-1} - 1.$$

Proof. In Appendix A. □

Intuitively, if $\theta = 0$, agents in state A only save capital for future realizations of state A. As a result, regardless of the value of $\bar{r}$, capital is priced as if the agents lived in a representative agent economy with no growth or risk.

4 Quantities and Social Welfare

In this section, I prove the main results about the welfare properties of stationary equilibria indexed by a time and state invariant interest rate $\bar{r}$ on public debt.

4.1 Capital

In Section 3, we derived a relationship between the interest rate on debt and the rental rate on capital in a stationary equilibrium. In this subsection, we use this result to establish a functional connection between the level of the physical capital stock and the interest rate $\bar{r}$. However, we also show that the derivative of this function, with respect to $\bar{r}$, is near zero for small values of θ (highly illiquid capital).

Agents rent capital to the representative firm in a competitive market. Hence the capital rental rate is equal to the marginal product of capital in any date and state:

$$r_{Kt} = f_k(K_t, \mu_A).$$

Here, $\mu_A = 1/(1+p)$ is the quantity of labor supplied inelastically to the firm. The restriction (5) then implies that, given a constant interest rate $\bar{r}$, the quantity K of capital is described by a function that satisfies:

$$f_k(K_{SS}(\bar{r}; \theta, C_{\nu\delta}), \mu_A) = \bar{r}_K(\bar{r}; \theta, C_{\nu\delta}). \tag{8}$$

The function K_{SS} is strictly decreasing in $\bar{r}$. This is the usual crowding out effect on physical investment: if government debt offers a higher interest rate, then agents allocate their wealth away from capital. However, the following proposition uses Proposition 4 to show that the crowding out effect vanishes when capital is highly illiquid.

Proposition 5. *For any $\bar{r} < 0$ and any $C_{\nu\delta} \geq 0$, K_{SS} is approximately independent of the interest rate $\bar{r}$ when θ is small, so that for any :*

$$\lim_{\theta \to 0} K_{SS,r}(\bar{r}; \theta, C_{\nu\delta}) = 0.$$

where $K_{SS,r}$ is the partial derivative of K_{SS} with respect to its first argument.

Proof. In Appendix A. □

Intuitively, when θ is near zero, the agents in state A only use capital as a way of generating consumption in future realizations of state A. The value of $\bar{r}$ does not affect the relevant trade-off.

4.2 Social Welfare

This section provides a simple formula for social welfare in a stationary equilibrium.

Given any $\bar{r}$, capital is constant. Hence, the output produced in any period is used for consumption and to replace depreciated capital. We have seen earlier that the consumption of accumulators satisfies:

$$\bar{c}_A(\bar{r}) \equiv u'^{-1}\left(\frac{\beta p(1+\bar{r})\bar{\nu}}{1-\beta(1-p)(1+\bar{r})}\right).$$

The average consumption of liquidators in aggregate state z is thus given by:

$$\bar{c}_L(z, \bar{r}; \theta, C_{\nu\delta}) = \frac{-\delta(z)K_{SS}(\bar{r}; \theta, C_{\nu\delta}) + f(K_{SS}(\bar{r}; \theta, C_{\nu\delta}), \mu_A) - \mu_A \bar{c}_A(\bar{r})}{\mu_L}.$$

The consumptions of the liquidators (agents in state L) depend on state z because the lump-sum transfers implicitly do. In particular, if $\delta(z)$ is low, the state L agents have more illiquid capital. The state A agents allocate less of their wealth to capital and more of their wealth to government bonds, which results in higher transfers.

We can then define welfare as:

$$W(\bar{r};\theta,C_{\nu\delta}) = \mu_A u(\bar{c}_A(\bar{r})) + \mu_L \int_0^1 \bar{c}_L(z,\bar{r};\theta,C_{\nu\delta})\nu(z)g(z)dz$$

This is the expectation of a utilitarian objective taken over the various aggregate states. It is helpful to rewrite welfare as the sum of two pieces:

$$\begin{aligned} W(\bar{r};\theta,C_{\nu\delta}) &= W_{cons}(\bar{r}) + W_{cap}(\bar{r};\theta,C_{\nu\delta}) \\ W_{cons}(\bar{r}) &= \mu_A(u(\bar{c}_A(\bar{r})) - \bar{\nu}\bar{c}_A(\bar{r})) \\ W_{cap}(\bar{r};\theta,C_{\nu\delta}) &= \bar{\nu}(f(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A) - K_{SS}(\bar{r};\theta,C_{\nu\delta})\int_0^1 \delta(z)\nu(z)g(z)dz) \\ &= \bar{\nu}(f(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A) - \bar{\delta}K_{SS}(\bar{r};\theta,C_{\nu\delta}) - C_{\nu\delta}K_{SS}(\bar{r};\theta,C_{\nu\delta})) \end{aligned}$$

The first term W_{cons} represents the welfare associated with consumption-smoothing from state A to state L. The second term W_{cap} represents the welfare associated with the net output produced by capital. Note that the positive covariance between δ and ν reduces this latter term.

4.3 Main Results

In this subsection, I prove the main results. I consider a stationary equilibrium in which $\bar{r} < 0$ and then investigate the impact on welfare of switching to another stationary equilibrium with a slightly larger $\bar{r}$. A key preliminary - but not surprising - finding is that increasing $\bar{r}$ from a negative value improves consumption-smoothing.

Proposition 6. *If* $\bar{r} < 0$, *then* $W'_{cons}(\bar{r}) > 0$.

Proof. As above, define:

$$\bar{c}_A(\bar{r}) \equiv u'^{-1}(\frac{\beta p(1+\bar{r})\bar{\nu}}{1-\beta(1-p)(1+\bar{r})}).$$

Note that $\bar{c}'_A(\bar{r}) < 0$.

