The Macroeconomics of Liquidity in Financial Intermediation[a]

Davide Porcellacchia[b] — European Central Bank

Kevin D. Sheedy — London School of Economics

30 September 2024

Abstract

In financial crises, the premium on liquid assets such as US Treasuries increases alongside credit spreads. This paper explains the link between the liquidity premium and spreads. We present a theory of endogenous bank fragility arising from a coordination friction among bank creditors. The theory's implications reduce to a single constraint on banks, which is embedded in a quantitative macroeconomic model to investigate the transmission of shocks to spreads and economic activity. Shocks that reduce bank net worth exacerbate the coordination friction. In response, banks lend less and demand more liquid assets. This drives up both credit spreads and the liquidity premium. By mitigating the coordination friction, expansions of public liquidity reduce spreads and boost the economy. Empirically, we identify high-frequency exogenous variation in liquidity by exploiting the time lag between auction and issuance of US Treasuries. We find a causal effect on spreads in line with the calibrated model.

Keywords: bank runs, bank-lending channel, liquidity.

JEL Codes: E40, E41, E44, E50, E51, G01, G21.

---

[a]We thank Mihaela Badanau Bujac for excellent research assistance. We also thank discussants Madalen Castells Jauregui, Lucia Esposito, Paweł Fiedor, Enisse Kharroubi, Oana Peia and Ansgar Walther, the audience at the Central Bank of Ireland, ECB, ESRB, Durham University, Ghent University, Goethe University, LSE, National Bank of Croatia, and University of Bern, and at the ECB's Money Market conference, EEA-ESEM congress, IBEFA Summer meeting, Oxford Saïd – ETH Zürich Macro-finance conference, T2M conference, RCEA conference, Sailing the Macro workshop and SED annual meeting. Opinions and findings in this paper reflect the authors' views, not necessarily those of the ECB.

[b]Contact: davide.porcellacchia@ecb.europa.eu

1 Introduction

Disruptions to financial intermediation make credit more expensive and thereby harm the economy. This pattern motivated the introduction of a specific banking friction in macroeconomic models. In their seminal contribution, Gertler and Kiyotaki (2010) introduce a problem of moral hazard between banks and their creditors. Consequently, banks' ability to fund themselves is limited by the value of their equity. The resulting leverage constraint leads to a powerful impact of bank net worth on macroeconomic outcomes via credit spreads. This explains the general observation of plummeting bank values, higher bank-funding costs and increased credit spreads in financial crises.[1] However, this approach to banking is silent on why we observe soaring demands for liquidity and hence liquidity premiums in times of financial stress.

We observe a heightened liquidity premium, defined as the difference between the 3-month general-collateral repo rate and the 3-month treasury-bill rate, during banking crises.[2] Figure 1 shows this for the global financial crisis.[3] More systematically, this paper documents a positive relationship over time between the liquidity premium and banks' funding costs, as measured by the difference between the 3-month LIBOR and the 3-month repo rate. Figure 2 shows the positive correlation between these two variables.[4]

Since policymakers often react to banking crises through expansions of liquidity, it is crucial to understand the causes of the empirical relationship between the liquidity premium and funding costs.[5] Existing research (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016) has shown the liquidity premium responds to government policies. The facts documented in this paper suggest that tight bank funding drives demand for liquid assets. This is consistent with a view that scarce liquidity impairs bank lending in times of stress, suggesting a channel through which a greater supply of public liquidity can benefit the economy.

Motivated by this, we do two things in the paper. First, we develop a novel financial friction based on coordination failure among bank creditors. Liquid-asset holdings and bank net worth both mitigate the coordination friction and are substitutes. Hence, when net worth is scarce, as in a financial crisis, banks demand more liquidity. This explains a high liquidity premium. It also implies policy can stabilize the economy by appropriately supplying liquid assets.

[1] Figure 8 and Figure 9 in appendix A shows the average dynamics of these variables in banking crises as identified by Baron et al. (2021).

[2] This definition of liquidity premium is standard in the literature (Nagel, 2016). More discussion on this point is provided in section 7.

[3] Figure 10 in appendix A zooms in on the recent period (2019–2023).

[4] Figure 12 shows that the positive correlation holds both in expansions and recessions. Figure 13 shows the scatterplot with data at monthly frequency instead of binned.

[5] There is a debate in the literature on the real effects of liquidity policies and the channels through which they operate (Kuttner, 2018).

Figure 1: Global financial crisis.

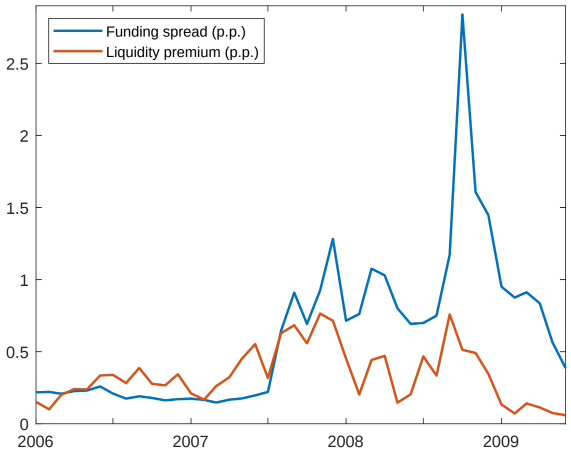
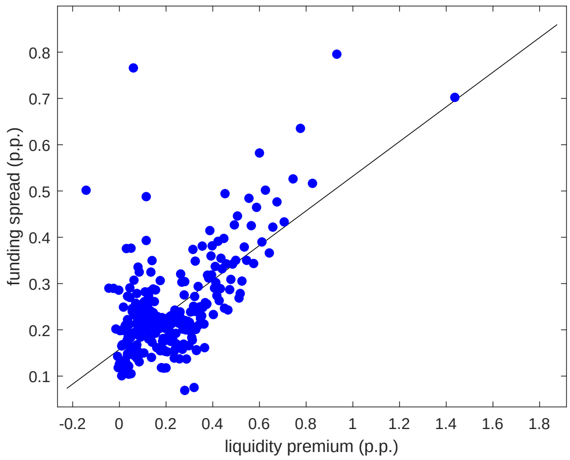


Figure 2: May 1991 – June 2023.



Note 1: Funding spread is 3-month (3M) LIBOR minus 3M general-collateral (GC) repo rate. Liquidity premium is 3M GC repo rate minus 3M T-bill rate.
Note 2: US daily data. Figure 1 plots at monthly frequency. Figure 2 plots a binned scatterplot with 300 quantile-based bins. Data sources in appendix B.

Second, we test whether the data supports the model's mechanism. In particular, the model implies that an increase in the liquidity premium pushes up the bank-funding spread. This is because it induces banks to economize on holding liquid assets. To identify exogenous variation in the liquidity premium, we use the quantity of outstanding US Treasuries as an instrumental variable. The instrument is strongly relevant and predetermined at daily frequency given the lag of a few days between the auction and issuance of Treasury securities. We find a significant positive effect.

Maturity transformation, a key function of financial intermediation, results in a mismatch on the balance sheets of banks.[6] This creates the conditions for coordination failures in the market for deposits (Diamond and Dybvig, 1983).[7] Such coordination failures take the form of "runs" by panicked creditors and played a central role in the global financial crisis in 2007, the crisis of US money-market funds in 2020 and the 2023 regional banking crisis (Shin, 2009; Bernanke, 2010; Li et al., 2021; Choi et al., 2023).

This paper models the deposits market as a coordination game. Strategic complementarities imply that under perfect information there are multiple equilibria. However, a deviation from common knowledge across depositors, which we introduce following the large literature on global games (Morris and Shin, 2003), leads to a unique equilibrium. Intuitively, without common knowledge it is impossible for depositors to coordinate on arbitrary equilibria. In the resulting unique equilibrium, depositors

[6]For simplicity, we use "banks" as a general label for financial intermediaries and "deposits" for their short-term debt. The analysis applies more broadly to financial intermediaries with a maturity mismatch on their balance sheet.

[7]Perfect deposit insurance rules out coordination failures in these models. However, in the period 1984–2023Q3 deposits made up 73% of banks' liabilities and only 62% of deposits were insured on average. These values are respectively 79% and 57% in 2023Q3 (data source: FDIC QBP).

demand a level of compensation that is commensurate to a bank's fragility, defined as the minimum share of depositors that must not run for the bank to survive. If the bank offers an insufficient deposit rate, then depositors run even though the bank is solvent. Intuitively, banks must compensate depositors for run risk. However, as long as the bank offers a sufficiently high deposit rate, no run takes place because no depositor has an incentive to start the run that they fear.

Bank fragility, the heart of the coordination friction, is endogenous. It is a function of the bank's balance-sheet fundamentals. In particular, more levered banks and banks with fewer liquid assets as a share of total assets are more fragile. Therefore, they face higher funding costs. In other words, the coordination friction results in a mapping from higher capital and liquidity ratios into a lower funding spread. The capital and liquidity ratios are bank choices. In equilibrium, these choices trade off the returns on illiquid assets against the increased funding costs due to more fragility.

With the coordination friction embedded in a standard real business cycle model, we can study its role quantitatively in the transmission of macroeconomic shocks. The banking friction can be calibrated using observations on the average size of the liquidity premium, the credit spread, and banks' return on equity. The parameters of the macroeconomic model are set following the literature.

The friction amplifies shocks that affect banks' net worth. By making it more costly for banks to fund themselves, a reduction in net worth weakens the supply of credit and reduces the economy's output. The friction amplifies the effect on output of capital-destruction shocks, commonly studied in the literature on financial crises, by about one third on impact. At longer horizons, the amplification is greater. This persistence comes from banks' funding costs rising alongside credit spreads, implying banks' net worth is rebuilt very slowly in contrast to models with a leverage constraint. Furthermore, the increase in fragility due to scarcer net worth gives banks an incentive to demand more liquid assets. This generates a countercyclical liquidity premium.

Monetary and fiscal liabilities of the government are the natural source of liquidity supply. Banks create liquid assets for other sectors of the economy but they cannot produce assets that maintain their value in case of a systemic run.[8] Therefore, the relevant supply of liquid assets is a policy variable. In the model, an increase in the supply of liquid assets is expansionary. The liquid assets are absorbed by banks' balance sheets and reduce their fragility. With lower fragility, banks have access to funding on better terms and thus find it optimal to lend more. In other words, the supply of liquidity crowds in private investment. In the calibrated model, a shock that reduces the liquidity premium by 15 basis points leads to an expansion of credit supply reducing credit spreads by 24 basis points. This generates a 2-percent increase in investment on

[8] This is related to the seminal finding in Holmström and Tirole (1998) of a role for public liquidity supply in the presence of aggregate risk.

impact, with GDP also going up by a quarter of one percent. Moreover, the supply of liquidity can be used as a stabilizing policy tool in the face of shocks. If the government responds to disruptions to financial intermediation by accommodating the increased demand for liquid assets, it can dampen the amplification of shocks.

We test the key implication of the model: an increase in the liquidity premium causes an increase in the funding spread.[9] The econometric challenge is to find exogenous variation in the liquidity premium. Our strategy is to run the analysis at daily frequency and use the quantity of outstanding US Treasuries as an instrument. The instrument is strongly relevant to the liquidity premium. As for its validity, the quantity of treasuries is predetermined at daily frequency because there is a lag of a few days from auction, the latest point at which it may be determined, to issuance.[10] Moreover, we include as controls 80 lags of financial and economic variables available at daily frequency, such as the dollar exchange rate and the liquidity premium itself. This cleans the autocorrelation out of the error term and ensures there is no endogeneity of the instrument driven by confounding variables or reverse causality. After all, if the error term only contains a non-autocorrelated daily shock, it cannot drive a variable determined on a previous day.

The empirical result is a robustly-significant positive effect of the liquidity premium on the funding spread. A 1-basis-point increase in the liquidity premium causes the funding spread to increase by about 1 basis point. This is in line with the size of the corresponding effect in the calibrated model. As a robustness check, we split the sample between expansions and recessions. We find no evidence of a different size of the effect according to the state of the economy.

Literature review. An extensive literature builds macroeconomic models around a leverage constraint on banks (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Boissay et al., 2016; Mendicino et al., 2020; Di Tella and Kurlat, 2021; Karadi and Nakov, 2021; Van der Ghote, 2021; Fernández-Villaverde et al., 2023).[11] This friction, based on moral hazard on bankers' part, does not naturally give a role to banks' liquid-asset holdings unlike this paper's friction based on coordination failure. Moreover, models with the moral-hazard friction generate limited shock propagation because adverse shocks to bank net worth push up bank profitability by increasing credit spreads with little response of funding costs. Also, they struggle to match the observed procyclicality of banks' book leverage (Nuño and Thomas, 2017). The coordination friction is an improvement on

[9] We measure the funding spread as the difference between the 3-month LIBOR and the 3-month GC repo rate. More discussion on the measurement is provided in section 7.

[10] A related high-frequency approach to treasury-market data is adopted in Ray et al. (2024).

[11] Holmström and Tirole (1997) is an early example of a model in which a leverage constraint on banks is micro-founded with a moral-hazard problem.

the latter two counts, too. In this paper's model, shock propagation is strong because the positive effect of higher credit spreads on bank profitability after adverse shocks is largely offset by increased funding costs. And we find that leverage is procyclical for standard shocks that affect credit demand, such as productivity shocks.

In this paper, banks demand liquid assets to mitigate the risk of coordination failures among their creditors. This is a novel source of demand for liquid assets in the macroeconomic literature.[12] The existing literature posits an exogenous risk that bank creditors withdraw their funds. Banks demand liquid assets as a precaution to limit the amount they must borrow from the central bank at a punitive interest rate (Poole, 1968; Arce et al., 2020; Bianchi and Bigio, 2022) or the amount of assets they must sell at fire-sale prices (Drechsler et al., 2018; d'Avernas and Vandeweyer, forthcoming; Li, forthcoming) if hit by an adverse liquidity shock. In our model, the risk of deposit-holder withdrawals is a fully endogenous function of bank fundamentals.[13]

Studies evaluating the effects of quantitative-easing programmes, recent examples of policies that increased the supply of liquid assets, find interest-rate reductions in line with our model (Gagnon et al., 2011; Krishnamurthy and Vissing-Jorgensen, 2011; Chodorow-Reich, 2014).[14] More recently, Acharya and Rajan (2022) and Diamond et al. (2023) have sounded a cautionary note on the effects of liquid-asset supply in the context of QE. The former paper stresses that some of the benefit to bank fragility of additional liquidity supply is undone by banks taking on extra leverage. This result conforms to this paper's mechanism. The latter contribution finds empirically that liquid-asset holdings increase banks' marginal cost of lending. The authors suggest the reason for this may be limited balance-sheet space due to regulation. While the effect of regulation is beyond the scope of our paper, the driving force behind our paper's results, i.e. the positive effect of liquid-asset holdings by banks on the demand for their debt, is not considered in Diamond et al. (2023).

Banks' vulnerability to runs has been first formalized in Diamond and Dybvig (1983). That paper illustrates the possibility of runs, but it does not speak to their determinants because it has multiple equilibria. A literature in macroeconomics has adopted the multiple-equilibrium approach to study the effects of bank runs (Gertler and Kiyotaki, 2015; Gertler et al., 2016, 2020; Amador and Bianchi, forthcoming). A limitation of this approach is the need to assume an arbitrary relationship of the probability of runs with fundamentals. Because of this limitation, the role of liquidity

[12] A strand of the banking literature formalizes this in static partial-equilibrium models (Rochet and Vives, 2004; Ahnert, 2016).

[13] A reduced-form approach to the demand for liquid assets is common in studies of the effects of liquidity supply (Krishnamurthy and Vissing-Jorgensen, 2012; Benigno and Benigno, 2022; Angeletos et al., 2023). Such approach may miss important characteristics of demand for liquid assets such as the substitutability of liquidity and bank capital, which is a feature of our model and which DeYoung et al. (2018) finds empirically.

[14] Ray et al. (2024) develop a theory with segmented asset markets in which QE has real effects.

in the determination of run risk does not emerge.

Leveraging theoretical results from Carlsson and van Damme (1993), Goldstein and Pauzner (2005) show that a small departure from perfect information produces a unique equilibrium in a bank-run game. This is an attractive feature because the evidence points to a strong relationship between poor bank fundamentals and banking crises (Gorton, 1988; Baron et al., 2021). A large literature in banking uses variations of such second-generation bank-run models to study optimal policy (Vives, 2014; Kashyap et al., 2024; Ikeda, 2024). Our paper is the first to integrate a second-generation bank-run model in a macroeconomic framework.[15]

Outline of the paper. The coordination game among depositors is laid out in section 2. This results in a constraint on bank behaviour, which is integrated in a standard macroeconomic model in the following two sections. The model is calibrated and quantitative experiments are carried out in section 5. In section 6, we discuss normative implications of the model. In section 7, the empirical results of the study are reported. The appendices contain: (A) figures, (B) details about data sources, (C) proofs, (D) details about the mechanics of a bank run in the macroeconomic model, (F) steady-state results, and (G) the full model solution.

2 Coordination game

This section sets up the coordination game played by bank depositors. It solves for the unique equilibrium, which implies a relationship between banks' balance sheets and the interest rates required to induce households to hold deposits. Since banks anticipate the outcome of the coordination game, in the remainder of the paper this relationship constrains the choices made by banks.

The economy contains a unit continuum of banks (more generally, financial intermediaries) indexed by $b \in [0,1]$. Deposits at bank b are demand deposits paying interest rate j_b if held to the next time period, but with an option to withdraw on demand. While referred to as 'demand deposits', this bank debt can be interpreted more broadly as short-term unsecured borrowing in money markets that is frequently rolled over.

A coordination game among depositors is played in each discrete time period, but time subscripts are omitted in this section given the essentially static nature of the game. Just before the coordination game begins, all deposits $D_b \geq 0$ at bank b are held equally by a unit continuum of households indexed by $h \in [0,1]$. Expected payoffs in the next time period are discounted at rate ρ by all households.[16]

[15] A small strand of the banking literature has studied the relationship of bank runs with selected macroeconomic variables (Ennis and Keister, 2003; Martin et al., 2014; Porcellacchia, 2020; Mattana and Panetti, 2021; Leonello et al., 2022).

[16] Since there is a continuum of banks, depositor behaviour can be analysed as if households were risk

Bank fragility. Before households decide whether to hold deposits in the coordination game, banks make portfolio and leverage decisions. Bank b chooses how much to invest in illiquid and liquid assets A_b and M_b respectively, where the notion of liquidity is defined below. Taking as given net worth (equity) N_b, these choices result in deposit creation up to a level of deposits D_b consistent with the balance-sheet identity $A_b + M_b = D_b + N_b$. These deposits are in the hands of households at the point where the coordination game among depositors is played.