The consumption-smoothing welfare term is given by:

$$W_{cons}(\bar{r}) = \mu_A u(\bar{c}_A(\bar{r})) - \mu_A \bar{\nu}\bar{c}_A(\bar{r}).$$

Differentiating, we obtain:

$$W'_{cons}(\bar{r}) = \mu_A u'(\bar{c}_A(\bar{r}))\bar{c}'_A(\bar{r}) - \mu_A \bar{\nu}\bar{c}'_A(\bar{r}).$$

Since $\bar{r} < 0:$

$$u'(\bar{c}_A(\bar{r})) = (\frac{\beta p(1+\bar{r})\bar{\nu}}{1-\beta(1-p)(1+\bar{r})}) < \bar{\nu}.$$

and so $W'_{cons}(\bar{r}; \theta, C_{\nu\delta}) > 0$. □

When $\bar{r} < 0$, the consumption of the agents in state A is too high, given their high consumption needs (that is, high marginal utility) in state L.

4.3.1 A Basic Result

We have seen in Proposition 6 that raising $\bar{r}$, when it is negative, always improves consumption-smoothing. We also know from (8) that the partial derivative $K_{SS,r}(\bar{r}; \theta, C_{\nu\delta}) < 0$, so that raising $\bar{r}$ crowds out capital. Thus, the core welfare questions are:

- does lowering capital raise or lower output?
- if lowering capital does lower output, how does that reduction compare with the welfare improvement engendered by better consumption-smoothing?

The result in this subsection is about the first issue and is essentially a proof-of-concept for the modeling framework. It shows that (as expected) lowering capital does increase output when $\bar{r} < 0$ if there is no aggregate risk or illiquidity issues.

Proposition 7. *Suppose* $\bar{r} = \bar{r}_{zero}(\bar{r};\theta) = \bar{r}_K(\bar{r};\theta, C_{\nu\delta}) - \bar{\delta} < 0$. *Then:*

$$W'_{cap}(\bar{r};\theta, C_{\nu\delta}) > 0$$

and $W'(\bar{r};\theta, C_{\nu\delta}) > 0$.

Proof. Definitionally:

$$W'_{cap}(\bar{r};\theta, C_{\nu\delta}) = \bar{\nu}(f_k(K_{SS}(\bar{r};\theta, C_{\nu\delta}), \mu_A) - \bar{\delta} - C_{\nu\delta})K_{SS,r}(\bar{r};\theta, C_{\nu\delta}).$$

If $r_K - \bar{\delta} = r_{zero}$, then $C_{\nu\delta} = 0$ and:

$$W'_{cap}(\bar{r};\theta, C_{\nu\delta}) = \bar{\nu}(f_k(K_{SS}(\bar{r};\theta, C_{\nu\delta}), \mu_A) - \bar{\delta})K_{SS,r}(\bar{r};\theta, C_{\nu\delta}).$$

Since $r_K = f_k < \bar{\delta}$, and $K_{SS,r}(\bar{r};\theta, C_{\nu\delta}) < 0$, it follows that:

$$W'_{cap}(\bar{r};\theta, C_{\nu\delta}) > 0.$$

The positivity of $W'(\bar{r};\theta, C_{\nu\delta}) > 0$ then follows from Proposition 6. □

4.3.2 Highly Liquid Capital

Proposition 7 assumes that there is no capital premium. Suppose instead that, as in the US data, $r_K - \bar{\delta} > 0 > \bar{r}$ (treating zero as the growth rate). The positive net expected return to capital could be due to risk or liquidity. The next proposition contemplates a situation in which the positivity of the net capital return is due solely to a risk premium. In this setting, the proposition shows that, as in Proposition 7, there is overaccumulation of capital.

Proposition 8. *Suppose* $\bar{r} < \bar{r}_{zero}(\bar{r};\theta) < 0$ *and* $C_{\nu\delta} < 0$. *Then* $W'_{cap}(\bar{r};\theta, C_{\nu\delta}) > 0$ *and* $W'(\bar{r};\theta, C_{\nu\delta}) > 0$.

Proof. In Appendix A. □

In their overlapping generations model, Abel and Panageas (2022) study the case in which physical capital has aggregate risk, but is just as liquid as government debt. These assumptions give rise to the restriction $r_K \geq r_{zero} = \bar{r}$. Proposition 8 then implies that, as they find, $W'(\bar{r}) > 0$ for any $\bar{r} < 0$.

4.3.3 Highly Illiquid Capital

Now suppose $r_{zero} > 0 > \bar{r}$. In this case, there can be a tension associated with raising $\bar{r}$ closer to zero, as the fall in capital may result in a reduction in output. However, the following proposition shows that this potential problem disappears when capital is highly illiquid. In that case, its quantity is essentially independent of the interest rate $\bar{r}$ on liquid instruments and so increasing $\bar{r}$, when it is negative, necessarily improves welfare.

Proposition 9. *Let $\bar{r} < 0$. Consider a sequence $\{\theta_n\}_{n=1}^{\infty}$ such that $\lim_{n\to\infty} \bar{r}_{zero}(\bar{r};\theta_n) = \beta^{-1} - 1$. Then:*

$$W'(\bar{r};\theta_n, C_{\nu\delta}) > 0.$$

Proof. In Appendix A. □

4.4 Evidence

Di Tella et al. (2023) define and estimate the zero-beta interest rate. They argue that this interest rate fits the Euler equation of a representative agent well. This empirical characterization parallels the hypothesis in Proposition 9 that r_{zero} is near $\beta^{-1} - 1$. Hence, in light of Proposition 9, di Tella et al.'s estimates suggest that it is optimal to use fiscal policy to induce an increase in the interest rate $\bar{r}$ on public debt in the circumstances described by Blanchard (2019). The key is that, if r_{zero} is so high, capital and public debt are not all that substitutable. As a result, we can safely abstract from the crowding-out of private capital when thinking about the welfare impact of an increase in $\bar{r}$.