If a positive fraction $1 - H_b$ of households chooses not to hold deposits D_b at bank b, the bank must make a total payment $(1 - H_b)D_b$ to these households by disposing of some assets. The full value M_b of the liquid assets acquired earlier can be obtained at this point, but disposal of illiquid assets A_b during the coordination game only recovers a fraction λ of their value at acquisition.[17] If the proceeds of these asset liquidations are insufficient to cover the withdrawals, then the bank fails. The condition for failure is given by

$$(1 - H_b)D_b > \lambda A_b + M_b. \tag{1}$$

The parameter $\lambda \in [0,1]$ measures the liquidity of assets A_b relative to the benchmark of the perfectly liquid asset M_b. Rearranging the condition above and using the balance-sheet identity, bank b does not fail if $H_b \geq F_b$, where fragility F_b is given by

$$F_b = \frac{(1-\lambda)A_b - N_b}{A_b + M_b - N_b}. \tag{2}$$

If net worth is positive, fragility is a number between 0 and $1 - \lambda$, and greater net worth lowers fragility. Increased holdings of liquid assets M_b reduce a bank's fragility where it is initially positive, while holding more illiquid assets A_b raises fragility when it is below $1 - \lambda$ initially. A noteworthy feature of fragility is that it can be expressed in terms of familiar liquidity and capitalization ratios, respectively

$$m_b = \frac{M_b}{A_b + M_b} \quad \text{and} \quad n_b = \frac{N_b}{A_b + M_b}, \tag{3}$$

as

$$F_b = \frac{(1-\lambda)(1-m_b) - n_b}{1 - n_b}. \tag{4}$$

Hence, the bank's scale plays no role in determining its fragility.

Notice that the illiquidity of assets A_b is key to the existence of a coordination problem. If the full value of any assets can always be realized (i.e., $\lambda = 1$), then banks with positive net worth can never be fragile. It is also important that banks' portfolio choice is made before people decide whether to hold deposits: once illiquid assets are

neutral and ρ taken as given. In the full model, the common discount rate ρ is an endogenous variable.

[17] A literature studies transaction costs (Grossman and Miller, 1988; Brunnermeier and Pedersen, 2008) and adverse selection (Eisfeldt, 2004) as sources of asset illiquidity.

funded by deposit creation, there is strategic complementarity in depositors' holding decisions. This timing assumption could capture the fact that banks create deposits when they make a loan and then someone in the economy must be willing to hold the deposits if the bank is to avoid having to dispose of assets. More generally, it could be interpreted as a mismatch between the timing of capital investment, which is typically long-term, and banks' more short-term funding sources.

Structure of the game. Independently for each bank b, households make a simultaneous binary choice whether to hold deposits D_b until the next period.[18] This choice is captured by the indicator function H_{bh}, which equals 1 if household h holds and 0 if it chooses to withdraw. Withdrawing households receive funds in the same time period.[19]

Holding bank deposits exposes households to credit risk because banks can fail. If this happens, those holding deposits recover the principal after incurring a cost θ per unit of deposits. The parameter $\theta > 0$ represents losses associated with the bankruptcy process, and these costs are paid by depositors at the beginning of the next time period.[20]

In this economy, banks fail because of 'runs' — too many depositors deciding to withdraw. The share of households who hold bank b's deposits is $H_b = \int_0^1 H_{bh} \, \mathrm{d}h$, and there is some endogenous minimum fraction F_b who must hold for the bank not to fail. Thus, the indicator function Φ_b for the failure of bank b is

$$\Phi_b(F_b, H_b) = \begin{cases} 0 & \text{if } H_b \geq F_b, \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

The variable F_b is bank b's fragility. This is the measure of the bank's fundamentals in the coordination game, and it depends on the liquidity of the bank's portfolio of assets and its leverage according to equation (2) as seen above. $F_b = 1$ means that bank b needs each and every household to trust it and hold its deposits in order to survive. On the other hand, an intermediary with $F_b = 0$ is not fragile at all — it will not fail even if all households refuse to hold its deposits.

Conditional on knowing a bank's fragility and the share of households holding its deposits, the net payoff per unit of deposits from holding versus withdrawing is

$$\pi(F_b, H_b) = \frac{(j_b - \rho)(1 - \Phi_b) - \theta \Phi_b}{1 + \rho}, \tag{6}$$

[18] To simplify the game, holding is a binary choice, but households would not gain from being able to make partial withdrawals here.

[19] To keep the analysis tractable, households would need to wait until the next time period to deposit these funds at another bank.

[20] This timing assumption is not essential, but is chosen for consistency with the full macroeconomic model presented later.

with Φ_b given by equation (5). Households want to hold deposits at bank b if $\pi(F_b, H_b) \geq 0$ given their discount rate ρ and the interest rate j_b offered by the bank.[21] Given $j_b \geq \rho$ as required for the net payoff to be non-negative, $\pi(F_b, H_b)$ is weakly decreasing in fragility F_b, representing a deterioration in the bank's fundamentals, and weakly increasing in the fraction H_b holding deposits, indicating the presence of strategic complementarity in the coordination game. With complete information, there would be multiple Nash equilibria whenever fragility is positive and $j_b \geq \rho$: an equilibrium where everyone holds with $H_{bh} = 1$, and a 'bank-run' equilibrium with $H_{bh} = 0$.

Incomplete information. In this paper, sunspot-driven bank runs are ruled out by a small deviation from complete information. Households cannot observe bank b's fragility F_b. Instead, each receives an independent private signal centred around the true fragility $\hat{F}_{bh} \sim \mathrm{U}[F_b - \omega, F_b + \omega]$ for some $\omega > 0$.

This information structure is a key ingredient of a global game. Even if the noise is vanishingly small and thus households are virtually certain about bank fundamentals, uncertainty about the information held by other households makes it hard to coordinate behaviour. Coordination on other publicly available information is ruled out here by assuming all depositors hold a uniform prior on F_b before observing their signals, and that it is common knowledge everyone holds this prior.[22] Formally, in the bank-b coordination game, all households' prior information is $\mathcal{I}_b = \{F_b \sim \mathrm{U}_{\mathbb{R}}, D_b, j_b\}$.[23] Household h updates this prior using signal $\hat{F}_{bh}$ to form expectations $\mathbb{E}_{bh}[\cdot] = \mathbb{E}\left[\cdot \middle| \hat{F}_{bh}, \mathcal{I}_b\right]$.

Equilibrium strategies. As is well-established in the literature, this small deviation from complete information in combination with well-behaved strategic complementarities in the net-payoff function rules out sunspot equilibria. The game is left with a unique Bayesian Nash equilibrium that depends on bank fundamentals.

Households follow strategies where conditional on their signals and prior information, deposits are held if and only if $\mathbb{E}_{bh}[\pi(F_b, H_b)] \geq 0$. If $j_b < \rho$ then the net payoff is negative irrespective of H_b, and hence withdrawing is a strictly dominant strategy for all signals $\hat{F}_{bh}$. In the interesting case where $j_b \geq \rho$, standard results from the literature on global games show that the unique strategy profile surviving rounds of iterated deletion of dominated strategies approaches a threshold rule.

Lemma 1. *In the unique equilibrium strategy, household h holds bank b's deposits if and*

[21] If indifferent, households are assumed to hold deposits to break ties.

[22] A literature studies conditions under which information provided by publicly observed endogenous variables such as policy (Angeletos et al., 2006) and prices (Atkeson, 2000; Angeletos and Werning, 2006) restores common knowledge.

[23] For simplicity, we specify an improper uninformative prior. The results are unchanged for a prior with positive density at least on the set $[-\omega, 1 + \omega)$.

only if $\hat{F}_{bh} \leq F_b^*$ *with*

$$F_b^* = \frac{j_b - \rho}{j_b - \rho + \theta} + \frac{j_b - \rho - \theta}{j_b - \rho + \theta}\omega \tag{7}$$

and $j_b \geq \rho$.

Proof. Please refer to appendix C. □

Given $j_b \geq \rho$, each household h compares its signal $\hat{F}_{bh}$ to a common threshold $F_b^* \in [-\omega, 1+\omega)$ and holds deposits if the signal indicates fragility is low enough. As the deposit interest-rate spread over ρ becomes larger, households use a higher threshold in (7) and thus accept greater levels of fragility while choosing to hold deposits.

To understand the threshold F_b^* intuitively, consider vanishing noise $\omega \to 0$. In this case, each household is near perfectly informed about bank fragility F_b but is uninformed about the ranking of its signal relative to other households'. With strategic complementarities, this implies a risk that many households are more pessimistic than oneself. This risk is higher when the bank is more fragile because fewer pessimistic households are enough to induce bank failure. At the common run threshold F_b^*, the interest-rate spread on deposits is enough to compensate for this risk. So, a household receiving a signal $\hat{F}_{bh} = F_b^*$ is indifferent between running or not.

It is possible to aggregate up the equilibrium behaviour of individual households and obtain the share of a bank's deposits that are held given the bank's fragility.

Lemma 2. *Consider a bank b with fragility* F_b *and* F_b^* *given by* (7)*. The share of households holding deposits in equilibrium is given by*

$$H_b = \begin{cases} 1 & \text{if } F_b \leq F_b^* - \omega, \\ \frac{1}{2} + \frac{F_b^* - F_b}{2\omega} & \text{if } F_b \in \left(F_b^* - \omega, F_b^* + \omega\right], \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

if $j_b \geq \rho$. *If* $j_b < \rho$, *then* $H_b = 0$.

Proof. Please refer to appendix C. □

The law of large numbers ensures that there is no uncertainty about the share of households holding the deposits. Liquidity and leverage, which determine fragility F_b, and the interest rate j_b, which determines households' tolerance of fragility F_b^*, deterministically pin down the share of households holding bank-b deposits in equilibrium.

In general, partial runs with $H_b \in (0,1)$ are possible. To keep the model tractable, we work under conditions that rule them out. These conditions are (1) small enough noise $\omega \to 0$ and (2) a degree of randomness in bank fragility. Vanishing noise implies that partial runs can only take place for $F_b = F_b^*$ and the randomness of bank fragility ensures that the probability of the realization of precisely this value is zero. The introduction of randomness in bank fragility is in the spirit of banks having a "trembling hand". The bank sets $\tilde{F}_b$ and true fragility F_b is uniformly distributed around this value with tremble $\tau > 0$.[24]

Proposition 1. *Consider $\omega/\tau \to 0$ and true fragility $F_b \sim U\left[\tilde{F}_b - \tau, \tilde{F}_b + \tau\right]$. Then, we have that:*

1. *Partial runs with $H_b \in (0,1)$ have probability zero for any $\tilde{F}_b$, F_b^* and j_b.*
2. *The probability of all households holding deposits is given by*

$$Pr(H_b = 1) = \begin{cases} 1 & \text{if } \tilde{F}_b \leq F_b^* - \tau, \\ \frac{1}{2} + \frac{F_b^* - \tilde{F}_b}{2\tau} & \text{if } \tilde{F}_b \in \left(F_b^* - \tau, F_b^* + \tau\right], \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

if $j_b \geq \rho$. If $j_b < \rho$, then $Pr(H_b = 1) = 0$.

Proof. Please refer to appendix C. □

For fragility $F_b \sim \mathrm{U}\left[\tilde{F}_b - \tau, \tilde{F}_b + \tau\right]$ and deposit rate j_b, we obtain as the game's outcome the probability of a run as given in Proposition 1.

No runs, equilibrium beliefs and funding costs. For the rest of the paper, we maintain $\omega/\tau \to 0$ to rule out partial runs. Moreover, we let the tremble $\tau \to 0$ so that banks find it infinitely costly in terms of the increase in the run probability to increase $\tilde{F}_b$ above $F_b^* - \tau$.[25] Hence, we effectively can treat

$$\tilde{F}_b \leq F_b^* - \tau \tag{10}$$

as a constraint on bank behaviour ensuring there is never a run on the bank. Additionally, profit-maximizing banks set

$$j_b \geq \rho \tag{11}$$

to avoid all households running with certainty.

Under the no-run conditions above, it is instructive to study the distribution of household beliefs in the game's equilibrium. Households have heterogeneous information. Do some believe that a run will take place?

[24] Because in the rest of the analysis we focus on an arbitrarily small tremble, it is not necessary to specify what components of true fragility, as given by equation (2), are random.

[25] Runs are costly for banks because their net worth, which is positive in equilibrium, is lost in case of run.

Corollary 1. *Under $\tilde{F}_b \leq F_b^* - \tau$ and $j_b \geq \rho$, in equilibrium all households are sure that no run takes place.*

Proof. Please refer to appendix C. □

No household receives a signal that makes it attach positive probability to a run. This is because banks' tremble is large relative to the noise in households' signals. Hence, average true fragility $\tilde{F}_b$ is set so far below the threshold F_b^* that all realized signals imply runs are impossible.

The equilibrium run threshold (7) combined with the equations that describe bank behaviour, (10) for $\tau \to 0$ and (11), yields a link between the required interest rate on deposits and a bank's fragility:[26]

$$j_b - \rho \geq \max\left\{\frac{F_b}{1-F_b}, 0\right\}\theta. \tag{12}$$

Banks pay a spread on their deposits in the game's equilibrium although all depositors are certain to be paid back in full as per Corollary 1. In this model, the interest-rate spread on deposits is not risk remuneration. The existence of the spread ensures that in equilibrium households do not run. So, the spread is necessary to make the deposits risk-free. Without a commensurate spread, the bank would suffer a run and fail for sure.

Substituting the determinants of bank fragility, given by equation (2), into condition (12), yields a mapping from the bank's balance sheet to the deposit rate required to avoid a run:

$$j_b - \rho \geq \max\left\{\frac{(1-\lambda)A_b - N_b}{\lambda A_b + M_b}, 0\right\}\theta. \tag{13}$$

In the full model, we use this equation as the no-run condition that restricts bank behaviour.

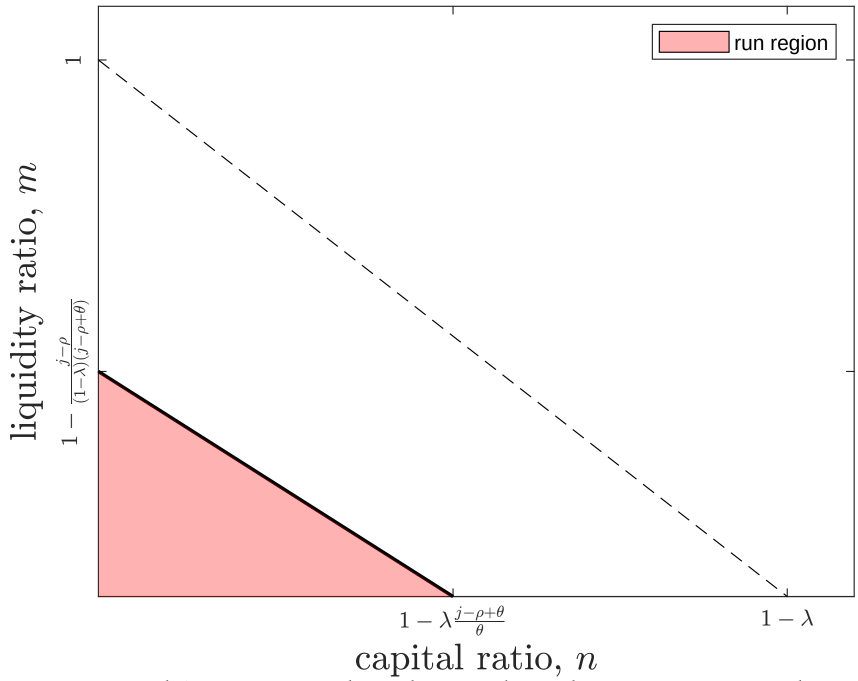
The substitution of familiar financial ratios (3) in the no-run condition gives further intuition into the result. The mapping from financial ratios to the deposit rate required to avoid bank runs is

$$j_b - \rho \geq \max\left\{\frac{(1-\lambda)(1-m_b) - n_b}{\lambda + (1-\lambda)m_b}, 0\right\}\theta. \tag{14}$$

A graphical representation is provided in Figure 3. In the figure, the dashed line depicts the combinations of capital ratio and liquidity ratio that rule out bank failure with zero spread on deposits. The region of bank fundamentals that lead to bank failure, coloured in red, is always within the dashed line for a positive spread on deposits. All else equal, a higher interest on deposits makes the failure region smaller. The key implication

[26] Working in the limiting case $\tau \to 0$ ensures that $\tilde{F}_b = F_b$.

Figure 3: Balance-sheet fundamentals and bank runs.



Note: For a given spread $j_b - \rho$, a run takes place in the red region as per condition (14). The dashed line contains the run region for a zero spread.

of this result is that there is a three-way substitutability from the bank's perspective between equity, liquidity and interest on deposits. For instance, a bank can lever up while keeping its interest-rate expenses in check by boosting its liquid-asset holdings.

The coordination game creates a motive for banks to hold liquid assets, keep a buffer of net worth, and pay extra interest on deposits. These three courses of action are substitutes and costly for banks. In section 4, we study how banks choose among these options on the basis of economic conditions. Analysing the implications for macroeconomic outcomes first requires integrating the coordination game with a full macroeconomic model.

3 Macroeconomic model

This section embeds the coordination friction faced by holders of bank deposits into a macroeconomic model. The core of the economy is a real business cycle model as in Kydland and Prescott (1982).

Timeline. Each discrete time period $t = 0, 1, 2, \ldots$ is divided into three stages. At the first stage, competitive markets for goods, labour, physical capital, liquid assets, and illiquid bonds are open. Aggregate shocks and households' signals are realized. Households choose labour supply and holdings of non-bank assets, and firms produce final goods and incomes are distributed. The government chooses a supply of liquid assets and adjusts fiscal policy. During this stage, banks choose deposit creation and deposit interest rates, select a portfolio of liquid and illiquid assets to hold, and pay dividends. At the second stage, households play the coordination game described in section 2, choosing whether to hold deposits at each bank. Bank failures may occur at this stage. In the last stage, on the basis of what happened at both previous stages of period t, households' consumption is determined.

Physical capital as the illiquid asset. The illiquid asset held by banks is physical capital. A surviving bank b holding illiquid assets $A_{b,t-1}$ at the end of period $t-1$ has a stock of physical capital $K_{bt} = X_t A_{b,t-1}$ to rent out to firms for use in production of final goods. The variable X_t, which has mean 1, represents an exogenous capital-quality shock common to all banks. Physical capital K_{bt} depreciates at rate δ during each time period.

At the first stage of period t, final goods can be transformed into new capital through investment $I_{bt} = A_{bt} - (1-\delta)K_{bt}$ financed by banks (or existing capital transformed back into final goods if investment is negative). Only goods transformed into capital by this stage can be stored and carried into period $t+1$ as physical capital.

Capital is illiquid at the second stage of period t in the sense that investment is partially irreversible at that point. Only a fraction λ of a bank's physical capital can be immediately converted back into goods usable for consumption without causing the bank to fail. More than this amount can be recovered, but at the cost of bank failure, with the wiping out of bank equity acting as an adjustment cost.[27] Those holding deposits at the point of bank failure must also incur a cost θ to recover each unit of deposits through the bankruptcy process described in section 2.

Other frictions. For the model of banks introduced earlier to be relevant for macroeconomics, three other frictions are needed. First, households cannot directly hold physical capital (banks' illiquid asset), so financial intermediation is necessary for capital accumulation and production. Second, bank debt takes the form of the short-term demand deposits described in section 2, so there is a mismatch between the liquidity of bank liabilities and assets. Third, banks require equity but face limits on accumulating net worth, and thus their asset holdings cannot be entirely financed through equity.

While the model does not speak to why such frictions are present, these are all standard assumptions in the existing macro-banking literature. The first could be justified if holding illiquid assets requires expertise possessed only by bankers, or diversification through the scale at which bankers operate. The second might come from some short-term liquidity needs of households that preclude them locking up wealth in a long-term asset.

[27] Banks will operate with positive net worth.

The third is often formally built into macro-banking models through exogenous exit of banks or bankers. Here, a simpler foundation for the assumption is a problem of separation of ownership and control of banks. Suppose bank employees are able to divert bank profits to their bonus pools if these funds are not swiftly returned to shareholders. Formally, suppose a constant fraction $\gamma/(1+\gamma)$ of pre-dividend net worth is vulnerable to diversion as bonuses G_{bt}, where γ is a positive parameter. Even if bank shareholders would otherwise prefer earnings to be retained, they need to pay out at least the funds vulnerable to diversion. This motivates a minimum dividend condition

$$\Pi_{bt} \geq \gamma N_{bt}, \tag{15}$$

where Π_{bt} is the dividend paid by bank b at the first stage of period t, and N_{bt} denotes net worth after distribution of dividends.

3.1 Production

Homogeneous final goods for consumption or investment are produced by a continuum of perfectly competitive firms $f \in [0,1]$. These firms hire homogeneous labour L_{ft} at wage w_t and rent physical capital K_{ft} from banks at price p_t in competitive markets. Firms face a constant-returns-to-scale Cobb-Douglas production function

$$Y_{ft} = Z_t K_{ft}^{\alpha} L_{ft}^{1-\alpha}, \tag{16}$$

where Z_t is the exogenous level of total factor productivity and α is the capital elasticity of output ($0 < \alpha < 1$). All prices and wages are fully flexible, and the purchase price of final goods is normalized to one so that all variables are in real terms.

Firms maximize profits $\Pi_{ft} = Y_{ft} - p_t K_{ft} - w_t L_{ft}$ that are immediately paid out as dividends. Profit maximization implies capital is used up to the point where the marginal product of capital equals the rental rate p_t, and labour is hired up to where the marginal product of labour equals the wage w_t:

$$\alpha Z_t \left(\frac{L_{ft}}{K_{ft}}\right)^{1-\alpha} = p_t, \quad \text{and} \quad (1-\alpha) Z_t \left(\frac{K_{ft}}{L_{ft}}\right)^{\alpha} = w_t. \tag{17}$$

With constant returns to scale, profits are equal to zero ($\Pi_{ft} = 0$) in equilibrium. The ex-post return received by owners of physical capital between $t-1$ and t is

$$R_t = X_t(1 - \delta + p_t) - 1.$$

3.2 Households

At the beginning of period t, household $h \in [0,1]$ has expected lifetime utility

$$\mathbb{E}_{ht}\left[\sum_{s=t}^{\infty}\beta^{s-t}\left\{\frac{C_{hs}^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}-\chi\frac{L_{hs}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right\}\right], \tag{18}$$

where C_{ht} is consumption and L_{ht} is labour supply, β is the subjective discount factor, χ is a parameter representing the disutility of labour, and σ and ψ are preference parameters that will be the elasticity of intertemporal substitution and Frisch elasticity of labour supply, respectively. All households have the same preferences, and start from equal wealth in an initial period 0. The only heterogeneity among households is in their signals about the fragility of banks.

The information set for the conditional expectation $\mathbb{E}_{ht}[\cdot]$ in (18) contains commonly known aggregate shocks, prices, and macroeconomic variables from date t and earlier. It also contains a sequence of household-specific signals about the fragility of each bank at date t and earlier. As analysed in section 2, each household uses its arbitrarily precise signals to form beliefs about the fragility of banks.[28]

Because banks comply with no-run condition (13) in all states of the world, by Corollary 1 all households have signals that make them believe correctly that no runs ever take place.[29] This implies that the heterogeneity in households' information sets is irrelevant for choices made at the competitive-markets stage, hence the household h subscript can be dropped from conditional expectations, replacing them by $\mathbb{E}_t[\cdot]$, which is conditional on just macroeconomic variables. Given that bank runs do not happen in equilibrium and no household attaches positive probability to them happening, this section analyses household behaviour abstracting from bank runs.[30]

Competitive-markets stage. Household h chooses labour supply L_{ht} and receives wage income $w_t L_{ht}$, and everyone pays a common lump-sum net tax T_t. Each household also receives a non-negative dividend $\mathit{\Pi}_t$ from owning an equal share of a non-tradable investment fund comprising all banks and non-financial firms,[31] and has a equal claim on the total bonus pool G_t across all banks. Bonuses are obtained through diversion of bank net worth, though these will be zero because (15) holds in equilibrium.[32]

[28]Households are restricted from using other information to inform their beliefs about fragility. As discussed in section 2, the use of public information by households could restore common knowledge and thus multiple equilibria.

[29]Bank net worth is non-negative in all states of the world on the equilibrium path as shown in section 4.

[30]For completeness, appendix D describes how actions taken in case of a bank run are integrated with the macroeconomic model.

[31]In equilibrium, there are no gains from trading shares in the investment fund among households.

[32]The total bonus pool is $G_t = \frac{1}{1+\gamma}\int_0^1 \max\{0, \gamma N_{bt} - \mathit{\Pi}_{bt}\}\,\mathrm{d}b$.

Household h may choose to borrow between periods t and $t+1$ an amount B_{ht} (or if negative, hold savings outside banks) in the form of a risk-free but illiquid bond with interest rate ρ_t.[33] Any past borrowing $(1+\rho_{t-1})B_{h,t-1}$ must be repaid, and a no-Ponzi condition must be respected.[34] Households may also hold non-bank liquid assets $M_{ht} \geq 0$ paying risk-free interest rate i_t.

Consumption. At the start of period t, households begin with deposits $(1+j_{b,t-1})D_{b,t-1}$ including accrued interest at bank b. The bank chooses net deposit creation $D_{bt} - (1 + j_{b,t-1})D_{b,t-1}$, which allow it to make purchases of physical capital and liquid assets. It is implicitly assumed bank deposits are accepted by firms and households as a means of payment and circulate at the competitive-markets stage.[35] Since non-financial firms are entirely static, paying out all sales revenue immediately as factor payments, any deposits must be in the hands of households once the competitive-markets stage is over. As banks treat all households symmetrically when deposits are created and since all ex-ante identical households will behave the same way at the competitive-markets stage, each household carries the same amount of deposits D_{bt} at bank b into the coordination game.

With banks' choices satisfying the no-run condition, households choose to hold the deposits in existence at each bank. Together with households' choices at the competitive-markets stage, the consumption enjoyed at the final stage of period t is determined by the following flow budget constraint conditional on everyone holding deposits:[36]

$$C_{ht} = w_t L_{ht} + \Pi_t + G_t - T_t + \int_0^1 \{(1+j_{b,t-1})D_{b,t-1} - D_{bt}\} \mathrm{d}b \\ - (1+\rho_{t-1})B_{h,t-1} + B_{ht} + (1+i_{t-1})M_{h,t-1} - M_{ht}. \quad (19)$$

Households directly holding physical capital is ruled out by assumption, so capital is excluded from (19). First-order conditions for maximizing utility with respect to B_{ht} and L_{ht} then determine optimal choices of consumption and labour supply.[37] Households can also choose to hold liquid assets $M_{ht} \geq 0$ directly. However, since $i_t \leq \rho_t$ must hold in equilibrium, utility is maximized by choosing $M_{ht} = 0$.[38]

[33] Illiquid in that no value from this asset can be realized until the $t+1$ competitive-markets stage.

[34] The no-Ponzi condition is $\lim_{s\to\infty} \frac{B_{hs}}{(1+\rho_t)\cdots(1+\rho_{s-1})} \leq 0$ in all states of the world.

[35] The medium-of-exchange role of deposits is not explicitly modelled here. Deposits would be accepted in exchange for goods if agents believe no bank failures will occur, as is true in equilibrium.

[36] At the final stage of period t, agents can also trade liquid assets for consumption goods, and the government can levy an additional lump-sum tax. However, in the absence of bank runs, households will not hold liquid assets at that stage and the government has no need to adjust taxes, so these possibilities can be ignored. For a full description of the final stage when runs occur, see appendix D.

[37] There is also a transversality condition $\lim_{s\to\infty} \beta^{s-t} C_{hs}^{-\frac{1}{\sigma}} (\int_0^1 D_{bs} \mathrm{d}b - B_{hs} + M_{hs}) \leq 0$ holding in all states.

[38] This can be interpreted as households choosing to deposit in banks any outside money obtained from fiscal transfers, and selling any liquid financial assets to banks. Note that if $i_t > \rho_t$, there would be

With no heterogeneity in wealth or preferences, and no relevant heterogeneity in their information set, all households make the identical consumption C_t and labour supply L_t choices satisfying:

$$C_t^{-\frac{1}{\sigma}} = \beta(1+\rho_t)\mathbb{E}_t\left[C_{t+1}^{-\frac{1}{\sigma}}\right], \quad \text{and} \quad \chi L_t^{\frac{1}{\psi}} = w_t C_t^{-\frac{1}{\sigma}}. \tag{20}$$

At the macroeconomic level, there is effectively a representative household.

Households' equilibrium strategies in the coordination game of section 2 maximized expected future payoffs discounted at a common rate. Since there is a continuum of banks $b \in [0,1]$, this is consistent with the concave utility function (18) because deposit-holding decisions and bank survival outcomes at any individual bank have only a negligible effect on any household's overall consumption. Household h can therefore act as if risk neutral in respect of deposits at a particular bank, discounting payoffs expected in period $t+1$ using discount factor $\beta\mathbb{E}_{ht}\left[(C_{h,t+1}/C_{ht})^{-\frac{1}{\sigma}}\right]$. Using the illiquid bond first-order condition in (20), this discount factor is equal to $1/(1+\rho_t)$, which is common to all households. The yield ρ_t on the illiquid bond is therefore the appropriate discount rate to apply to expected future payoffs in the coordination game.

3.3 Government

The government issues liabilities M_t that are liquid assets in that they can be exchanged one-for-one with consumption goods at any stage of time period t.[39] These liabilities are broadly interpretable as government bonds, reserves, or outside money more generally, though the model has a single type of liquid government liability for simplicity. This liquid asset offers a risk-free return i_t between periods t and $t+1$.

The government is able to levy lump-sum taxes on all households or make transfers. At the competitive-markets stage of period t, the net lump-sum is T_t.[40] The government can also choose to purchase illiquid bonds B_t (or if negative, issue illiquid bonds). Changes in fiscal and monetary policy are represented through different combinations of M_t, B_t, and T_t.[41] Consolidating across all branches of government, the flow budget constraint necessary to deliver a risk-free return of i_{t-1} on M_{t-1} is

$$T_t = (1+i_{t-1})M_{t-1} - (1+\rho_{t-1})B_{t-1} - M_t + B_t. \tag{21}$$

an unlimited demand for liquid assets, so this case cannot occur in equilibrium.

[39] The liquidity of government bonds ultimately derives from the government's ability to adjust the supply of liquid assets and taxes after the coordination game as described in appendix D.

[40] Assuming no runs occur, date-t tax revenue can be collected at the first stage of that time period without loss of generality.

[41] Purchases of $B_t > 0$ financed by issuing M_t can be interpreted as a form of unconventional monetary policy. However, the government never buys physical capital here, so it does not ever directly take on the financial intermediation role performed by banks.

It is necessary to add at least one further equation to describe how the government determines the supply of liquidity M_t.

3.4 Banks

Investment fund. Each bank $b \in [0,1]$ is owned by an investment fund that is itself owned equally by all households. Dividends Π_{bt} from banks are aggregated and passed on to households as Π_t along with any dividends Π_{ft} from non-financial firms:

$$\Pi_t = \int_0^1 \Pi_{bt}\,\mathrm{d}b + \int_0^1 \Pi_{ft}\,\mathrm{d}f. \tag{22}$$

Assume that any bank b that has negative net worth and is unable to distribute positive dividends must be recapitalized by the investment fund. A resource cost $\varkappa$ is incurred per unit of recapitalization funds provided. The funds are raised by requiring other banks owned by the fund to pay higher dividends.[42] As seen later, in the presence of a recapitalization cost $\xi > 0$, banks acting in the interests of their owners avoid insolvency with probability one.

Objective. Banks act in the interests of their owners, and as there is effectively a representative household in equilibrium, the objective function of bank b is $\Pi_{bt} + V_{bt}$, where V_{bt} is the present value of future dividends (the ex-dividend value of bank b) obtained using a stochastic discount factor P_{ts} given by households' common marginal rate of substitution between consumption at date t and (state-contingent) consumption at date $s > t$:

$$V_{bt} = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty} P_{ts}\Pi_{bs}\right], \quad \text{where } P_{ts} = \beta^{s-t}\left(\frac{C_s}{C_t}\right)^{-\frac{1}{\sigma}}. \tag{23}$$

At each date t, each bank makes an interest rate j_{bt} and deposit creation decision that results in a stock of deposits D_{bt}. In addition, the bank chooses the amount of physical capital A_{bt} and liquid assets M_{bt} to hold, and the dividend Π_{bt} to distribute.[43] Each bank is competitive in goods and asset markets and hence takes prices (p_t, R_t, i_t), and the stochastic discount factor $P_{t,s}$ as given. There is no competitive market for deposits, so banks can choose the quantity supplied D_{bt} and the interest rate j_{bt}, but they know that households will decide whether to hold or not during the coordination game.

Constraints. If bank b avoids failure prior to date t, its net worth (bank equity or capital) N_{bt} after paying dividend Π_{bt} depends as follows on past decisions and the

[42] A failure of the whole banking system where recapitalization through this means is impossible will not occur on the equilibrium path.

[43] Banks do not want to hold illiquid bonds, and by assumption, cannot fund themselves by issuing illiquid bonds.

realized return on physical capital R_t:

$$N_{bt} = (1+R_t)A_{b,t-1} + (1+i_{t-1})M_{b,t-1} - (1+j_{b,t-1})D_{b,t-1} - \Pi_{bt}. \tag{24}$$

This assumes no diversion of funds to employee bonuses, which requires the minimum-dividend condition (15) to hold. Given net worth N_{bt}, the balance-sheet identity is

$$A_{bt} + M_{bt} = D_{bt} + N_{bt}. \tag{25}$$

Since failure wipes out a bank's ability to pay dividends, banks want to ensure the no-run condition (13) derived in section 2 holds.[44] The present value of dividends (23) is maximized subject to (13), (15), (24), and (25).[45] Since (13) specifies a minimum value for j_{bt} and net worth $N_{b,t+1}$ is decreasing in j_{bt}, the no-run condition must bind:

$$j_{bt} = \rho_t + \max\left\{ \frac{1}{\lambda + \frac{\lambda N_{bt} + (1-\lambda)M_{bt}}{D_{bt}}} - 1, 0 \right\} \theta. \tag{26}$$

3.5 Aggregation and market clearing

Equilibrium in factor markets requires that firms rent the physical capital owned by banks and hire the labour supplied by households:

$$\int_0^1 K_{ft}\,\mathrm{d}f = \int_0^1 K_{bt}\,\mathrm{d}b = K_t = X_t A_{t-1}, \quad \text{and} \quad \int_0^1 L_{ft}\,\mathrm{d}f = \int_0^1 L_{ht}\,\mathrm{d}h = L_t, \tag{27}$$

where the supply of capital $K_t = X_t A_{t-1}$ depends on banks' past total holdings of physical capital $A_{t-1} = \int_0^1 A_{b,t-1}\mathrm{d}b$ adjusted for the capital quality shock X_t. Aggregating (16) and (17) across firms:

$$Y_t = Z_t K_t^{\alpha} L_t^{\alpha}, \quad w_t = (1-\alpha)\frac{Y_t}{L_t}, \quad \text{and} \quad R_t = X_t\left(1 - \delta + \alpha\frac{Y_t}{K_t}\right). \tag{28}$$

Equilibrium in financial markets requires banks' demand for liquid assets equals the amount supplied by the government, and households' supply of illiquid bonds equals the amount purchased by the government:

$$\int_0^1 M_{bt}\,\mathrm{d}b = M_t, \quad \text{and} \quad B_t = \int_0^1 B_{ht}\,\mathrm{d}h, \tag{29}$$

[44] For completeness, the actions taken by bank b in case of a run are described in appendix D.

[45] The transversality condition $\lim_{s\to\infty} P_{ts}\Pi_{bs} = 0$ in all states of the world is necessary for a maximum. Given the minimum-dividend constraint (15), this implies the restriction $\lim_{s\to\infty} P_{ts}N_{bs} = 0$ on net worth.

recalling that household choose not to hold liquid bonds and banks choose not to hold illiquid bonds. There is not a competitive market deposits, but households hold the amount supplied by banks given the no-run condition (26), as has been assumed in the budget constraint (19).[46] Combining household, firm, bank, investment fund, and government budget constraints implies market clearing for final goods:

$$C_t + I_t = Y_t, \quad \text{where } I_t = A_t - (1-\delta)K_t. \tag{30}$$

4 Bank behaviour

This section analyses banks' profit-maximizing choices of asset liquidity, the creation of deposits, the supply of credit, and the distribution of dividends subject to the friction developed in the coordination game of section 2.

The full dynamic optimization problem is solved as a series of static problems in liquidity and leverage choices taking as given the path of net worth, and finally considering dividend policy to characterize the evolution of net worth. For bank b with net worth N_{bt} at date t, liquid asset demand M_{bt}, credit supply A_{bt}, and the total quantity of deposits D_{bt} created must maximize the expected discounted value of $N_{b,t+1} + \Pi_{b,t+1}$ using a stochastic discount factor Ψ_{t+1} common to all banks that is explained below. The objective function is $\Omega_{bt} = \mathbb{E}_t[\Psi_{t+1}(N_{b,t+1} + \Pi_{b,t+1})]/\mathbb{E}_t[\Psi_{t+1}]$, and using the evolution of net worth (24) and the balance-sheet identity (25) this is

$$\Omega_{bt} = (1 + r_t)N_{bt} + (r_t - j_{bt})D_{bt} - (r_t - i_t)M_{bt}, \tag{31}$$

where $r_t = \mathbb{E}_t[\Psi_{t+1}R_{t+1}]/\mathbb{E}_t[\Psi_{t+1}]$ denotes the risk-adjusted expected value of R_{t+1}. From the above, the no-run condition must bind whenever deposits are positive. In maximizing Ω_{bt} with net worth N_{bt} and market prices r_t, i_t, and ρ_t given, there are three choice variables j_{bt}, D_{bt}, and M_{bt} and one binding no-run condition (26).

Demand for liquid assets. An increase in M_{bt} given D_{bt} and N_{bt} means switching from illiquid to liquid assets while keeping the size of the balance sheet unchanged. This has a cost $r_t - i_t$, as can be seen from (31), reflecting the difference in (risk-adjusted) expected returns between the two assets. The benefit of more liquidity comes from the resulting fall in bank fragility F_{bt}, which lowers bank b's funding cost. The binding no-run constraint (26) gives the deposit interest rate j_{bt} as a function of M_{bt} and D_{bt}, and (31) implies the marginal benefit is equal to $-\partial j_{bt}/\partial M_{bt}$ multiplied by deposits D_{bt}.