5 Conclusions

This paper makes two contributions. First, it constructs an analytically tractable class of models of public debt bubbles in which physical capital is both more risky and less liquid than government bonds. As a result, the return to capital can exceed the growth rate even when the interest rate does not. Second, it uses the model to highlight the significance of the zero-beta interest rate r_{zero} (di Tella et al. (2023, 2024)) in welfare assessments. In particular, a government can improve social welfare by increasing the interest rate $\bar{r}$ on public debt if:

- $\bar{r} \leq r_{zero} < 0$
- $\bar{r} < 0$ and $r_{zero} \approx \beta^{-1} - 1$ (the rate of time preference of agents in the economy).

The estimates in di Tella et al. (2023) suggest that the latter case is consistent with US data.

The results described above are driven by simple intuitions, and so seem likely to be robust to a number of possible generalizations of the models in the paper. In terms of the first result, if $r_{zero} < 0$, then the risk-adjusted net return to capital is negative. As long as private risk assessments align with social risk assessments, it makes sense that in this case, output will rise if capital falls. In terms of the second result, increasing $\bar{r}$ above a negative value always makes public debt a better vehicle for precautionary savings. But, if r_{zero} is near the rate of time preference, then precautionary savings is playing little role in the determination of the level of capital. In this context, increasing $\bar{r}$ has only a slight deleterious effect on the quantity of capital in the economy. This kind of asset segregation could have material consequences for the analysis of other macroeconomic issues, including monetary policy.

As I write, the real return on US government debt (as measured via ten-year TIPs) has been close to 2% over the past couple years. This level is (arguably) slightly above reasonable expectations for future growth. In a similar vein, di Tella et al. (2023)'s estimates imply that r_{zero} is highly variable. Hence, it would be useful to extend the analysis in this paper (along the lines of Kocherlakota (2023b)) to deal with shocks to $\bar{r}$ and r_{zero}.

Appendix A

This appendix contains the proofs of Propositions 1, 4, 5, 8 and 9.

Proof of Proposition 1

If $C_{\nu\delta} = 0$ and $\theta = 1$, then $\bar{r} = r_K - \bar{\delta}$ uniquely solves (5). For the remainder of the proof, suppose either $C_{\nu\delta} > 0$ or $\theta < 1$ or both.

Consider the quadratic in the denominator of the RHS:

$$\psi(x) = 1 - \beta(1-p)x - \beta^2 p(1-\theta)x^2.$$

If $\theta < 1$, its unique positive root is given by:

$$x^* = \frac{-\beta(1-p) + \sqrt{\beta^2(1-p)^2 + 4\beta^2 p(1-\theta)}}{2\beta^2 p(1-\theta)}.$$

and if $\theta = 1$, its unique positive root is given by:

$$x^* = \beta^{-1}(1-p)^{-1}.$$

Note that, for any $\theta > 0$, $\psi(\beta^{-1}) = \theta > 0$, and so $x^* > \beta^{-1} > (1 - \bar{\delta})$.

The numerator of (5) is positive because it equals:

$$\int_0^1 \frac{\nu(z)}{\bar{\nu}}(1 - \delta(z) + r_K)g(z)dz$$

which, given the upper bound on δ, is positive for any $r_K \geq 0$. Hence, the RHS of (5) is larger than the LHS if r_K is close to (but less than) $x^* - (1 - \bar{\delta})$.

If $r_K = \bar{r} + \bar{\delta}$, then the RHS of (5) is less than (equal to):

$$\frac{\beta\theta(1-p)(1+\bar{r})}{1 - \beta(1-p)(1+\bar{r}) - \beta^2 p(1-\theta)(1+\bar{r})^2} \tag{9}$$

if $C_{\nu\delta} > (=)0$. The numerator of the derivative of (9) with respect to θ is positive:

$$\begin{aligned}\beta(1-p)(1+\bar{r})(1-\beta(1-p)(1+\bar{r})-\beta^2p(1-\theta)(1+\bar{r})^2)-\beta\theta(1-p)(1+\bar{r})\beta^2p(1+\bar{r})^2\\ =\beta(1-p)(1+\bar{r})-\beta^2(1-p)^2(1+\bar{r})^2-\beta^3p(1-p)(1+\bar{r})^3\\ >\beta(1+\bar{r})(1-p)(1-(1-p)-p)\\ =0.\end{aligned}$$

where the inequality is a consequence of $\beta(1+\bar{r}) < 1$. Hence, the RHS of (5) is less than the LHS if $\theta < 1$ or $C_{\nu\delta} > 0$.

From the Intermediate Value Theorem, it follows that there exists some r_K such that $r_K \in [\bar{r} + \bar{\delta}, x^* - 1 + \bar{\delta}]$ that satisfies (5). The RHS of (5) is strictly increasing in r_K and so the solution for r_K is unique.