[46] The no-Ponzi condition on household borrowing, the transversality condition on households' asset holdings, $M_{ht} = 0$, and the non-negativity of deposits imply $\lim_{s\to\infty} \mathbb{E}_t\left[P_{ts}\int_0^1 D_{bs}\,\mathrm{d}\,b\right] = 0$ using the formula for the stochastic discount factor in (23).

Where fragility is positive, the marginal benefit is

$$-D_{bt}\frac{\partial j_{bt}}{\partial M_{bt}} = (1-\lambda)\theta\left(\lambda + \frac{(1-\lambda)M_{bt} + \lambda N_{bt}}{D_{bt}}\right)^{-2} = \frac{(1-\lambda)\theta}{(1-F_{bt})^2}, \tag{32}$$

but if fragility is already negative then the marginal benefit is zero. The marginal benefit of an extra unit of liquidity depends only fragility because the reduction in fragility is inversely proportional to deposits, but the resulting improvement in a bank's funding cost is multiplied by the size of the deposit base.

Assuming $r_t - i_t > (1-\lambda)\theta$, for which the optimal demand for liquid assets implies positive fragility, the first-order condition for maximizing (31) with respect to M_{bt} is $r_t - i_t = -D_{bt}\partial j_{bt}/\partial M_{bt}$.[47] If $r_t - i_t = 0$ then it is optimal to hold enough liquid assets to ensure fragility is negative, while if $0 < r_t - i_t \leq (1-\lambda)\theta$, the bank targets exactly zero fragility when choosing liquid assets. Hence, together with (2), bank b's demand for liquid assets is

$$M_{bt}\begin{cases} = \frac{1}{1-\lambda}\left(\sqrt{\frac{(1-\lambda)\theta}{r_t-i_t}} - \lambda\right)D_{bt} - \frac{\lambda}{1-\lambda}N_{bt} & \text{if } r_t - i_t > (1-\lambda)\theta \\ = D_{bt} - \frac{\lambda}{1-\lambda}N_{bt} & \text{if } 0 < r_t - i_t \leq (1-\lambda)\theta \cdot \\ \geq D_{bt} - \frac{\lambda}{1-\lambda}N_{bt} & \text{if } r_t - i_t = 0 \end{cases} \tag{33}$$

Demand for liquidity is decreasing in the cost $r_t - i_t$ of holding liquid assets, increasing in deposits D_{bt} because more leverage increases fragility, and decreasing in net worth N_{bt} because that is a substitute for liquidity in reducing fragility.[48]

As banks all face the same cost $r_t - i_t$ and the marginal benefit of a unit of liquidity depends only on a bank's fragility F_{bt} and parameters, banks trade liquidity in money markets up to the point where fragility is equalized across banks. This is analogous to the demand for reserves in the Poole model arising from payments risk. With $F_{bt} = F_t$ for all b, the level of systemic bank fragility F_t is derived from (2) and the balance-sheet identity (25):

$$F_t = 1 - \lambda - \frac{(1-\lambda)M_t + \lambda N_t}{D_t}, \tag{34}$$

where N_t, M_t, and D_t are the aggregate amounts of equity, liquid assets, and deposits in the banking system. An immediate consequence of $F_{bt} = F_t$ is that all banks face the same minimum funding cost $j_t = j_{bt}$ consistent with a binding no-run constraint. Using

[47] The second-order condition is satisfied because (32) shows $-D_{bt}\partial j_{bt}/\partial M_{bt}$ is decreasing in M_{bt}.

[48] The non-negativity constraint $M_{bt} \geq 0$ means it is necessary to check for corner solutions. However, cases where there is a corner solution for some banks but not others can be ruled out because a binding non-negativity constraint reduces the maximum attainable Ω_{bt}, but it will be seen that banks are indifferent about the size of D_{bt}, and the non-negativity constraint is slack for sufficiently large D_{bt}. Furthermore, a positive aggregate supply of liquidity means that there cannot be a corner equilibrium for all banks.

(12), this interest rate satisfies $j_t - \rho_t + \theta = \theta/(1 - F_t)$ for non-negative fragility F_t, and by combining with (32) shows that the marginal benefit of liquidity common to all banks after optimal trades is positively related to the funding cost j_t. Equating this to the cost of liquidity shows that

$$r_t - i_t = \frac{(1-\lambda)\theta}{(1-F_t)^2} = \frac{(1-\lambda)}{\theta}(j_t - \rho_t + \theta)^2, \tag{35}$$

which holds whenever systemic fragility F_t is positive.

Deposit creation. An increase in deposits D_{bt} given M_{bt} and N_{bt} means greater leverage, with bank b increasing the size of its balance sheet (25). Since banks trade liquid assets so as to equalize fragility and funding costs for any given level of deposits D_{bt}, the objective function (31) can be written in terms of the common funding cost j_t and systemic fragility F_t using (34)

$$\Omega_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)N_{bt} + \left(\frac{r_t - i_t}{1-\lambda}F_t + i_t - j_t\right)D_{bt}. \tag{36}$$

Intuitively, if deposits increase by one unit, but fragility remains unchanged at F_t after adjusting liquid assets, (2) implies the composition of the rise in total assets is A_{bt} increasing by $F_t/(1-\lambda)$ and M_{bt} increasing by $1 - F_t/(1-\lambda)$. This delivers an additional payoff of $(r_t - i_t)F_t/(1-\lambda) + i_t$ for the bank at the cost of paying extra interest j_t, but with no further effect on overall funding costs, hence the coefficient of D_{bt} above.

The objective function (36) is linear in deposits D_{bt} with a coefficient that is common to all banks. If the coefficient is positive, there is no limit to banks' desire to create deposits, while if negative, no deposit creation occurs, hence equilibrium with a positive but finite supply of deposits requires the coefficient on D_{bt} is zero.[49] If fragility is zero, this immediately implies that $i_t = j_t = \rho_t$ must hold. With $F_t > 0$, a rearrangement of the coefficient of D_{bt} shows that it is zero when $(r_t - \lambda i_t)/(1-\lambda) = j_t + (r_t - i_t)(1-F_t)/(1-\lambda)$, which in combination with the implication (35) of liquidity demand, is equivalent to:

$$\frac{r_t - \lambda i_t}{1-\lambda} = j_t + (j_t - \rho_t + \theta). \tag{37}$$

The two terms on the right-hand side are respectively the direct cost of funding the additional deposit and the cost of holding the additional liquid assets to avoid raising fragility, which is positively related to the level of banks' funding costs. Intuitively, the aggregate supply of deposits adjusts until the deposit rate j_t satisfies the equation above.[50] Taking as given r_t, i_t, and ρ_t, higher aggregate deposits increase systemic

[49] Deposits are zero in equilibrium only if $r_t \leq \rho_t$. This is because fragility must be negative if $D_t = 0$, hence $r_t = i_t$ and $j_t = \rho_t$, so the coefficient of D_{bt} is $r_t - \rho_t$.

[50] Note that the exact distribution of deposits D_{bt} across banks is not uniquely determined, only the

fragility (34) and push up banks' funding cost j_t, raising the right-hand side of (37) until the equation holds. The explicit implications of this for the supply of bank credit are derived further below.

The liquidity premium and the aggregate demand for liquidity. The zero coefficient on D_{bt} from (36) in equilibrium implies $j_t - i_t = (r_t - i_t)F_t/(1-\lambda)$. This says that the spread between banks' funding cost j_t and the interest rate on government bonds i_t (sometimes referred to as the 'TED' spread $j_t - i_t$), is positively related to fragility F_t and the difference in return $r_t - i_t$ between illiquid and liquid assets, intuitively because banks' incentive to create deposits rises with $r_t - i_t$. Banks face high funding spreads $j_t - \rho_t$ when fragility is high, and with $r_t - i_t$ being the marginal cost of shifting from illiquid to liquid assets, equal to the marginal benefit of liquidity that rises with the funding spread, it follows that the TED spread $j_t - i_t$ must rise with $j_t - \rho_t$. Mathematically, from (35), the TED spread is $j_t - i_t = (j_t - \rho_t)(\theta + j_t - \rho_t)/\theta$.

Using the interest rate ρ_t on a risk-free but illiquid bond, the TED spread $j_t - i_t$ can be decomposed into the sum of a pure funding spread $j_t - \rho_t$ and the liquidity premium $\rho_t - i_t$. This definition of the liquidity premium is analogous to the empirical GC repo-Tbill spread in that both ρ_t and i_t are risk-free yields, but with government bonds having the advantage of liquidity. Taking the formula linking $j_t - i_t$ to $j_t - \rho_t$ and rearranging to obtain $j_t - \rho_t$, the funding spread is a geometric average of the liquidity premium $\rho_t - i_t$ and the parameter θ measuring the loss given default for depositors:

$$j_t - \rho_t = \sqrt{\theta}\sqrt{\rho_t - i_t}. \tag{38}$$

This equation implies a positive relationship between the funding spread and the liquidity premium consistent with the empirical evidence presented in Figure 2. Intuitively, a high liquidity premium reflects a high demand for liquid assets, which is associated with high bank fragility and funding spreads.

The liquidity premium $\rho_t - i_t$ is also connected to the difference between the risk-adjusted returns r_t and i_t on banks' assets A_t and M_t for which there a difference in liquidity. Combining equations (35) and (38):

$$r_t - i_t \begin{cases} = 4(1-\lambda)\left(\frac{1}{2}\sqrt{\theta} + \frac{1}{2}\sqrt{\rho_t - i_t}\right)^2 & \text{if } F_t > 0 \\ \in [0, (1-\lambda)\theta) & \text{if } F_t = 0\,, \\ = 0 & \text{if } F_t < 0 \end{cases} \tag{39}$$

the spread $r_t - i_t$ being a multiple $4(1-\lambda)$ of a generalized mean of $\rho_t - i_t$ and θ when

aggregate amount of deposits D_t consistent with (37). With reference to (33), this ensures that the earlier non-negativity constraint $M_{bt} \geq 0$ on liquid assets can be ignored without loss of generality.

fragility is positive, the term $1-\lambda$ capturing the relative liquidity of the two assets.

Taking as given the aggregate supply of credit A_t and other macroeconomic variables, the liquidity premium and other spreads are jointly determined by banks' aggregate demand for liquid assets and the supply M_t of these assets that results from government policy. In the case $F_t > 0$, aggregating (33) and using equation (39), banks' total demand for liquid assets is[51]

$$M_t = \frac{\sqrt{\theta}\,((1-\lambda)A_t - N_t)}{\sqrt{\rho_t - i_t}} - \lambda A_t, \tag{40}$$

noting that $(1-\lambda)A_t > N_t$ if and only if fragility is positive. The aggregate demand for liquidity is decreasing in the liquidity premium $\rho_t - i_t$, with a horizontal asymptote as the $\rho_t - i_t$ approaches zero. The demand curve shifts to the right as bank holdings of illiquid assets A_t increase, and to the left if net worth N_t is higher. When $(1-\lambda)A_t \leq N_t$, which means fragility is non-positive, the liquidity premium must be zero, but (33) is consistent with banks holding any level of liquid assets, so the demand curve for M_t is horizontal at $\rho_t - i_t = 0$.

The supply curve for M_t is determined by government policy. This can be inelastic to interest-rate spreads such as $\rho_t - i_t$, $j_t - i_t$, and $r_t - i_t$, or could involve some elastic response of M_t. Since spreads move together with the liquidity premium $\rho_t - i_t$, the supply curve can be represented as a relationship between M_t and $\rho_t - i_t$ without loss of generality.

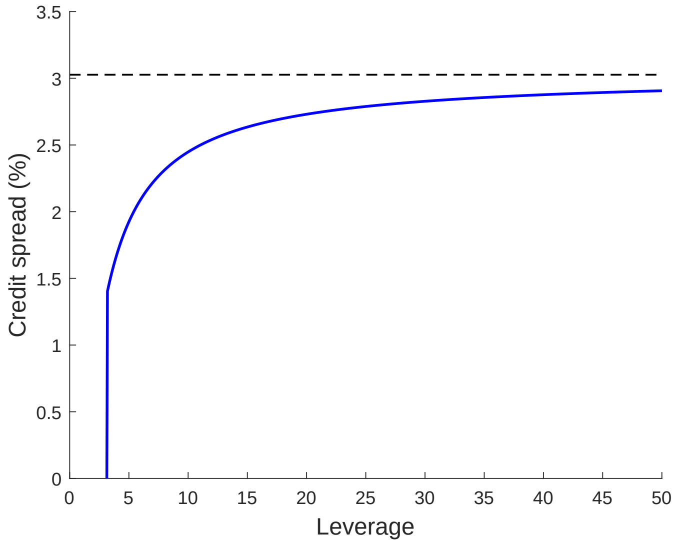
The credit supply curve. While individual banks can choose holdings of liquid assets, in equilibrium, the banking system must hold the amount of liquidity M_t supplied by the government. Hence, banks' optimal deposit supply can be seen as determining the supply of credit A_t taking as given M_t and the aggregate net worth N_t of banks. From (33), the credit supply curve is

$$A_t = \begin{cases} \frac{\left(\sqrt{r_t - i_t} - \sqrt{(1-\lambda)\theta}\right)M_t + \sqrt{(1-\lambda)\theta}N_t}{\sqrt{(1-\lambda)\theta} - \lambda\sqrt{r_t - i_t}} & \text{if } r_t - i_t > (1-\lambda)\theta \\ \frac{N_t}{1-\lambda} & \text{if } 0 < r_t - i_t \leq (1-\lambda)\theta \end{cases}. \tag{41}$$

A higher credit spread makes banks choose to increase their leverage, $1/n_t = (A_t + M_t)/N_t$, and thus increase their fragility according to equation (26). The credit supply curve is represented in Figure 4 for an inelastic supply of liquid assets M_t. For low levels of the credit spread, the supply of credit (i.e., investment in physical capital by banks) is inelastic at the level that ensures banks are not fragile, and they pay the risk-free rate

[51] This is derived noting that (33) holds for aggregates because the coefficients are the same for all banks, and then substituting $D_t = A_t + M_t - N_t$ and $\sqrt{\frac{r_t - i_t}{(1-\lambda)\theta}} - 1 = \sqrt{\frac{\rho_t - i_t}{\theta}}$ from the formula in (39).

Figure 4: Credit supply



Note 1: The credit spread is $r_t - i_t$ and leverage is $1/n_t$.
Note 2: Annualized calibrated parameter values from Table 2 and the implied steady-state M are used.
Note 3: The dashed line is the spread at which credit supply is infinite.

on their deposits. Above a given credit spread, banks have the incentive to lever up and become fragile. In this region, the supply of credit is elastic. Increases in the supply of liquid assets expand banks' credit supply in the fragile region. They are irrelevant when banks are not fragile.

Dividend policy and net worth. Bank behaviour has been studied taking as given net worth N_{bt}. The remaining decision to analyse is the distribution of dividends Π_{bt}.

Since the coefficient of deposits D_{bt} in (36) is zero (or if not, deposits are zero), the static objective function Ω_{bt} is linear in net worth only:

$$\Omega_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right) N_{bt}. \tag{42}$$

The evolution of net worth is affected by the return on equity and the distribution of dividends. Defining $Q_{b,t+1}$ to be the ex-post return on bank b's book equity between dates t and $t+1$:

$$Q_{b,t+1} = \frac{\Pi_{b,t+1} + (N_{b,t+1} - N_{bt})}{N_{bt}}, \quad \text{or} \quad Q_{b,t+1} = R_{t+1}\frac{A_{bt}}{N_{bt}} + i_t \frac{M_{bt}}{N_{bt}} - j_t \frac{D_{bt}}{N_{bt}}. \tag{43}$$

The risk-adjusted expected return on bank b's book equity is $q_{bt} = \mathbb{E}_t[\Psi_{t+1} Q_{b,t+1}]/\mathbb{E}_t[\Psi_{t+1}]$ evaluated using the stochastic discount factor Ψ_{t+1} introduced earlier in this section. The definition $\Omega_{bt} = \mathbb{E}_t[\Psi_{t+1}(N_{b,t+1} + \Pi_{b,t+1})]/\mathbb{E}_t[\Psi_{t+1}]$ of the static objective function and (43) imply that $q_{bt} = (\Omega_{bt} - N_{bt})/N_{bt}$, so the analysis of bank behaviour up to this point can be understood in terms of maximizing the risk-adjusted expected return on book equity conditional on initial net worth N_{bt}. Equation (42) shows this maximized expected return is the same for all banks, $q_t = q_{bt}$, and is equal to the following:

$$q_t = \frac{r_t - \lambda i_t}{1 - \lambda}. \tag{44}$$

Bank b's actual objective function in choosing the path of dividends and other variables is the present value of current and future dividends discounted using the representative-household stochastic discount factor P_{ts}. This means maximizing $\Pi_{bt} + V_{bt}$, where V_{bt} is the present value of future dividends from (23). The solution of this problem is derived fully in Appendix E, with just the key results presented here.

Optimization by banks implies the present value of future dividends V_{bt} is proportional to net worth N_{bt}, with the market-to-book ratio $v_t = V_{bt}/N_{bt}$ being common to all banks. The profit-maximizing choices of bank b's portfolio of assets and deposit creation also maximize the static objective function Ω_{bt} from (31) defined in terms of a stochastic discount factor Ψ_{t+1}, hence the earlier analysis of bank behaviour correctly characterizes the solution to the full dynamic optimization problem. The appropriate stochastic discount factor Ψ_{t+1} modifies the representative-household stochastic discount factor $P_{t,t+1}$ between dates t and $t+1$ depending on the future market-to-book ratio v_{t+1} of banks. An expression for Ψ_{t+1} and the expectational difference equation satisfied by the market-to-book ratio v_t are

$$\Psi_{t+1} = \left(1 + \frac{v_{t+1} - 1}{1 + \gamma}\right) P_{t,t+1}, \quad \text{and} \;\; v_t = \left(\frac{1 + \frac{r_t - \lambda i_t}{1 - \lambda}}{1 + \rho_t}\right)\left(1 + \frac{\mathbb{E}_t\left[P_{t,t+1}(v_{t+1} - 1)\right]}{(1 + \gamma)\mathbb{E}_t[P_{t,t+1}]}\right). \tag{45}$$

A key result is that the market-to-book ratio v_t is never lower than 1, and $v_t > 1$ implies the minimum dividend constraint (15) is binding. If this constraint is binding at date $t+1$, the evolution of net worth is

$$N_{b,t+1} = \left(\frac{1 + Q_{b,t+1}}{1 + \gamma}\right) N_{bt}, \tag{46}$$

where Q_{bt} is the ex-post return on book equity from (43).

The minimum dividend constraint will bind when the parameter γ is sufficiently large. There is a range of γ values for which net worth converges to a positive steady state in the absence of shocks, which is assumed in the remainder of the paper.[52] Starting from that steady state, the minimum dividend constraint will always be binding for some bounds on the size of aggregate shocks.

[52] This range is analysed in Appendix F.

5 Quantitative analysis

This section quantifies the importance of bank fragility in the transmission of economic shocks. The model is simulated using a log linearization around its non-stochastic steady state. This steady state itself is analysed in appendix F, and the log-linearized equations are given in appendix G.

5.1 Calibration

The banking sector of the economy is described by the three parameters λ, θ, and γ. These are calibrated using information on the average values of the liquidity premium, credit spread, and return on bank equity. The parameter β is calibrated using the average level of interest rates. The approach is to choose parameters to match the model's implications for targeted variables in a non-stochastic steady state to the average values observed. The policy-determined supply of liquid assets in steady state consistent with liquidity premium can be inferred from the average capitalization ratio of banks. Finally, the other macroeconomic parameters α, δ, σ, and ψ are set to conventional values following the literature.

The model is calibrated to U.S. economy using data from 1991 up to the 2007–8 financial crisis. Data availability for banking variables determines the start of the sample in 1991Q3, and stopping in 2008Q4 accounts for the substantially different provision of liquidity after 2008 resulting from the many policy responses to the crisis.

The liquidity premium is defined with reference to the 3-month Treasury Bill as the most liquid asset. The average T-Bill yield over the period 1991Q3–2008Q4 is 3.7% in nominal terms. In the model, all interest rates are real interest rates, so the average 2.2% rate of inflation according to the personal consumption expenditure (PCE) over the same period is subtracted, leaving a real yield of 1.5%. The macroeconomic model is formulated in discrete time, and it is natural to align the length of one period with the 3-month maturity of the T-Bill. The steady-state quarterly real interest rate on the liquid asset is i, so $i = 1.5\%/4$, where a variable without a time subscript denotes its non-stochastic steady-state value. The liquidity premium as measured by the 3-month GC repo rate minus the T-Bill yield is 28 basis points on average, therefore $\rho = i + 0.28\%/4$.

The credit spread $r - i$ for illiquid bank assets is proxied by the yield on Moody's seasoned Baa-rated corporate bonds over 10-year Treasuries, which is 2.2% annual, hence $r = i + 2.2\%/4$. The return on bank equity q is measured by the average ratio of cash dividends to equity for commercial banks covered by the FDIC, which is 8.4% at an annual rate, giving $q = 8.4\%/4$. In the model, the real return on bank equity coincides with the dividend-net worth ratio.[53]

[53]The nominal return on book equity for FDIC banks is 11.6%, implying an annual real return of 9.4%, which is close to the dividend-equity ratio.

Since $r = (1-\lambda)q + \lambda i$ from (44), the parameter λ measuring the liquidity of bank assets is calibrated as $\lambda = (q-r)/(q-i)$. As the formula shows, a low value of λ arises if r is large relative to i, because the illiquidity of assets makes it challenging for the bank to lend without increasing fragility. The calibration targets imply $\lambda = 0.681$.

The parameter θ measuring the costs of bank failure for depositors can be calibrated with information on ρ, i, and q. Using equations (39) and (44), $q - i = (\sqrt{\theta} + \sqrt{\rho - i})^2$, so θ can be set as $\theta = (\sqrt{q-i} - \sqrt{\rho - i})^2$. High values of θ arise when the return on bank equity q is far above the risk-free interest rate ρ because a more severe credit friction between banks and their depositors increases spreads. The value resulting from the calibration targets is $\theta = 4.4\%/4$.

In a steady state where the return on bank equity exceeds the risk-free rate, the return on equity q is equal to the minimum fraction γ of equity distributed as dividends. This immediately implies $\gamma = 8.4\%/4$. Households' Euler equation from (20) in steady state implies the discount factor β satisfies $\beta = 1/(1+\rho)$. With $\rho = 1.78\%/4$, the implied value of the discount factor is $\beta = 0.996$. In summary, the calibration makes use of the following equations linking the model parameters to the targets:

$$\lambda = \frac{r-i}{q-i}, \quad \theta = \left(\sqrt{q-i} - \sqrt{\rho-i}\right)^2, \quad \gamma = q, \quad \text{and} \quad \beta = \frac{1}{1+\rho}.$$

Information on all the calibration targets is collected in Table 1, and the implied banking parameters are shown in Table 2.

The observed liquidity premium as the price of liquidity effectively pins down, along with the other spreads, the quantity of liquidity supplied in the steady state by the government. Using equation (34) for bank fragility, the steady-state liquidity ratio is

$$m = 1 - \left(\frac{q-i}{r-i}\right)\left(n + (1-n)\sqrt{\frac{\rho-i}{q-i}}\right),$$

where n is the steady-state capital ratio of banks. Using data on total equity capital and total assets from the Federal Deposit Insurance Corporation, the average bank capital ratio is 8.8% from 1991Q3 to 2008Q4. This and the other calibration targets implies $m = 0.148$.

The parameters describing the macroeconomic features of the model are set following the literature. The elasticity of intertemporal substitution σ is 1 and the Frisch elasticity of labour supply ψ is 3. The capital elasticity of output α is set to 1/3 to match the capital share of national income. The depreciation parameter δ is chosen to give a 7.5% annualized depreciation rate.

Table 1: Targets used to calibrate the parameters of the model

Description	Notation	Value
Liquidity premium	$\rho - i$	0.28%/4
Credit spread	$r - i$	2.2%/4
Real return on bank equity	q	8.4%/4
Real Treasury Bill rate	i	1.5%/4
Bank capital ratio	n	8.8%

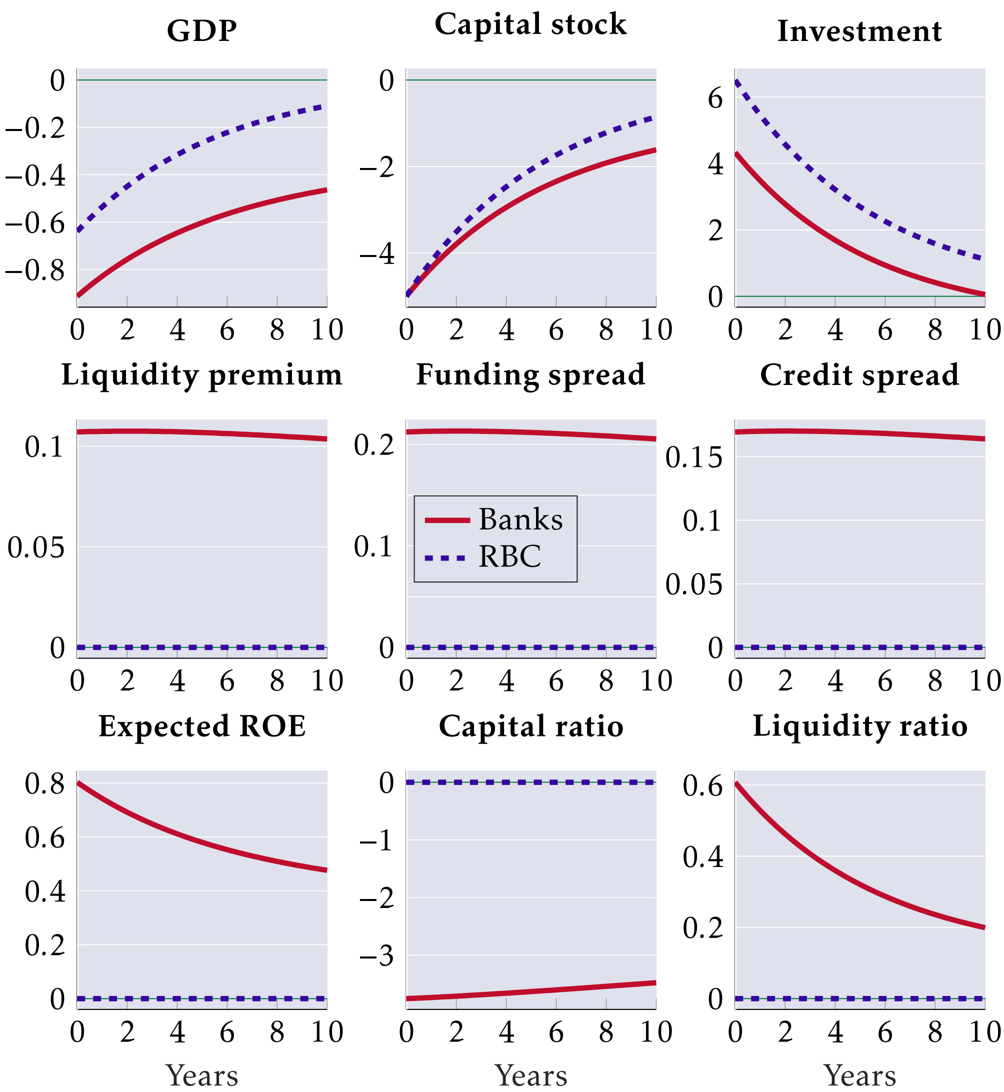
Table 2: Calibrated parameters of the model

Description	Notation	Value
Bank-asset liquidity relative to T-bills	λ	0.681
Loss given bank default	θ	4.4%/4
Minimum dividend distribution	γ	8.4%/4
Subjective discount factor	β	0.996
Elasticity of intertemporal substitution	σ	1
Frisch elasticity of labour supply	ψ	3
Capital elasticity of output	α	1/3
Depreciation	δ	7.5%/4
Steady-state liquidity ratio	m	0.148

5.2 Results

Capital destruction shock. We simulate the model to show the effects of a one-off capital destruction shock, that is, an unexpected negative shock to X_t (see 27; formally, $X_t = 1 + v_t$, where v_t is a zero-mean i.i.d. shock). The impulse response functions are shown in Figure 5 alongside those for an RBC model with the same macroeconomic features but no banking sector. In the RBC model, households can directly invest in capital. To make the models comparable, the RBC model includes an exogenous but time-invariant spread between the return on capital and the marginal rate of substitution. Variables such as interest rates, spreads, and ratios are percentage point deviations from steady state (annualized for interest rates and spreads), with 1 meaning 1 percentage point. All other variables are percentage deviations from steady state, with 1 denoting 1%. To begin with, we assume policy is completely passive and the supply of liquid assets is not adjusted.

Figure 5: Impulse response functions for a capital destruction shock



Consider first the effectively frictionless responses of variables in the RBC model. The shock directly reduces the capital stock by 5%, which brings down GDP. Investment rises to equate the marginal product of capital to the interest rate (in the RBC model, all interest rates move together one-for-one).

In the model with banks, the loss of some of the assets held by banks reduces their equity, which increases fragility and causes them to demand more liquid assets, pushing up the liquidity premium $\rho_t - i_t$ by 11 basis points. Banks must offer a higher interest rate on deposits to avoid runs, and the funding spread $j_t - \rho_t$ rises by 21 basis points. The increase in funding costs reduces lending, and the credit spread $r_i - i_t$ rises by 17 basis points. This results in less investment and a slower recovery of the capital stock compared to the RBC model. Consequently, GDP is lower and returns to its steady state at a slower rate.

Liquidity premium shock. The no-run constraint implies that the quantity of liquid assets held by banks has the effect of reducing fragility. We simulate the effects of an increase in liquidity by considering an exogenous shift in policy such that there is an unexpected 15 basis points decline in the liquidity premium $l_t = \rho_t - i_t$ with a half-life of 5 years.[54] The impulse response functions are shown in Figure 6.

The reduction in fragility brought about by the shock allows banks to take on more leverage and pay a lower interest rate on their debt. The funding spread falls by 30 basis points, and the credit spread by 24 basis points. This leads to a rise in investment, which raises GDP.

Stabilizing the liquidity premium. We can also study the supply of liquid assets as a systematic response to shocks. The optimal policy response to shocks is the elastic supply of enough liquid assets M_t to keep the liquidity premium $l_t = \rho_t - i_t$ constant at its initial steady-state value l. Impulse responses to a one-off 5% capital destruction shock under such perfectly elastic policy are represented by the blue dashed line in Figure 7. The red solid line represents the case of an inelastic supply of liquid assets. The elastic policy completely stabilizes the liquidity premium and hence the bank funding spread and the credit spread. To accomplish this, the quantity of liquid assets must increase significantly and persistently. The high persistence is necessary because in the absence of spreads bank equity does not fully recover. The greater supply of liquid assets leads to banks' liquidity ratio going up by 12 percentage points, which reduces bank fragility so much that the responses of macroeconomic variables are the same as those of the RBC model seen in Figure 5.

[54]Formally, the liquidity premium l_t is assumed to follow an exogenous AR(1) process $l_t = a l_{t-1} + v_t$, where v_t is a zero-mean i.i.d. shock. The autoregressive parameter a is calibrated so the persistence matches the 5-year half-life. The supply of liquid assets M_t adjusts so that the equilibrium liquidity premium is equal to the exogenous AR(1) process.

Figure 6: Impulse response functions for an expansion of liquid assets

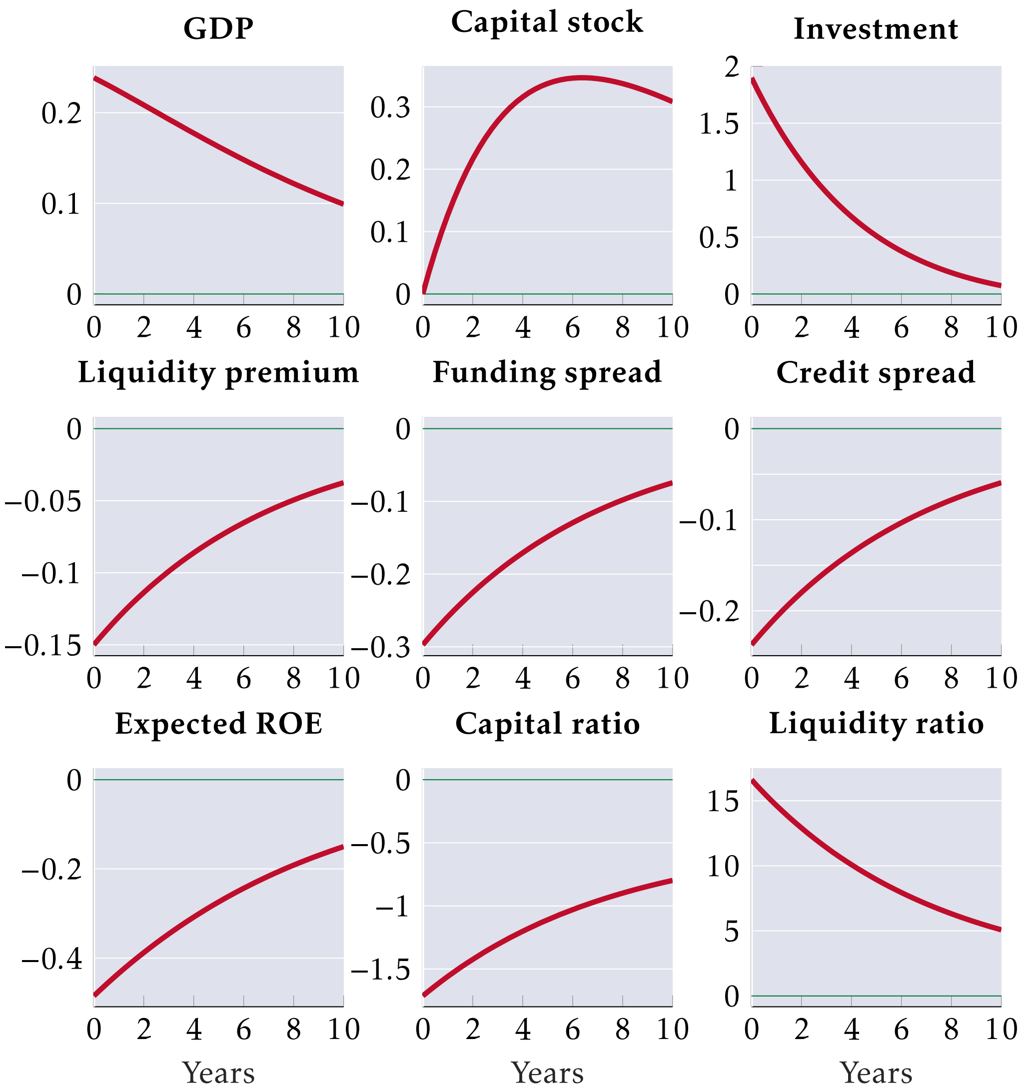
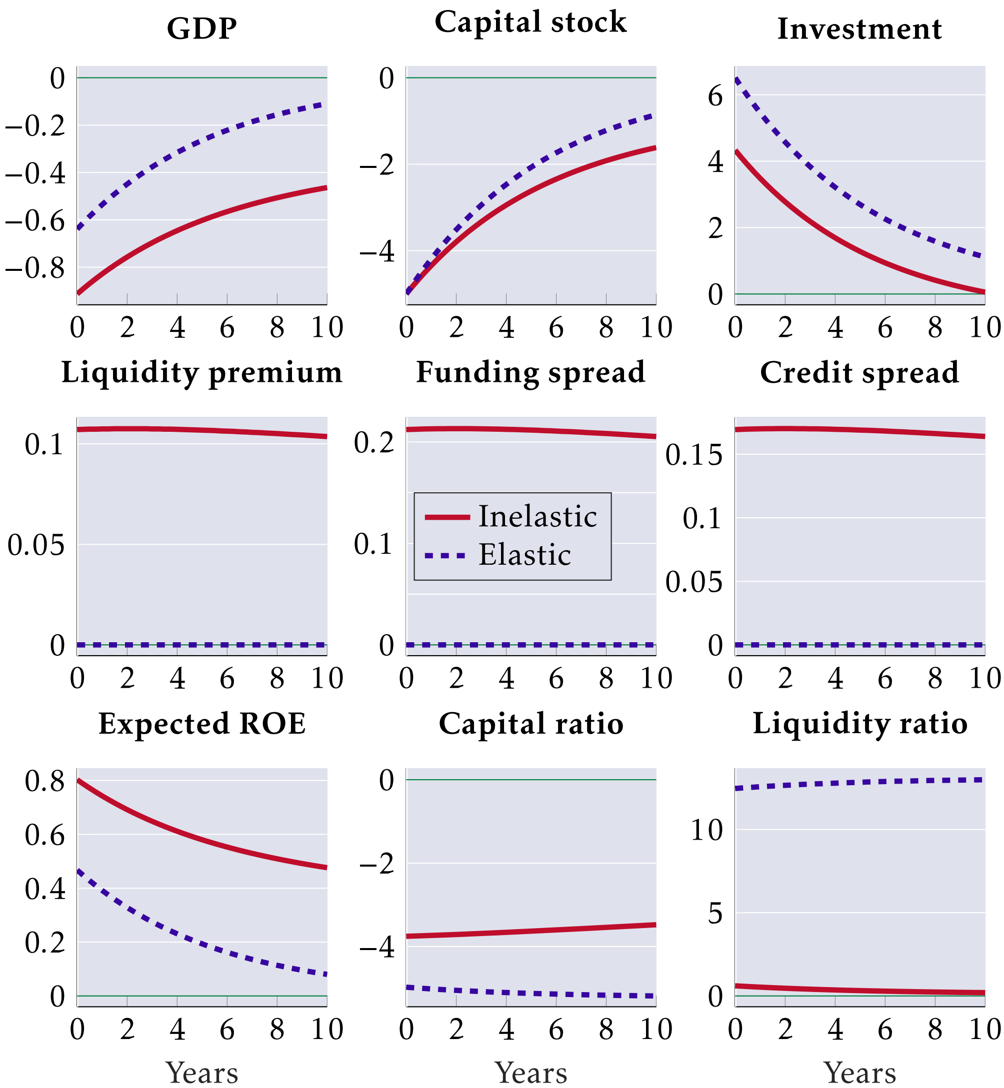


Figure 7: IRFs for capital-destruction shock by elasticity of liquidity supply



6 Liquidity policy

This section studies the supply of liquidity from a normative perspective.