Proof of Proposition 4

$$\gamma(\bar{r}) \equiv \frac{\beta p(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})}. \tag{10}$$

Recall that function $\bar{r}_{zero}$ satisfies:

$$\frac{\beta p(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})} = \frac{\beta\theta p(1+\bar{r}_{zero}(\bar{r};\theta))}{1-\beta(1-p)(1+\bar{r}_{zero}(\bar{r};\theta))-\beta^2p(1-\theta)(1+\bar{r}_{zero}(\bar{r};\theta))^2}$$

By cross-multiplying, we can show that the return $\bar{r}_{zero}$ satisfies:

$$\gamma(\bar{r})(1-\beta(1-p)(1+\bar{r}_{zero}(\bar{r};\theta))-\beta^2p(1-\theta)(1+\bar{r}_{zero}(\bar{r};\theta))^2) = \beta p\theta(1+\bar{r}_{zero}(\bar{r};\theta)). \tag{11}$$

where γ is defined as in (10). Since $\beta^{-1} > (1+\bar{r}_{zero}(\bar{r};\theta))$:

$$\lim_{\theta\to 0}\beta p\theta(1+\bar{r}_{zero}(\bar{r};\theta)) = 0.$$

It follows from (11) that:

$$\lim_{\theta\to 0}(1-\beta(1-p)(1+\bar{r}_{zero}(\bar{r};\theta))-\beta^2 p(1-\theta)(1+\bar{r}_{zero}(\bar{r};\theta))^2)=0$$

which means:

$$\lim_{\theta\to 0}\beta(1+\bar{r}_{zero}(\bar{r};\theta))$$

is a non-negative root of:

$$\begin{aligned}&(1-x(1-p)-px^2)\\&\quad=(1-x)(1+px)\end{aligned}$$

Hence:

$$\lim_{\theta\to 0}(1+\bar{r}_{zero}(\bar{r};\theta))=\beta^{-1}.$$

Similarly, $\bar{r}_K$ satisfies:

$$\begin{aligned}\gamma(\bar{r})(1-\beta(1-p)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})-\beta^2 p(1-\theta)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})^2).\\=\beta p\theta(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta}-C_{\nu\delta})\end{aligned}$$

For any $\theta>0$, the RHS is positive because it equals:

$$\begin{aligned}&\int_0^1\frac{\nu(z)}{\bar{\nu}}(1-\delta(z)+\bar{r}_K)g(z)dz\\\geq&\int_0^1\frac{\nu(z)}{\bar{\nu}}(1-\delta(z)+\bar{r}+\bar{\delta})g(z)dz\\>&\int_0^1\frac{\nu(z)}{\bar{\nu}}(1-\delta(z))g(z)dz.\end{aligned}$$

As well, the LHS is bounded from above by:

$$\gamma(\bar{r})(1-\beta(1-p)(1+\bar{r}_{zero}(\bar{r};\theta))-\beta^2 p(1-\theta)(1+\bar{r}_{zero}(\bar{r};\theta))^2).$$

It follows that:

$$
\begin{aligned}
&\lim_{\theta\to 0}\gamma(\bar{r})(1-\beta(1-p)(1+\bar{r}_{zero}(\bar{r};\theta))-\beta^2 p(1-\theta)(1+\bar{r}_{zero}(\bar{r};\theta))^2)\\
\geq &\lim_{\theta\to 0}\gamma(\bar{r})(1-\beta(1-p)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})-\beta^2 p(1-\theta)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})^2)\\
&\geq 0
\end{aligned}
$$

We showed above that the first limit is zero. Hence:

$$
\lim_{\theta\to 0}\gamma(\bar{r})(1-\beta(1-p)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})-\beta^2 p(1-\theta)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})^2)=0
$$

which implies (as above) that:

$$
\lim_{\theta\to 0}(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})=\beta^{-1}.
$$

Proof of Proposition 5

Define K^*_β as the unique solution to:

$$
1+f_k(K^*_\beta,\mu_A)-\bar{\delta}=\beta^{-1}
$$

(Such a solution exists since $\beta^{-1}>(1-\bar{\delta})$.) Proposition 4 implies that for all $\bar{r}<0$ and any $C_{\nu\delta}\geq 0$:

$$
\lim_{\theta\to 0}K_{SS}(\bar{r};\theta,C_{\nu\delta})=K^*_\beta.
$$

Note that:

$$
K_{SS,r}(\bar{r};\theta,C_{\nu\delta})f_{kk}(K^*_\beta,\mu_A)=\bar{r}'_K(\bar{r};\theta,C_{\nu\delta}).
$$

Hence, we need only show that the derivative on the RHS is zero.

As before, let γ be defined as:

$$\gamma(\bar{r}) \equiv \frac{\beta p(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})}.$$

The function $\bar{r}_K(\bar{r};\theta, C_{\nu\delta})$ satisfies (5):

$$\gamma(\bar{r})(1-\beta(1-p)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})-\beta^2 p(1-\theta)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})^2)$$
$$= \beta\theta p(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})-\beta\theta p C_{\nu\delta}$$

Take the derivative of this equation with respect to $\bar{r}$:

$$\gamma'(\bar{r})(1-\beta(1-p)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})-\beta^2 p(1-\theta)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})^2)$$
$$+\gamma(\bar{r})(-\beta(1-p)\bar{r}'_K(\bar{r};\theta,C_{\nu\delta})-2\beta^2 p(1-\theta)(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})\bar{r}'_K(\bar{r};\theta,C_{\nu\delta}))=\beta\theta p\bar{r}'_K(\bar{r};\theta,C_{\nu\delta})$$

which implies:

$$r'_K(\bar{r};\theta,C_{\nu\delta}) = \frac{\gamma'(\bar{r})(1-\beta(1-p)(1+r_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})-\beta^2 p(1-\theta)(1+r_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})^2)}{\beta\theta p+\gamma(\bar{r})\beta(1-p)+2\beta^2 p\gamma(\bar{r})(1-\theta)(1+r_K(\bar{r};\theta,C_{\nu\delta})-\bar{\delta})}$$