First best. As a benchmark, consider a social planner assigning an equal amount of consumption C_t and labour supply L_t to each household so as to maximize expected lifetime utility (18) subject only to resource constraints. The constraints faced by the planner are an aggregate production function $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$, a constraint $C_t + I_t = Y_t$ on utilization of the economy's output, and the capital accumulation equation $K_{t+1} = X_{t+1}(I_t + (1-\delta)K_t)$. The first-order conditions for this problem are $(1-\alpha)Y_t/L_t = \chi C_t^{-\frac{1}{\sigma}} L_t^{\frac{1}{\psi}}$ and $\beta \mathbb{E}_t\left[(C_{t+1}/C_t)^{-\frac{1}{\sigma}} X_{t+1}(\alpha Y_{t+1}/K_{t+1} + 1 - \delta)\right] = 1$. Except for the final one, all of these are equilibrium conditions of the market economy with banks (see 20, 28, and 30).

To judge whether the planner's first-order for capital holds in the economy with banks, define $r_t^* = \mathbb{E}_t[P_{t,t+1}R_{t+1}]/\mathbb{E}_t[P_{t,t+1}]$ to be the risk-adjusted expected return on physical capital using the representative household's stochastic discount factor $P_{t,t+1}$ from (23). The first best is attained in the market economy if and only if $r_t^* = \rho_t$, where ρ_t is the yield on an illiquid but risk-free bond.

The liquidity premium as a capital wedge. From the credit spread formula in equation (39) assuming positive fragility, $r_t - \rho_t = (1-\lambda)\left(\sqrt{\theta} + \sqrt{l_t}\right)^2 - l_t$, where $l_t = \rho_t - i_t$ is the liquidity premium and $r_t = \mathbb{E}_t[\Psi_{t,t+1}R_{t+1}]/\mathbb{E}_t[\Psi_{t,t+1}]$ is the risk-adjusted expected return on physical capital using banks' stochastic discount factor Ψ_{t+1}. The spread between $r_t - \rho_t$ is increasing in the liquidity premium l_t. Taking a second-order approximation around a steady state with no aggregate risk, $r_t^* \approx r_t$, thus the wedge between r_t^* and ρ_t is approximately equal to $r_t - \rho_t$, which is a function of the liquidity premium. Therefore, in the economy with banks, a large liquidity premium acts as a wedge between the expected return on capital and households' discount rate. Government policy that increases the supply of liquidity and reduces the liquidity premium thus acts to move the economy closer to first best by reducing the size of the capital wedge.

Liquidity policy cannot implement the first best. While a lower liquidity premium improves efficiency, liquidity policy cannot implement a first-best allocation of resources. Note that even if the liquidity premium were zero, the wedge $r_t - \rho_t$ remains positive (assuming bank net worth is scarce, so fragility is not negative, see 39). Moreover, the shape of the aggregate demand curve for liquidity (40) shows that the liquidity premium cannot be reduced to zero with any large but finite supply of liquid assets. Therefore, the capital wedge cannot be entirely eliminated by government increasing the quantity of liquid assets banks are able to hold.

Stabilizing spreads. While the capital wedge cannot be closed with liquidity policy, the government is able to stabilize the size of the wedge by having an elastic supply of liquid assets. By adjusting M_t to target the steady-state positive liquidity premium l, banks' funding cost, the credit spread, and the capital wedge also remain at their steady-state levels (see 38 and 39). This policy generally requires permanent changes in liquidity supply following temporary shocks. The reason is that the expected return on bank equity (44) is also held at its steady-state level, so bank equity does not revert to its mean after a shock.

Substitutability between liquidity and bank capital. The policy described above is based on there being some substitutability between liquid assets and bank capital in managing bank fragility. The credit supply function (41) can be expressed equivalently as follows using (40) for a given supply of liquid assets M_t:

$$A_t = \frac{\sqrt{\rho_t - i_t} M_t + (1-\lambda)\sqrt{\theta} N_t}{(1-\lambda)\sqrt{\theta} - \lambda\sqrt{\rho_t - i_t}}.$$

If a shock causes a change in net worth N_t, the adjustment of the supply of liquidity M_t needed to maintain the same supply of credit at the same liquidity premium, and hence other spreads, is

$$\left.\frac{\partial M_t}{\partial N_t}\right|_{\rho_t - i_t, A_t} = -\frac{(1-\lambda)\sqrt{\theta}}{\sqrt{\rho_t - i_t}}. \tag{47}$$

The required size of the liquidity response to change in net worth is therefore decreasing in the liquidity premium $\rho_t - i_t$. When liquidity is abundant and the premium is low, a larger response of liquid assets is needed to stabilize spreads. This reflects a form of diminishing returns to liquid assets.[55]

Fiscal implications of liquidity policy. Iterating forwards the government's flow budget constraint (21) and using $\lim_{s\to\infty} \mathbb{E}_t[P_{ts}(M_s - B_s)] = 0$ implied by the transversality and no-Ponzi conditions, yields a present-value government budget constraint:

$$\sum_{s=t}^{\infty} \mathbb{E}_t[P_{t,s} T_s] = (1 + i_{t-1}) M_{t-1} - (1 + \rho_{t-1}) B_{t-1} - \sum_{s=t}^{\infty} \mathbb{E}_t[P_{t,s} g_s], \quad \text{where } g_t = \frac{(\rho_t - i_t) M_t}{1 + \rho_t}.$$

This states that the present value of current and future taxes T_t must be equal to initial government liabilities $(1+i_{t-1})M_{t-1}$ net of initial government assets $(1+\rho_{t-1})B_{t-1}$, minus the present-value of the fiscal gain g_t from the government's ability to supply liquid assets.[56] This fiscal gain derives from a positive liquidity premium $\rho_t - i_t$, which means

[55] Figure 11 in appendix A provides a graphical representation of equation (47).

[56] Note that Ricardian equivalence holds in respect of tax policy T_t and the government's supply or purchase of illiquid bonds B_t. Only government policies that affect M_t have an impact on the economy,

the government is able to borrow at a lower rate i_t than issuers of illiquid bonds.

Policies that reduce the liquidity premium $\rho_t - i_t$, which we have seen move the economy closer to first best, can have a fiscal cost. If the present value of g_t falls, the present value of taxes T_t must increase.[57]

The liquidity Laffer curve. Using the aggregate demand curve for liquid assets (40):

$$(\rho_t - i_t)M_t = \sqrt{\theta}\sqrt{\rho_t - i_t}((1-\lambda)A_t - N_t) - \lambda(\rho_t - i_t)A_t.$$

This shows that as M_t rises to reduce $\rho_t - i_t$ towards zero, the fiscal gain g_t to the government approaches zero. In other words, the elasticity of demand for liquidity with respect to the liquidity premium is less than unity, so eventually a large enough supply of liquidity pushes the government's total fiscal gain towards zero. This points to a cost of liquidity policies in terms of higher government borrowing costs.

Note however that more liquidity does not always mean lower fiscal gains. As M_t approaches zero, the liquidity premium $\rho_t - i_t$ rises, but only by a finite amount, which means g_t would also approache zero. Therefore, there is a 'Laffer curve' for the fiscal gains deriving from the government's supply of liquid assets.

Ex-ante versus ex-post supply of liquidity. The liquidity policies analysed so far are essentially changes in the supply of liquid assets held ex ante by banks before run risk materializes. However, it is also possible to analyse ex-post provision of liquidity within this framework.

Suppose that the government or central bank offers a discount window facility whereby banks can exchange illiquid assets for liquid assets. Assume the central bank applies a 'haircut' λ_t^* at date t, where $\lambda_t^* > \lambda$.[58] All the analysis of section 2 and 4 goes through as before with the parameter λ replaced by the policy variable λ_t^*.[59]

Along similar lines, the government could also set up a system of deposit insurance whereby those holding deposits at banks that fail now suffer a smaller loss θ_t^* than θ.[60] This can also be analysed by replacing the parameter θ by the policy variables θ_t^* in all earlier equations.

---

and whatever combination of changes in T_t or B_t are implemented to satisfy the budget constraint does not matter.

[57]While this model does not have any distortions arising from the (lump-sum) taxes T_t, more realistic representations of the tax system will entail deadweight losses from increases in the fiscal needs of the government.

[58]Implicitly, such a facility is backed by the government's tax-raising powers at the final stage of period t. See appendix D.

[59]Since banks will still want to avoid runs, the facility is not used on the equilibrium path, but its presence affects the outcome of the coordination game.

[60]Again, backed by the government's tax-raising powers. The cost θ_t^* for bank failures at date t would be paid at the beginning of period $t+1$. On the equilibrium path, the deposit insurance is not used, but its presence affects the coordination game.

With these new policy instruments, the aggregate supply of credit (41) is now

$$A_t = \frac{\sqrt{\rho_t - i_t} M_t + (1 - \lambda_t^*)\sqrt{\theta_t^*} N_t}{(1 - \lambda_t^*)\sqrt{\theta_t^*} - \lambda_t^* \sqrt{\rho_t - i_t}},$$

which is decreasing in λ_t^* and θ_t^*. Hence, an increase in credit supply, or equivalently, a lower credit spread and liquidity premium, can also be achieved by lower λ_t^* or θ_t^*, that is, more ex-post liquidity, as well as higher M_t, that is, more ex-ante liquidity.

Since the ex-post liquidity facilities are not used on the equilibrium path, there is no direct change to the earlier government budget constraint, and $g_t = (\rho_t - i_t)M_t/(1 + \rho_t)$ remains the fiscal gain the government derives from supplying liquid assets. But as all liquidity policies affect the liquidity premium $\rho_t - i_t$ in equilibrium, ex-post liquidity is not a free lunch for the government, even though it does not actually get used. Greater ex-post liquidity reduces the desire for banks to hold liquid assets, reducing the liquidity premium. By effectively raising the government's borrowing costs, ex-post liquidity policies have a fiscal cost.

Moreover, if the same reduction in $\rho_t - i_t$ is achieved through λ_t^* or θ_t^* without an increase in M_t, the reduction in g_t is larger than when higher M_t is used to reduce $\rho_t - i_t$. It follows that ex-post liquidity provision to reduce $\rho_t - i_t$ is more expensive to the government than the same change brought about through an expansion of liquidity ex ante.

7 Empirical Analysis

In this section, we empirically test the key prediction that distinguishes our model from other macroeconomic models with financial frictions. Our model predicts that liquidity is an important factor for banks' ability to fund lending. Specifically, it predicts that an increase in the liquidity premium increases banks' funding spread.

Specification. Equation (38) in the model describes the equilibrium relationship between liquidity premium and funding spread. If we linearize the equation and add an error term ϵ_t that captures possible drivers of the funding spread not considered in the model, we can write

$$\mathrm{FS}_t = \alpha + \beta \, \mathrm{LP}_t + \epsilon_t. \tag{48}$$

We allow the error term to be autocorrelated with L lags of a data vector $\boldsymbol{y}_t$, to contain time fixed effects $\boldsymbol{d}_t$ and a linear trend.[61] Thus, we can re-write the empirical

[61] The data vector also contains the funding spread and liquidity premium.

specification as

$$\mathrm{FS}_t = \alpha + \beta\,\mathrm{LP}_t + \sum_{l=1}^{L} \mathbf{y}_{t-l}^{\top} \zeta_{t-l} + \mathbf{d}_t^{\top} \boldsymbol{\eta} + \kappa t + \nu_t, \tag{49}$$

where ν_t is a stochastic innovation that is not autocorrelated but is potentially heteroskedastic. Vectors ζ_{t-l} and $\boldsymbol{\eta}$ and scalar κ contain parameters.

Data. We include in the data vector $\mathbf{y}_t$ eleven variables at daily frequency with the first observation on 3 January 2006 and the last on 30 June 2023.[62] (1) The funding spread measured as the difference between 3-month LIBOR and 3-month general-collateral (GC) repo rate. (2) The liquidity premium measured as the difference between the 3-month GC repo rate and the 3-month T-bill rate.[63] (3) The log-transformed quantity of outstanding treasuries. (4) The log-transformed balance on the Treasury General Account. 5) The spread between Moody's seasoned Baa corporate bond yield and the 10-year treasury rate. (6) The log-transformed value of the S&P 500 stockmarket index. (7) The log-transformed value of the S&P 500 financials stockmarket index. (8) The log-transformed VIX. (9) The level of the 3-month GC repo rate. (10) The level of the 10-year treasury rate. (11) The trade-weighted exchange rate of the US dollar. We set $L = 80$ to ensure we control for at least one quarter of data as lags. Our vector $\mathbf{d}_t$ includes time dummies for (1) weekday, (2) day of the month, (3) month, and (4) NBER recessions. The linear time trend does not allow for gaps in the observed dates.[64]

Identification. The econometric challenge is to find exogenous variation in the liquidity premium to estimate our coefficient of interest β. Because of omitted variables, measurement error and reverse causality, OLS estimates are unlikely to be consistent. For example, it is possible that unobserved shocks to uncertainty are driving both the funding spread and the liquidity premium. Or perhaps the GC repo rate is a noisy measure of the risk-free rate, and measurement error is driving a correlation between the measured liquidity premium and funding spread. It is also possible that shocks to the funding spread are driving demand for liquidity and thus the liquidity premium.[65]

Our identification strategy is to instrument the liquidity premium with the quantity of outstanding treasury debt. The quantity of treasuries is relevant to the liquidity premium as shown in a vast literature studying the convenience yield on treasuries

[62] Before 2006 we do not have daily data for the dollar's trade-weighted exchange rate. The dataset's end date coincides with the final discontinuation date of LIBOR in the US. After merging the series, we are left with 4157 observations over the period. Data sources are reported in appendix B.

[63] Our adopted measure of the liquidity premium is standard in the literature (Nagel, 2016; Krishnamurthy and Li, 2023). The funding spread is the difference between the rate at which banks can borrow without collateral and the risk-free rate as measured by the GC repo rate.

[64] On average, our dataset contains 59 observations per quarter, nearly the universe of business days.

[65] The results from OLS, reported in table 5 in appendix A, are consistent with measurement error in the risk-free rate as a driver of endogeneity.

(Krishnamurthy and Li, 2023). We confirm the instrument's relevance in the first-stage regression.

As for the instrument's validity, treasury debt is issued a few days after it is auctioned as can be seen in figure 15 in appendix A.[66] This institutional feature makes outstanding treasury debt predetermined at daily frequency. This rules out confounding variables in the error term ν_t driving treasury debt and thus making it invalid. It also rules out reverse causality.

Another threat to the instrument's validity are alternative mechanisms through which the quantity of treasuries affects the funding spread for a given liquidity premium. We can assuage this concern by noting that an implication of outstanding treasuries being predetermined at daily frequency is that they are also perfectly anticipated. In other words, there is no new information revealed when treasuries are issued and mature. All the information, for instance regarding fiscal policy, is revealed at the latest during the auction. This rules out a direct information effect of the quantity of treasuries.

Finally, treasury debt is a highly persistent variable. To rule out a persistent omitted variable driving both treasury debt and the funding spread, it is important that the controls included in the regression succeed in removing the autocorrelation from the residual. For that, a rich lag structure is needed. Suppose we omitted lags of an element of the true data vector $\boldsymbol{y}_t$ from the analysis. Then, the residual would contain the omitted lags as well as the stochastic innovation. If in addition to driving the funding spread the omitted lags are also driving treasury debt, because for instance they drive fiscal policy, then the instrument is no longer valid.[67] As described above, we include as controls 80 lags of eleven variables available at daily frequency. As a result, the estimated residuals are not autocorrelated as can be seen in figure 14 in appendix A.

Key result. Table 3 contains the results of the benchmark IV regression. An exogenous one basis-point increase in the liquidity premium increases banks' funding spread by 1 basis point.[68] The effect is robustly significant with a p-value of 2.8%.[69]

The instrument is highly relevant as confirmed by the first-stage F statistic of 15. In the first-stage regression, we find that a one-percent increase in treasuries reduces the liquidity premium by 2.1 basis points (p-value is 0.3%). The direction is consistent

[66] Even more days pass from announcement to auction.

[67] For example, the policymaker could use private information available to him at the auction date to anticipate the funding spread on the issuance date. If the policy-maker used this private information to stabilize the funding spread with their treasury issuance, then our estimates would be biased downwards.

[68] The size of the effect in the calibrated model is 2 basis points, which is in the 99% confidence interval of the estimate.

[69] We use heteroskedasticity-consistent standard errors although a Pagan-Hall general test overwhelmingly fails to reject homoskedasticity of the residuals (the test's p-value is 100%). With regular standard errors, the p-value is 0.4%.

Table 3: Regression table

	Funding spread
Liquidity premium	0.99**
	(0.45)
Lags	Y
Time dummies	Y
Linear trend	Y
R-squared	97%
Observations	4077
1^{st}-stage F statistic	15

Note 1: Outstanding treasuries as external instrument.
Note 2: Heteroskedasticity-consistent standard errors in parentheses.
Note 3: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

with a movement along the downward-sloping demand for treasuries.

To check the robustness of the results, we look for evidence of state-dependence in the effect of the liquidity premium on the funding spread. We add as regressor an interaction term of the liquidity premium with the recession dummy to see to what extent the effect differs according to the state of the economy. As an additional instrument, we use the interaction of treasuries with the recession dummy. As reported in table 6 in appendix A, the effect of the liquidity premium on the funding spread in recessions is not significantly different from the same effect in expansions.

In table 6 in appendix A, we check alternative specifications and find that excluding the time dummies or the lag structure does not affect the results. Finally, to understand the import of lag selection for the result, we run the benchmark regression with different values for the number of lags and report the results for the coefficient of interest in figure 16 in appendix A.

8 Conclusion

This paper has developed a novel financial friction based on coordination failure in the market for bank deposits. The friction implies that fragile banks borrow on worse terms. Liquid-asset holdings and net worth are substitutable factors that keep banks' fragility in check. Hence, when net worth is scarce, banks demand more liquid assets. Introducing this friction in a canonical macroeconomic model, we have found that the model matches the positive correlation of the liquidity premium with indicators of financial stress. This is a fact that current macroeconomic models with financial frictions do not speak to. Moreover, the friction gives a role for policy in adjusting the supply of liquid assets to stabilize the economy. Finally, we have tested empirically a key prediction of the model: a high liquidity premium leads to high funding costs for banks. Exploiting exogenous variation in the liquidity premium at daily frequency due to predetermined changes in the supply of treasuries, we find a robustly-significant positive effect. The corresponding effect in the calibrated model is within the 99% confidence interval of the empirical estimate.