Take limit of the RHS of the above expression with respect to θ. In light of Proposition 4, the numerator converges to 0. The denominator converges to:

$$\gamma(\bar{r})(\beta(1-p)+2\beta p) = \gamma(\bar{r})(\beta+\beta p) > 0.$$

Hence, we can conclude that:

$$\lim_{\theta\to 0} K_{SS,r}(\bar{r};\theta,C_{\nu\delta}) = \frac{\lim_{\theta\to 0} r'_K(\bar{r};\theta)}{f_{kk}(K^*_\beta,\mu_A)}$$
$$= 0.$$

Proof of Proposition 8

Suppose $-1 < \bar{r}_{zero}(\bar{r};\theta) < 0$. Then definitionally:

$$\frac{\beta\theta p}{1-\beta(1-p)-\beta^2 p(1-\theta)}$$
$$> \frac{\beta\theta(1+\bar{r}_{zero}(\bar{r};\theta))}{1-\beta(1-p)(1+\bar{r}_{zero}(\bar{r};\theta))-\beta^2 p(1-\theta)(1+\bar{r}_{zero}(\bar{r};\theta))^2}$$
$$= \frac{\beta\theta p(1+f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta})-\beta\theta p C_{\nu\delta}}{1-\beta(1-p)(1+f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta})-\beta^2 p(1-\theta)(1+f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta})^2}$$

I claim:

$$f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta}-C_{\nu\delta} < 0.$$

Suppose not and:

$$f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta}-C_{\nu\delta} \geq 0.$$

Then:

$$f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta} \geq C_{\nu\delta} \geq 0.$$

It follows that:

$$\frac{\beta\theta p(1+f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta})-\beta\theta p C_{\nu\delta}}{1-\beta(1-p)(1+f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta})-\beta^2 p(1-\theta)(1+f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta})^2}$$
$$\geq \frac{\beta\theta p}{1-\beta(1-p)-\beta^2 p(1-\theta)}.$$

But this is a contradiction of the original inequality in the proof and so:

$$f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta}-C_{\nu\delta} < 0.$$

We can then conclude that:

$$W'_{cap}(\bar{r};\theta,C_{\nu\delta}) = \bar{\nu}(f_k(K_{SS}(\bar{r};\theta,C_{\nu\delta}),\mu_A)-\bar{\delta}-C_{\nu\delta})K_{SS,r}(\bar{r};\theta,C_{\nu\delta}) > 0.$$

The positivity of $W'(\bar{r};\theta, C_{\nu\delta})$ follows from Proposition 6.

Proof of Proposition 9

The proof of Proposition 6 shows that $W'_{cons}(\bar{r})$ is independent of θ.

Fix $\bar{r}$ and, for notational convenience, define $\hat{r}(\theta) = \bar{r}_{zero}(\bar{r};\theta)$. Then:

$$\gamma(\bar{r})(1-\beta(1-p)(1+\hat{r}(\theta)) - \beta^2 p(1-\theta)(1+\hat{r}(\theta))^2) = \beta\theta p(1+\hat{r}(\theta))$$

where γ is defined as before:

$$\gamma(\bar{r}) \equiv \frac{\beta p(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})}.$$

Take derivatives with respect to θ.

$$\gamma(\bar{r})(-\beta(1-p)\hat{r}'(\theta) + \beta^2 p(1+\hat{r}(\theta))^2 - 2\beta^2 p(1+\hat{r}(\theta))\hat{r}'(\theta)) = \beta p(1+\hat{r}(\theta)) + \beta\theta p\hat{r}'(\theta)$$

Re-arranging, we obtain for $\theta > 0$:

$$\begin{aligned}
\hat{r}'(\theta) &= \frac{\beta^2\gamma(\bar{r})p(1+\hat{r}(\theta))^2 - \beta p(1+\hat{r}(\theta))}{\beta\theta p + \gamma(\bar{r})(\beta(1-p) + 2\beta^2 p(1+\hat{r}(\theta))} \\
&< \frac{\beta^2 p(1+\hat{r}(\theta))^2 - \beta p(1+\hat{r}(\theta))}{\beta\theta p + \gamma(\bar{r})(\beta(1-p) + 2\beta^2 p(1+\hat{r}(\theta))} \\
&< 0
\end{aligned}$$

since $\gamma(\bar{r}) < 1$ and $(1+\hat{r}(\theta)) < \beta^{-1}$.

Since $\hat{r}$ is strictly decreasing and continuous as a function of θ, and given Proposition 4:

$$\lim_{n\to\infty} \hat{r}(\theta_n) = \beta^{-1} - 1$$

implies that $\lim_{n\to\infty}\theta_n = 0$. Proposition 5 states that $\lim_{n\to\infty} K_{SS,r}(\bar{r};\theta_n, C_{\nu\delta}) = 0$ and hence

$\lim_{n\to\infty} W'_{cap}(\bar{r};\theta_n, C_{\nu\delta}) = 0$. It follows from Proposition 6 that:

$$\lim_{n\to\infty} W'(\bar{r};\theta_n, C_{\nu\delta}) = W'_{cons}(\bar{r}) > 0.$$

Appendix B

This appendix sets forth a set of sufficient conditions for the existence of a stationary equilibrium. The key conditions are as follows:

- $f(1, \hat{k}_{SS}^{-1}) - \bar{\delta} > p\beta^{-1}$, where $\hat{k}_{SS}$ satisfies $f_k(1, \hat{k}_{SS}^{-1}) = (\bar{r} + \bar{\delta})$
- $\bar{\nu}$ is sufficiently large (in a sense to be made precise)
- $p < \beta$
- Aggregate shocks to δ are sufficiently small (in a sense to be made precise).