The paper provides a quantitative framework to understand and evaluate policies that change the quantity of liquid assets in the economy. A case in point is quantitative easing, as enacted in response to the financial disruptions of the global financial crisis. The current generation of macroeconomic models largely appraise such policy as a credit policy: QE is effective because the central bank makes loans that banks cannot make on account of a binding leverage constraint. In this paper's framework, the real effects of QE stem from the liability-side of the central-bank balance sheet regardless of its asset holdings. Lots of liquid reserves on banks' balance sheets make creditors willing to lend to banks at more favourable conditions. The two effects are not exclusive. Hence, there is scope for studying moral-hazard and coordination frictions together for a rounder account of central-bank balance-sheet policies. More generally, the interaction of liquid-asset supply with other policy levers warrants further investigation. For this, the introduction of additional frictions from the literature, such as distortionary taxes or nominal rigidities, will be necessary.

References

Acharya, Viral V, and Raghuram Rajan. 2022. "Liquidity, Liquidity Everywhere, Not a Drop to Use – Why Flooding Banks with Central Bank Reserves May Not Expand Liquidity." Working paper 29680, National Bureau of Economic Research.

Ahnert, Toni. 2016. "Rollover Risk, Liquidity and Macroprudential Regulation." *Journal of Money, Credit and Banking*, 48(8): 1753–1785.

Amador, Manuel, and Javier Bianchi. forthcoming. "Bank Runs, Fragility, and Credit Easing." *American Economic Review*.

Angeletos, George-Marios, Fabrice Collard, and Harris Dellas. 2023. "Public Debt as Private Liquidity: Optimal Policy." *Journal of Political Economy*, 131(11): 3233–3264.

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan. 2006. "Signaling in a Global Game: Coordination and Policy Traps." *Journal of Political economy*, 114(3): 452–484.

Angeletos, George-Marios, and Iván Werning. 2006. "Crises and Prices: Information Aggregation, Multiplicity, and Volatility." *american economic review*, 96(5): 1720–1736.

Arce, Oscar, Galo Nuño, Dominik Thaler, and Carlos Thomas. 2020. "A Large Central Bank Balance Sheet? Floor vs Corridor Systems in a New Keynesian Environment." *Journal of Monetary Economics*, 114 350–367.

Atkeson, Andrew. 2000. "Discussion on Morris and Shin." *NBER Macroeconomics Annual* 161–164.

Baron, Matthew, Emil Verner, and Wei Xiong. 2021. "Banking Crises without Panics." *The Quarterly Journal of Economics*, 136(1): 51–113.

Benigno, Gianluca, and Pierpaolo Benigno. 2022. "Managing Monetary Policy Normalization." Staff report 1015, Federal Reserve Bank of New York.

Bernanke, Ben S. 2010. "Causes of the recent financial and economic crisis.." Testimony before the Financial Crisis Inquiry Commission, Washington, D.C.

Bianchi, Javier, and Saki Bigio. 2022. "Banks, Liquidity Management, and Monetary Policy." *Econometrica*, 90(1): 391–454.

Boissay, Frédéric, Fabrice Collard, and Frank Smets. 2016. "Booms and Banking Crises." *Journal of Political Economy*, 124(2): 489–538.

Brunnermeier, Markus K., and Lasse Heje Pedersen. 2008. "Market Liquidity and Funding Liquidity." *The Review of Financial Studies*, 22(6): 2201–2238.

Brunnermeier, Markus K., and Yuliy Sannikov. 2014. "A Macroeconomic Model with a Financial Sector." *American Economic Review*, 104(2): 379–421.

Carlsson, Hans, and Eric van Damme. 1993. "Global Games and Equilibrium Selection." *Econometrica* 989–1018.

Chodorow-Reich, Gabriel. 2014. "Effects of Unconventional Monetary Policy on Financial Institutions." *Brookings Papers on Economic Activity*(1): 155–227.

Choi, Dong Beom, Paul Goldsmith-Pinkham, and Tanju Yorulmazer. 2023. "Contagion effects of the silicon valley bank run." Working paper 31772, National Bureau of Economic Research.

d'Avernas, Adrien, and Quentin Vandeweyer. forthcoming. "Treasury Bill Shortages and the Pricing of Short-Term Assets." *Journal of Finance*.

DeYoung, Robert, Isabelle Distinguin, and Amine Tarazi. 2018. "The joint regulation of bank liquidity and bank capital." *Journal of Financial Intermediation*, 34 32–46.

Di Tella, Sebastian, and Pablo Kurlat. 2021. "Why Are Banks Exposed to Monetary Policy?" *American Economic Journal: Macroeconomics*, 13(4): 295–340.

Diamond, Douglas, and Philip Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–419.

Diamond, William F., Zhengyang Jiang, and Yiming Ma. 2023. "The Reserve Supply Channel of Unconventional Monetary Policy." Working paper 31693, National Bureau of Economic Research.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl. 2018. "A Model of Monetary Policy and Risk Premia." *Journal of Finance*, 73(1): 317–373.

Eisfeldt, Andrea L. 2004. "Endogenous Liquidity in Asset Markets." *Journal of Finance*, 59(1): 1–30.

Ennis, Huberto M., and Todd Keister. 2003. "Economic Growth, Liquidity, and Bank Runs." *Journal of Economic Theory*, 109(2): 220–245.

Fernández-Villaverde, Jesús, Samuel Hurtado, and Galo Nuño. 2023. "Financial Frictions and the Wealth Distribution." *Econometrica*, 91(3): 869–901.

Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian P. Sack. 2011. "Large-scale Asset Purchases by the Federal Reserve: Did They Work?" *Economic Policy Review*, 17(1): , p. 41.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58(1): 17–34.

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*. 3 547–599.

Gertler, Mark, and Nobuhiro Kiyotaki. 2015. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy." *American Economic Review*, 105(7): 2011–2043.

Gertler, Mark, Nobuhiro Kiyotaki, and Andrea Prestipino. 2016. "Anticipated Banking Panics." *American Economic Review: Papers & Proceedings*, 106(5): 554–559.

Gertler, Mark, Nobuhiro Kiyotaki, and Andrea Prestipino. 2020. "A Macroeconomic Model with Financial Panics." *Review of Economic Studies*, 87(1): 240–288.

Van der Ghote, Alejandro. 2021. "Interactions and Coordination between Monetary and Macroprudential Policies." *American Economic Journal: Macroeconomics*, 13(1): 1–34.

Goldstein, Itay, and Ady Pauzner. 2005. "Demand-deposit Contracts and the Probability of Bank Runs." *Journal of Finance*, 60(3): 1293–1327.

Gorton, Gary. 1988. "Banking Panics and Business Cycles." *Oxford Economic Papers*, 40(4): 751–781.

Grossman, Sanford J., and Merton H. Miller. 1988. "Liquidity and Market Structure." *Journal of Finance*, 43(3): 617–633.

He, Zhiguo, and Arvind Krishnamurthy. 2013. "Intermediary Asset Pricing." *American Economic Review*, 103(2): 732–770.

Holmström, Bengt, and Jean Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector." *Quarterly Journal of Economics*, 112(3): 663–691.

Holmström, Bengt, and Jean Tirole. 1998. "Private and Public Supply of Liquidity." *Journal of Political Economy*, 106(1): 1–40.

Ikeda, Daisuke. 2024. "Bank Runs, Prudential Tools and Social Welfare in a Global Game General Equilibrium Model." *Journal of Financial Stability*, 72, p. 101236.

Karadi, Peter, and Anton Nakov. 2021. "Effectiveness and Addictiveness of Quantitative Easing." *Journal of Monetary Economics*, 117 1096–1117.

Kashyap, Anil K., Dimitrios P. Tsomocos, and Alexandros P. Vardoulakis. 2024. "Optimal Bank Regulation in the Presence of Credit and Run Risk." *Journal of Political Economy*, 132(3): 772–823.

Krishnamurthy, Arvind, and Wenhao Li. 2023. “The Demand for Money, Near-money, and Treasury Bonds.” *Review of Financial Studies*, 36(5): 2091–2130.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2011. “The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy.” *Brookings Papers on Economic Activity*, 42(2 (Fall)): 215–287.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012. “The Aggregate Demand for Treasury Debt.” *Journal of Political Economy*, 120(2): 233–267.

Kuttner, Kenneth N. 2018. “Outside the Box: Unconventional Monetary Policy in the Great Recession and Beyond.” *Journal of Economic Perspectives*, 32(4): 121–146.

Kydland, Finn E., and Edward C. Prescott. 1982. “Time to build and aggregate fluctuations.” *Econometrica* 1345–1370.

Leonello, Agnese, Caterina Mendicino, Ettore Panetti, and Davide Porcellacchia. 2022. “Savings, Efficiency and the Nature of Bank Runs.” ECB Working Paper 2636.

Li, Lei, Yi Li, Marco Macchiavelli, and Xing (Alex) Zhou. 2021. “Liquidity Restrictions, Runs, and Central Bank Interventions: Evidence from Money Market Funds.” *Review of Financial Studies*, 34(11): 5402–5437.

Li, Wenhao. forthcoming. “Public Liquidity and Financial Crises.” *American Economic Journal: Macroeconomics*.

Martin, Antoine, David Skeie, and Ernst-Ludwig von Thadden. 2014. “The Fragility of Short-term Secured Funding Markets.” *Journal of Economic Theory*, 149 15–42.

Mattana, Elena, and Ettore Panetti. 2021. “The Welfare Costs of Self-Fulfilling Bank Runs.” *Journal of Money, Credit and Banking*, 53(2-3): 401–440.

Mendicino, Caterina, Kalin Nikolov, Javier Suarez, and Dominik Supera. 2020. “Bank Capital in the Short and in the Long Run.” *Journal of Monetary Economics*, 115 64–79.

Morris, Stephen, and Hyun Song Shin. 2003. “Global games: Theory and Applications.” In *Advances in Economics and Econometrics*. eds. by Mathias Dewatripont, Lars P. Hansen, and Stephen J. Turnovsky, Chap. 3 56–114.

Nagel, Stefan. 2016. “The Liquidity Premium of Near-money Assets.” *Quarterly Journal of Economics*, 131(4): 1927–1971.

Nuño, Galo, and Carlos Thomas. 2017. “Bank Leverage Cycles.” *American Economic Journal: Macroeconomics*, 9(2): 32–72.

Poole, William. 1968. "Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy." *Journal of Finance*, 23(5): 769–791.

Porcellacchia, Davide. 2020. "The Tipping Point: Interest Rates and Financial Stability." ECB Working Paper 2447.

Ray, Walker, Michael Droste, and Yuriy Gorodnichenko. 2024. "Unbundling Quantitative Easing: Taking a Cue from Treasury Auctions." *Journal of Political Economy*, 132(9): 000–000.

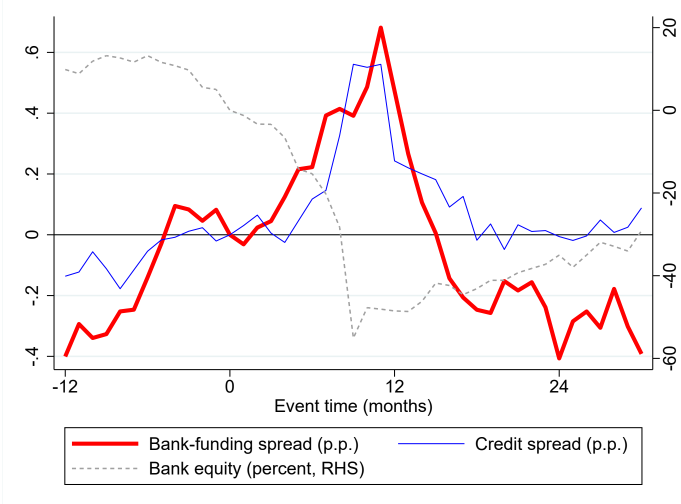
Rochet, Jean-Charles, and Xavier Vives. 2004. "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?" *Journal of the European Economic Association*, 2(6): 1116–1147.

Shin, Hyun Song. 2009. "Reflections on Northern Rock: the Bank Run that Heralded the Global Financial Crisis." *Journal of Economic Perspectives*, 23(1): 101–119.

Vives, Xavier. 2014. "Strategic Complementarity, Fragility, and Regulation." *Review of Financial Studies*, 27(12): 3547–3592.

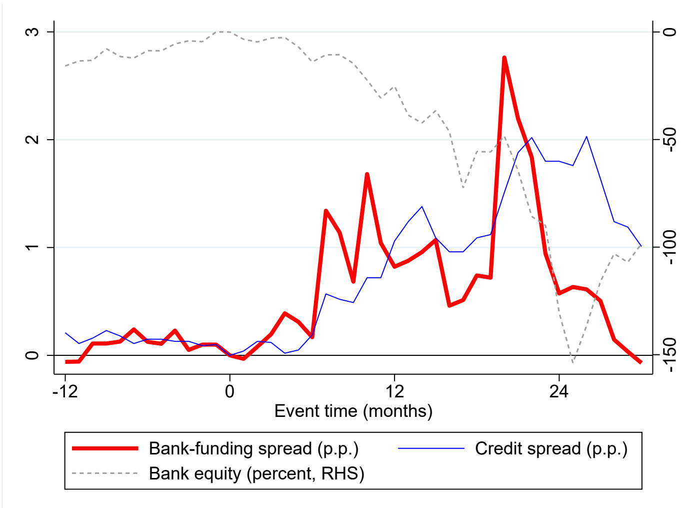
A Figures

Figure 8: Interest-rate spreads in banking crises.



Note 1: Because of data limitations, bank-funding spread is defined as the 3-month interbank rate minus 3-month government-bond rate (funding spread plus liquidity premium in the paper's definition). *Note 2:* The figure plots the average evolution of bank-funding spreads, credit spreads and bank equity (cumulated bank-share returns in log points) around banking-crisis years identified in Baron et al. (2021). The variables are normalized to 0 at event time 0, which is January of the banking-crisis year. The plots are averages over 66 episodes for which data is available. *Note 3:* The list of included banking crises is reported in table 4 in appendix A.

Figure 9: The dynamics of the global financial crisis in the US.



Plot of the evolution of the US bank-funding spread (the TED spread between 3-month LIBOR and 3-month T-bill rate), the US credit spread (Moody's Aaa corporate yield minus the 10-year treasury rate) and bank equity (cumulated bank-share returns in log points). Event time 0 is January 2007.

Table 4: List of banking crises underlying figure 8.

Country	Year
Australia	1989
Austria	2008, 2011
Belgium	2008, 2011
Czechia	1995
Denmark	1877, 1885, 1907, 1992, 2008, 2011
Finland	1990
France	1882, 1889, 1937, 2008
Germany	1891, 1901, 1930, 2008
Hong Kong	1998
Hungary	1995, 2008
Iceland	2008
Ireland	2007
Italy	1992, 2008, 2011, 2016
Japan	1990, 1997, 2001
Korea	1997
Luxembourg	2008
Malaysia	1985, 1997
Norway	1987, 2008
Philippines	1997
Portugal	2008, 2011
Russia	2008
Spain	2008, 2010
Sweden	1991, 2008
Switzerland	1990, 2008
Taiwan	1998
Thailand	1979, 1983, 1997
UK	1878, 1890, 1914, 1991, 2008
US	1890, 1893, 1907, 1930, 1984, 1990, 2007

Figure 10: Pandemic and tightening cycle.

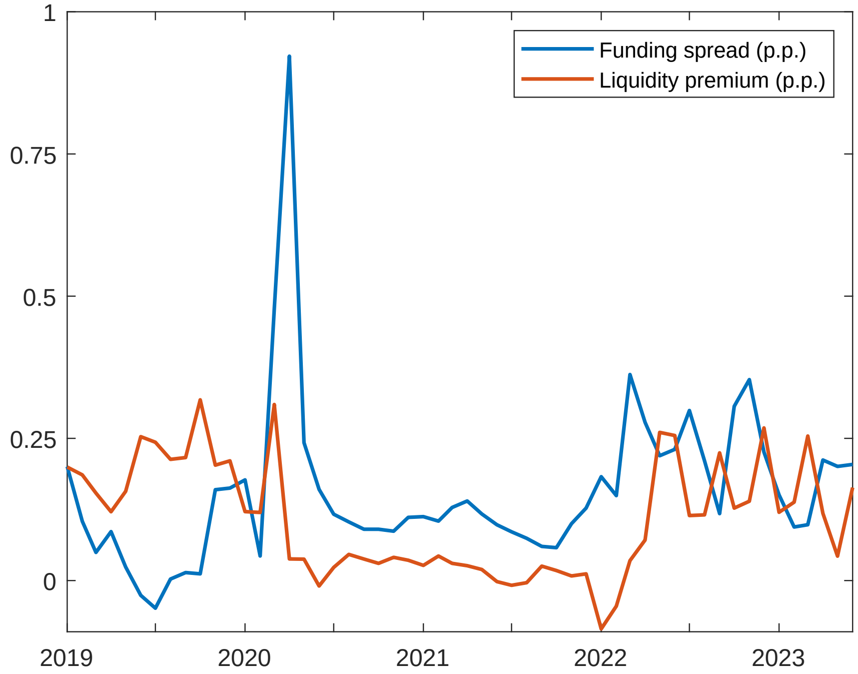
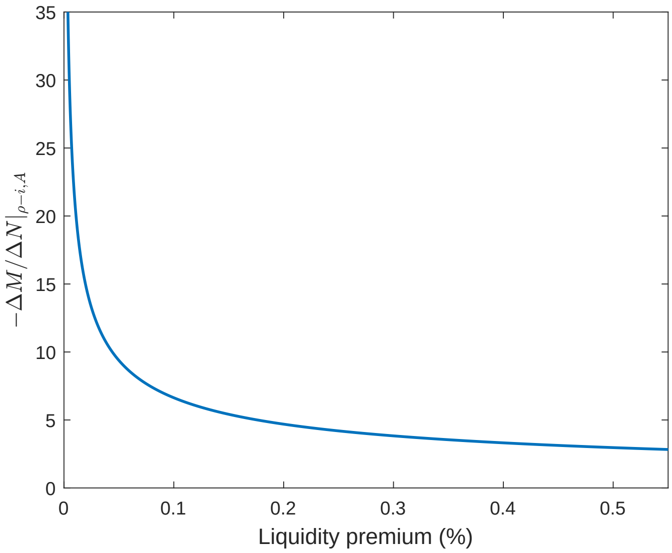


Table 5: OLS

Funding spread	OLS	OLS	OLS	OLS
Liquidity premium	0.75***	0.40***	-0.30***	-0.30***
	(0.06)	(0.04)	(0.06)	(0.06)
Lags	N	N	Y	Y
Time dummies	N	Y	N	Y
Linear trend	Y	Y	Y	Y
R-squared	23%	57%	99%	99%
Observations	4157	4157	4077	4077