The first two conditions ensure the existence of a stationary equilibrium in the case without aggregate risk (so that $\delta(z) = \bar{\delta}$ for all z). The last two conditions ensures the existence of a stationary equilibrium in the presence of aggregate shocks.

Riskfree Stationary Equilibrium

We first construct a stationary equilibrium with constant interest rate $\bar{r} < 0$ in an economy in which $\delta(z) = \bar{\delta}$ for all z.

Aggregates

In this subsection, we construct the aggregates in the riskfree stationary equilibrium.

In this riskless economy, (5) implies that r_{zero} satisfies:

$$\frac{\beta p(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})} = \frac{\beta\theta p(1+r_{zero})}{1-\beta(1-p)(1+r_{zero}) - \beta^2 p(1-\theta)(1+r_{zero})^2}.$$

and the rental rate on capital satisfies:

$$r_K = r_{zero} + \bar{\delta}.$$

The constant level of capital in the stationary equilibrium satisfies:

$$f_k(\bar{K}_{SS}, \mu_A) = r_{zero} + \bar{\delta}.$$

Since f_k is homogeneous of degree zero, the capital-labor ratio $\hat{k}_{SS} = \bar{K}_{SS}/\mu_A$ is defined so that:

$$\hat{f}_k(1, \hat{k}_{SS}^{-1}) = (r_{zero} + \bar{\delta}).$$

Recall that $\bar{c}_A$ is pinned down by $\bar{r}$ and $\bar{\nu}$ via (3):

$$u'(\bar{c}_A) = \frac{\beta p(1+\bar{r})\bar{\nu}}{1 - \beta(1-p)(1+\bar{r})}$$

I assume that, given $\bar{r}$, $\bar{\nu}$ is sufficiently large that:

$$f_n(\bar{K}_{SS}, \mu_A) - \bar{c}_A > 0.$$

(Note that $\bar{K}_{SS}$ is independent of $\bar{\nu}$.) This guarantees that accumulators always have enough labor income to fund their consumption purchases, regardless of their asset position.

Let k_{SS}^A be the per-capita capital holdings of accumulators (at the end of the period) in the stationary equilibrium. Then:

$$\bar{K}_{SS} = \mu_A k_{SS}^A + \mu_L(1-\theta)k_{SS}^A(1 + r_{zero})$$
$$k_{SS}^A = \frac{\bar{K}_{SS}(1+p)}{1 + p(1-\theta)(1 + r_{zero})}.$$

Let B_{SS} be the steady-state level of per-capita bondholdings. Then B_{SS} satisfies:

$$\begin{aligned} B_{SS}/\mu_A + k^A_{SS} = {} & f_n(\bar{K}_{SS}, \mu_A) - \bar{c}_A \\ & - \bar{r} B_{SS} \\ & + (1-p)(B_{SS}/\mu_A)(1+\bar{r}) \\ & + (1-p)k^A_{SS}(1+r_{zero}) + p k^A_{SS}(1-\theta)(1+r_{zero})^2 \end{aligned} \tag{12}$$

We can solve for B_{SS} as:

$$\begin{aligned} B_{SS}(p + p^2(1+\bar{r})) = {} & (f_n(\bar{K}_{SS}, \mu_A) - \bar{c}_A) \\ & + (1-p)k^A_{SS}(1+r_{zero}) + p k^A_{SS}(1-\theta)(1+r_{zero})^2 - k^A_{SS} \\ = {} & (f_n(\bar{K}_{SS}, \mu_A) - \bar{c}_A) \\ & + \bar{K}_{SS}(1+r_{zero})(1+p) - (1+p(1+r_{zero}))k^A_{SS} \\ = {} & (f_n(\bar{K}_{SS}, \mu_A) - \bar{c}_A) \\ & + \bar{K}_{SS}(1+r_{zero})(1+p) - (1+p(1-\theta)(1+r_{zero}))k^A_{SS} - p\theta(1+r_{zero})k^A_{SS} \\ = {} & (f_n(\bar{K}_{SS}, \mu_A) - \bar{c}_A) \\ & + \bar{K}_{SS}(1+p)r_{zero} - p\theta(1+r_{zero})k^A_{SS}. \end{aligned}$$

The steady-state level of bonds B_{SS} is positive if:

$$\mu_A f_n(\bar{K}_{SS}, \mu_A) - \mu_A \bar{c}_A + \bar{K}_{SS} r_{zero} - p\mu_A \theta(1+r_{zero})k^A_{SS} > 0 \tag{13}$$

Given that $\bar{r} < 0$, the positivity of bonds ensures that taxes are in fact transfers.

We can rewrite the condition (13) as:

$$\mu_A f_n(K_{SS}, \mu_A) + K_{SS} r_K - \mu_A \bar{c}_A - \bar{\delta} K_{SS} - p\mu_A \theta(1+r_{zero})k^A_{SS} > 0$$

or:

$$f(\bar{K}_{SS}, \mu_A) - \bar{\delta}\bar{K}_{SS} - \frac{p\theta(1 + r_{zero})\bar{K}_{SS}}{1 + p(1-\theta)(1 + r_{zero})} > \mu_A \bar{c}_A \tag{14}$$

We know that $\theta \leq 1$ and (from Proposition 2) that $(1 + r_{zero}) \leq \beta^{-1}$. Hence, (14) is implied by:

$$f(\bar{K}_{SS}, \mu_A) - \bar{\delta}\bar{K}_{SS} - p\beta^{-1}\bar{K}_{SS} > \mu_A \bar{c}_A. \tag{15}$$

The left-hand side of (15) is positive because we assumed above that:

$$f(1, \mu_A/\bar{K}_{SS}) - \bar{\delta} - p\beta^{-1} > 0$$

Hence, we can ensure B_{SS} to be positive by choosing $\bar{\nu}$ to be sufficiently large (which lowers the RHS of (15) without affecting the LHS).

Individual Evolution

We now construct the evolution of individual assetholdings and consumptions. I assume that the initial distribution of assets satisfies the following restrictions:

- The per-capita endowment of agents in state L in period 1 is $B_{SS}(1+p)$ units of bonds and k^A_{SS} units of capital.
- The per-capita endowment of agents in state A in period 1 is $(1-p^2)B_{SS}$ units of bonds and $((1-p)k^A_{SS} + p(1-\theta)k^A_{SS}(1 + r_{zero}))$ units of capital.

We shall see that this specification is recursive, in the sense that these restrictions are also satisfied in all following periods. (Note that these restrictions allow for a wide range of initial distributions of capital across agents in the two different states.)

As is shown in Appendix C, at the individual level, any agent who enters state L with bondholdings b and capitalholdings k sets consumption equal to:

$$c_L = b(1 + \bar{r}) + \theta(1 + r_{zero})k - B_{SS}\bar{r}$$

so that they consume all of their liquid wealth and all of their transfer income. At the end of the period, they have no bondholdings and have capitalholdings equal to $(1-\theta)(1+r_{zero})k$.

An agent who enters state A with bondholdings b and capitalholdings k sets their consumption equal to $\bar{c}_A$. Since they are saving for a state in which they are risk-neutral, they can split their other resources between bonds and capital in a number of ways that are consistent with equilibrium and optimality. One way is linearly. Define $1 > \omega_B > 0$ such that:

$$\begin{aligned} B_{SS}(1+p) = \omega_B(&f_n(\bar{K}_{SS}, \mu_A) - \bar{r}B_{SS} - \bar{c}_A \\ &+ (1-p^2)B_{SS}(1+\bar{r}) + (1-p)k^A_{SS}(1+r_{zero}) + pk^A_{SS}(1-\theta)(1+r_{zero})^2). \end{aligned}$$

Then, agents in state A set their end-of-period bond and capitals equal to:

$$\begin{aligned} b' &= \omega_B(f_n(\bar{K}_{SS}, \mu_A) - \bar{r}B_{SS} - \bar{c}_A + b(1+\bar{r}) + k(1+r_{zero})) \\ k' &= (1-\omega_B)(f_n(\bar{K}_{SS}, \mu_A) - \bar{r}B_{SS} - \bar{c}_A + b(1+\bar{r}) + k(1+r_{zero})). \end{aligned}$$

We now verify that the initial endowment specification replicates under these individual laws of motion, by plugging the initial assetholdings into these individual decision rules. The per-capita bondholdings of agents in state L are given by:

$$\begin{aligned} \omega_B(f_n(\bar{K}_{SS}, \mu_A) - \bar{r}B_{SS} - \bar{c}_A + B_{SS}(1+\bar{r})(1-p^2) \\ +(1-p)k^A_{SS}(1+r_{zero}) + pk^A_{SS}(1-\theta)(1+r_{zero})^2) \end{aligned}$$

which, from the definition of ω_B, is $B_{SS}(1+p)$. The per-capita capitalholdings of agents in state L are given by:

$$\begin{aligned} (f_n(\bar{K}_{SS}, \mu_A) - \bar{r}B_{SS} - \bar{c}_A(1-\omega_B) + (1-p^2)B_{SS}(1+\bar{r}) \\ +(1-p)k^A_{SS}(1+r_{zero}) + pk^A_{SS}(1-\theta)(1+r_{zero})^2) - B_{SS}(1+p) \end{aligned}$$

which, from (12), equals k^A_{SS}. The per-capita bondholdings of agents in state A are given by:

$$(1-p^2)B_{SS},$$

and their per-capita capitalholdings are given by:

$$(1-p)k^A_{SS} + p(1+r_{zero})k^A_{SS}(1-\theta).$$

Adding Aggregate Shocks

We now add aggregate shocks to the analysis in the prior subsection. I suppose that the depreciation rate is indexed by a parameter $\epsilon > 0$:

$$\delta(z,\epsilon) = \bar{\delta} + \epsilon h(z)$$

where $h : [0,1] \to [0,1]$ is a continuous function. Then, I show that there is a stationary equilibrium for ϵ sufficiently close to zero.

Aggregate Quantities

Recall that Proposition 1 allowed us to conclude that (for ϵ sufficiently close to zero, so that agents in state L find it optimal to not hold capital) there exists a function $\bar{r}_K(\bar{r};\theta,C_{\nu\delta})$ that satisfies (5) and is strictly increasing in its first argument. Given the above specification of δ, for a fixed function ν, $C_{\nu\delta}$ is a continuous function of ϵ that equals zero when $\epsilon = 0$. Hence, as we vary ϵ, we can define the rental rate on capital $\hat{r}_K(\epsilon)$ as a continuous function of ϵ where $\hat{r}_K(0) = r_K$. This relationship in turn implies a continuous function $K_{SS}(\epsilon)$, where $K_{SS}(0) = \bar{K}_{SS}$, and:

$$f_k(K_{SS}(\epsilon),\mu_A) = \hat{r}_K(\epsilon).$$

Fix any ϵ. Then, for any z^∞ in $[0,1]^\infty$, define:

$$\begin{aligned}
k^L_{t+1}(\epsilon) &= (1-\theta)k^A_t(\epsilon)(1-\delta(z_t,\epsilon)+\hat{r}_K(\epsilon)), t \geq 1 \\
k^A_{t+1}(\epsilon) &= (K_{SS}(\epsilon) - k^L_{t+1}(\epsilon)\mu_L)/\mu_A \\
&= K_{SS}(\epsilon)/\mu_A - (1-\theta)(\mu_L/\mu_A)k^A_t(\epsilon)(1-\delta(z_t,\varepsilon)+\hat{r}_K(\epsilon)) \\
k^A_{t+1}(\epsilon) &= K_{SS}(\epsilon)(1+p) - p(1-\theta)(1-\delta(z_t,\epsilon)+\hat{r}_K(\epsilon)k^A_t(\epsilon), t \geq 1
\end{aligned}$$

to be the per-capita end-of-period capital holdings for agents in state L and agents in state A. We have assumed that $p\beta^{-1} < 1$. Hence, for ϵ sufficiently near zero:

$$p(1-\theta)(1-\delta(z_t,\epsilon)+\hat{r}_K(\epsilon)) < 1.$$

for all z_t in $[0,1]$. It follows that, given any upper bound M, there exists k^A_1 sufficiently near k^A_{SS} and ϵ sufficiently near zero such that the sequence $|k^A_t(\epsilon) - k^A_{SS}|$ is uniformly bounded from above over $z^\infty \in [0,1]^\infty$.

Conditional on $(k^A_t(\epsilon), z_t)^\infty_{t=1}$ and an initial B_1, we can calculate the implied per-capita bond supply sequence $(B_t(\epsilon))^\infty_{t=1}$ as:

$$\begin{aligned}
(k^A_{t+1}(\epsilon) + B_{t+1}(\epsilon)/\mu_A) &= (1-p)(B_t(\epsilon)/\mu_A)(1+\bar{r}) + ((1-p)k^A_t(\epsilon) + pk^L_t(\epsilon))(1-\delta(z_t,\epsilon)+\hat{r}_K(\epsilon)) \\
&+ f_n(K_{SS}(\epsilon),\mu_A) - \bar{c}_A + B_{t+1}(\epsilon) - B_t(\epsilon)(1+\bar{r})
\end{aligned}$$

For $t \geq 1$:

$$\begin{aligned}
(k^A_{t+1}(\epsilon) + B_{t+1}(\epsilon)/\mu_A) &= (1-p)(B_t(\epsilon)/\mu_A)(1+\bar{r}) + ((1-p)k^A_t(\epsilon) + pk^L_t(\epsilon))(1-\delta(z_t)+r_K(\epsilon)) \\
&+ f_n(K_{SS}(\epsilon),\mu_A) - \bar{c}_A + B_{t+1}(\epsilon) - B_t(\epsilon)(1+\bar{r}).
\end{aligned}$$

and so:

$$B_{t+1}(\epsilon) = -p(1+\bar{r})B_t(\epsilon) + ((1-p)k_t^A(\epsilon) + pk_t^L(\epsilon))(1-\delta(z_t,\epsilon) + r_K(\epsilon)) + f_n(K_{SS}(\epsilon), \mu_A) - \bar{c}_A.$$

Note $p(1+\bar{r}) < 1$. Hence, for any B_1 sufficiently near B_{SS} and ϵ sufficiently small, then $|B_{t+1}(\epsilon) - B_{SS}|$ is uniformly bounded from above over $[0,1]^\infty$.

Appendix C

This appendix shows that the two conditions:

$$\nu_{min} > \frac{\bar{\nu}\beta p}{1-\beta(1-p)} \tag{16}$$

$$C_{\nu\delta} < C_{max} \equiv \frac{1-\beta}{1-\beta(1-p)} \tag{17}$$

ensure that, for any realization of $\nu(z)$, agents in state L never want to hold assets (in excess of the relevant lower bound).

The first condition (16) implies that for any $\bar{r} < 0$:

$$\begin{aligned}
\nu_{min} &> \frac{\beta p \bar{\nu}}{1-\beta(1-p)} \\
&> \frac{\beta p \bar{\nu}(1+\bar{r})}{1-\beta(1-p)(1+\bar{r})} \\
&= u'(\bar{c}_A(\bar{r})) \\
&> \beta(1+\bar{r})u'(\bar{c}_A(\bar{r})).
\end{aligned}$$

It follows that, regardless of their realization of $\nu(z)$, it is optimal for agents in state L to set their bondholdings equal to zero.

The second condition (17) implies that if $\bar{r} < 0$:

$$
\begin{aligned}
\frac{\beta(1+\tilde{r})p}{1-\beta(1+\bar{r})(1-p)} &< \frac{\beta p}{1-\beta(1-p)} \\
&= (1 - C_{max}) \\
&= \frac{p\theta(1-C_{max})}{1-(1-p)-p(1-\theta)} \\
&< \frac{p\theta(\frac{\nu_{min}}{u'(\bar{c}_A(\bar{r}))}) - C_{\nu\delta}p\theta}{1-(1-p)(\frac{\nu_{min}}{u'(\bar{c}_A(\bar{r}))}) - p(1-\theta)(\frac{\nu_{min}}{u'(\bar{c}_A(\bar{r}))})^2}
\end{aligned}
$$

The last inequality makes use of (16) which implies that:

$$\nu_{min} > u'(\bar{c}_A(\bar{r})).$$

It then follows that:

$$\beta(1+\bar{r}_K(\bar{r};\theta,C_{\nu\delta}) - \bar{\delta}) < \frac{\nu_{min}}{u'(\bar{c}_A(\bar{r}))},$$

and so all agents in state L choose not to hold excess capital, regardless of their current marginal utility.

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