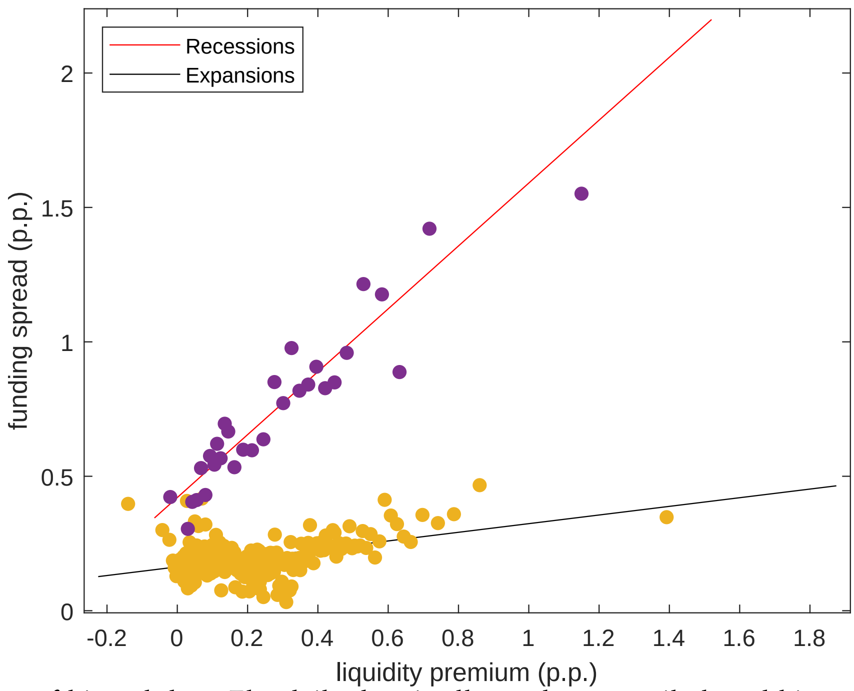
Note 1: Heteroskedasticity-consistent standard errors in parentheses.
Note 2: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

Figure 11: Substitutability of net worth for liquidity



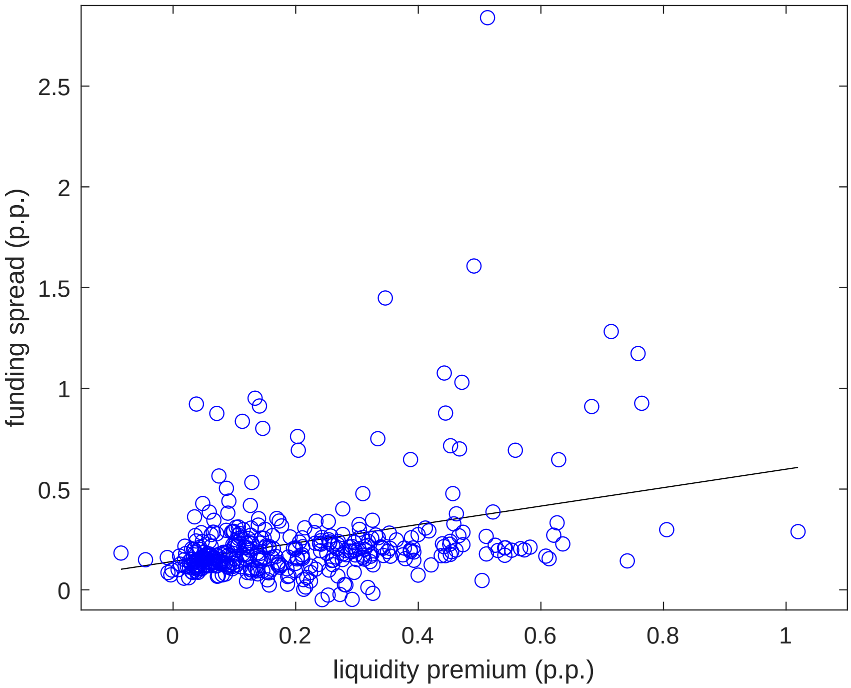
Note: Annualized calibrated parameter values from Table 2 are used.

Figure 12: Expansions vs recessions.



Note: Scatterplot of binned data. The daily data is allocated to quantile-based bins according to the liquidity premium. There are 270 bins for expansions and 30 bins for recessions.

Figure 13: May 1991 – June 2023.



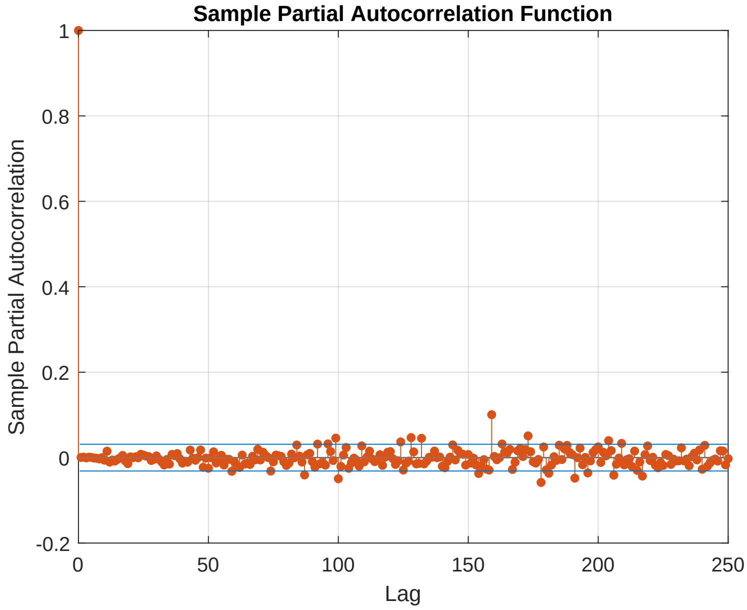
Note: Scatterplot of data at monthly frequency.

Table 6: IV with alternative specifications

Funding spread	IV	IV	IV	IV	IV
Liquidity premium	1.4	1.0**	0.31***	1.28***	0.99**
	(1.0)	(0.48)	(0.04)	(0.06)	(0.45)
Liquidity premium × Recession	-0.54				
	(1.0)				
Lags	Y	Y	N	N	Y
Time dummies	Y	N	Y	N	Y
Linear trend	Y	Y	Y	Y	Y
R-squared	96%	96%	57%	17%	97%
Observations	4077	4077	4157	4157	4077
1st-stage F statistic	3.9	13	1560	1823	15

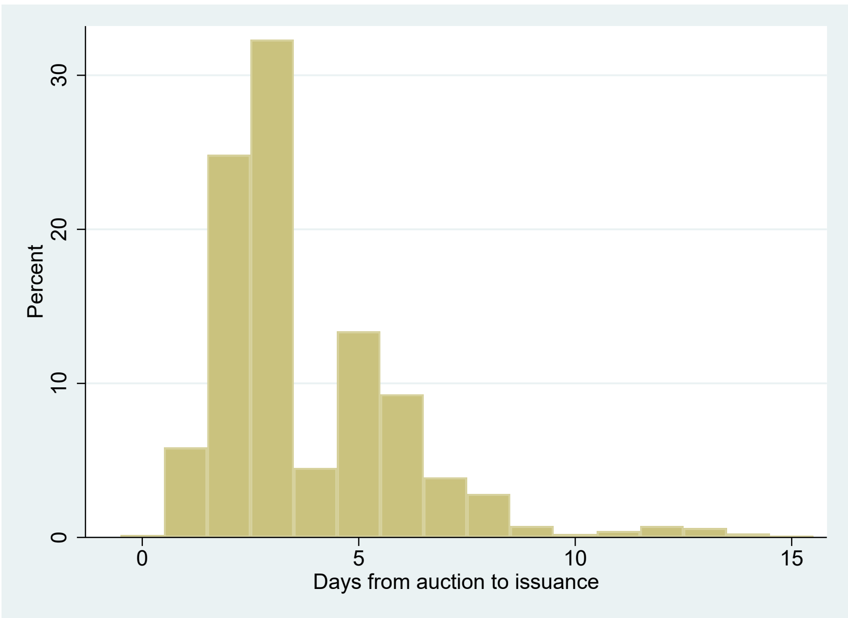
Note 1: IV estimation uses outstanding treasuries as external instrument.
Note 2: In regression with interaction term, estimation uses outstanding treasuries × recession as additional external instrument.
Note 3: Heteroskedasticity-consistent standard errors in parentheses.
Note 4: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

Figure 14: Partial autocorrelation function of funding-spread innovations.



Note 1: The residuals are estimated in the benchmark IV regression.
Note 2: The blue lines are 95% confidence intervals for estimates of sample partial autocorrelation with a white-noise process.

Figure 15: Time from treasury auction to issuance



Note 1: Data source is TreasuryDirect.gov.
Note 2: There are 5320 observations of CUSIP-level treasury bills, notes and bonds issued between January 2006 and June 2023.
Note 3: The instances of auction and issuance on the same day are 7, corresponding to 0.1% of observations.

Figure 16: Robustness to lag selection.

Note: The blue bar represents the 95% confidence interval for the coefficient of interest β estimated with IV.

B Data sources

We obtain the 3-month GC repo rate (mid-price from ticker ”USRGCGC ICUS Currncy”) and the 3-month LIBOR from Bloomberg. Daily data on quantity of outstanding treasuries (series ”Debt held by the public” in dataset ”Debt to the Penny”) and on the TGA closing balance (series ”Treasury General Account (TGA) Closing Balance” in dataset ”Daily Treasury Statement (DTS)”) is available on the website Fiscaldata maintained by the US Treasury Department. From the website FRED maintained by the Federal Reserve Bank of St. Louis, we retrieve the 3-month T-bill rate (series ”DTB3”), the spread between Moody’s seasoned Baa corporate bond yield and the 10-year treasury rate (series ”BAA10Y”), the 10-year treasury rate (series ”DGS10”), the VIX (series ”VIXCLS”), and the nominal broad US dollar index (series ”DTWEXBGS”). The closing values of the S&P 500 stockmarket index and the S&P 500 financials stockmarket index are downloaded from the website Yahoo! Finance.

C Proofs

Proof of Lemma 1. A strategy in the coordination game is a correspondence that maps a household’s signal $\tilde{F}_h$ into deposit-holding decision H_h.
Consider other households playing the same threshold strategy such that they hold a bank’s deposits with $H_h = 1$ if they receive signal $\hat{F}_h \leq k$ and do not hold the deposits otherwise. Given a household i’s (improper) uniform prior and signal $\hat{F}_i$ about the fundamental, its expected net payoff of holding deposits can be written as

$$\tilde{\pi}^*(\hat{F}_i, k) = \int_{\hat{F}_i-\omega}^{\hat{F}_i+\omega} \frac{\tilde{\pi}(F,k)}{2\omega}\, \mathrm{d}F, \tag{50}$$

where $\tilde{\pi}(F,k)$ is the payoff of holding deposits for a given fundamental F.
Using (6), we can show that $\tilde{\pi} < 0$ for $j < \rho$. Hence, it is a dominant strategy for households not to hold deposits. Focusing on the interesting case $j \geq \rho$, we have that net payoff $\tilde{\pi}(F,k)$ is $-\theta < 0$ for $F > 1$, because in this case the bank fails even if everyone else’s strategy implies holding for any signal. It is $j - \rho \geq 0$ for $F \leq 0$ because in this case the bank does not fail regardless of everyone else’s strategy. As for the intermediate range, notice that by the law of large numbers the share of households holding is

$$H = \begin{cases} 1 & \text{if } F \leq k - \omega, \\ \frac{1}{2} + \frac{k-F}{2\omega} & \text{if } F \in (k-\omega, k+\omega] \\ 0 & \text{otherwise.} \end{cases} \tag{51}$$

This result implies that

$$\tilde{\pi}(F,k) = \begin{cases} -\theta & \text{if } F > 1, \\ -\theta & \text{if } F \in (0,1] \text{ and } F > \frac{k+\omega}{1+2\omega}, \\ j-\rho & \text{if } F \in (0,1] \text{ and } F \leq \frac{k+\omega}{1+2\omega}, \\ j-\rho & \text{otherwise.} \end{cases} \tag{52}$$

Using (50) and (52), we can verify that a dominant strategy sets $H_i = 0$ for $\hat{F}_i > 1+\omega$. A dominant strategy also sets $H_i = 1$ for $\hat{F}_i < -\omega$ if $j > \rho$. If $j = \rho$, then it is only weakly dominant to set $H_i = 1$ for $\hat{F}_i < -\omega$. Nonetheless, the equilibrium strategy must imply this behaviour because of the tie-breaking assumption that households hold deposits if indifferent.

Now, we start to iteratively delete strictly dominated strategies. We start with a strategy of holding deposits if and only if $\hat{F}_i \leq 1+\omega$. As already noticed, a strategy that implies holding for $\hat{F}_h > 1+\omega$ is dominated. This restricts the strategies we consider for other households. Let us consider other households playing a threshold strategy with $k = 1+\omega$. Because of strategic complementarity, if a strictly better strategy for household i can be found under this conjecture about other households' behaviour, then household i's strategy under consideration is strictly dominated. The expected net payoff of holding for a household receiving signal $1+\omega$ given $k = 1+\omega$ is $\tilde{\pi}^*\left(\hat{F}_i, k\right) = -\theta$. Hence, the strategy is dominated.

This logic can be extended by studying

$$\tilde{\pi}^*(z,z) = \int_{z-\omega}^{\frac{z+\omega}{1+2\omega}} \frac{j-\rho}{2\omega} \, \mathrm{d}F - \int_{\frac{z+\omega}{1+2\omega}}^{z+\omega} \frac{\theta}{2\omega} \, \mathrm{d}F = \frac{j-\rho+\omega(j-\rho-\theta)}{1+2\omega} - \frac{j-\rho+\theta}{1+2\omega} z. \tag{53}$$

The function is monotonically decreasing and crosses zero at

$$z^* = \frac{j-\rho}{j-\rho+\theta} + \frac{j-\rho-\theta}{j-\rho+\theta}\omega. \tag{54}$$

This allows us to delete as dominated all strategies such that a household holds deposits with $\hat{F}_i > z^*$. We can apply an analogous analysis in reverse to delete as dominated all strategies that set $H_h = 0$ for $\hat{F}_i < z^*$.

Deletion of strictly dominated strategies and the assumption that households hold a bank's deposits if indifferent give us the strategy played in the unique Bayesian Nash equilibrium of the coordination game as

$$H_h^* = \begin{cases} 1 & \text{if } \hat{F}_h \geq \frac{j-\rho}{j-\rho+\theta} + \frac{j-\rho-\theta}{j-\rho+\theta}\omega, \\ 0 & \text{otherwise.} \end{cases} \tag{55}$$

□

Proof of Lemma 2. The lemma is proven by substituting the equilibrium run threshold (7) into equation (51), which gives the share of households holding deposits for a general run threshold k. The result for $j_b < \rho$ follows directly from Lemma 1. □

Proof of Proposition 1. We start with the proof of statement 1. First, consider the case with $(\tilde{F}_b - F_b^*)/\tau \in (-1 + \omega/\tau, 1 - \omega/\tau)$, a non-empty set since $\omega/\tau \to 0$. Using equation (8), we can compute $\Pr(H_b \in (0,1)) = \omega/\tau \to 0$. Second, consider the case with $(\tilde{F}_b - F_b^*)/\tau = -1 + \omega/\tau$, which implies $(\tilde{F}_b - F_b^*)/\tau < 1 - \omega/\tau$ since $\omega/\tau \to 0$. In this case too, we have that $\Pr(H_b \in (0,1)) = \omega/\tau \to 0$. Third, consider the case with $(\tilde{F}_b - F_b^*)/\tau \in (-1 - \omega/\tau, -1 + \omega/\tau)$. In this case, we can compute $\Pr(H_b \in (0,1)) = (1/2)\left[1 + (\tilde{F}_b - F_b^*)/\tau + \omega/\tau\right]$, which is smaller than $3\omega/2\tau \to 0$. For the last case with $(\tilde{F}_b - F_b^*)/\tau \leq -1 - \omega/\tau$, it is trivially true that $\Pr(H_b \in (0,1)) = 0$.
Second, we prove statement 2. We work with $j_b \geq \rho$. For $j_b < \rho$, there probability of $H_b = 1$ is trivially zero due to Lemma 2. Given statement 1 of the proposition, for $(\tilde{F}_b - F_b^*)/\tau \leq -1 + \omega/\tau$ we have that $\Pr(H_b = 1) \to 1$ because clearly $H_b = 0$ is impossible under equation (8). In the alternative case $(\tilde{F}_b - F_b^*)/\tau > -1 + \omega/\tau$, we have that

$$\Pr(H_b = 1) = \max\left\{0, \frac{1}{2}\left(1 - \frac{\tilde{F}_b - F_b^*}{\tau} - \frac{\omega}{\tau}\right)\right\} < 1. \tag{56}$$

For $\omega/\tau \to 0$, this is equivalent to statement 2. □

Proof of Corollary 1. If a household is sure that in equilibrium $H_b = 1$ and $F_b \leq 1$, then it is also sure that there is no run. By Lemma 2, for given F_b^* a household can be sure that in equilibrium $H_b = 1$ if

$$\hat{F}_{bh} + 2\omega \leq F_b^* \tag{57}$$

and $j_b \geq \rho$. Because equation (7) implies $F_b^* < 1$, condition (57) is also sufficient for $F_b \leq 1$.
For any realization of true fragility $F_b + \tilde{F}_b + \mu_b$ with $\mu_b \sim \mathrm{U}[-\tau, \tau]$, we know the distribution of signals $\hat{F}_{bh} \sim \mathrm{U}[F_b - \omega, F_b + \omega]$. With this, we can calculate the share of households violating condition (57) as

$$\Pr\left(\hat{F}_{bh} + 2\omega > F_b^*\right) = \min\left\{\max\left\{0, \frac{\tilde{F}_b + \mu_b - F_b^* + 3\omega}{2\omega}\right\}, 1\right\}. \tag{58}$$

Re-writing as $\Pr\left(\hat{F}_{bh} + 2\omega > F_b^*\right) = \min\left\{\max\left\{0, 3/2 + \tau/2\omega\left[(\tilde{F}_b - F_b^*)/\tau + \mu_b/\tau\right]\right\}, 1\right\}$ clarifies that under $\omega/\tau \to 0$ and $\tilde{F}_b \leq F_b^* - \tau$ we have $\Pr\left(\hat{F}_{bh} + 2\omega > F_b^*\right) = 0$ for any $\mu_b < \tau$. The corollary follows from $\Pr(\mu_b \geq \tau) = 0$. □

D Bank runs in the macroeconomic model

Since bank runs do not happen in equilibrium and households anticipate this, section 3 abstracts from bank runs in the analysis of household, firm, bank, and government behaviour. Nonetheless, the equilibrium of the coordination game depends on understanding what would happen if a run were to occur, and this appendix shows how this outcome is consistent with the macroeconomic model.

Banks. Each bank b makes choices at the competitive-markets stage that determine the variables A_{bt}, M_{bt}, D_{bt}, and N_{bt} on its balance sheet $A_{bt} + M_{bt} = D_{bt} + N_{bt}$, where $N_{bt} > 0$. If a positive fraction of households choose not to hold the bank's deposits ($H_{bt} < 1$) in the coordination game, its balance sheet must adapt. In what follows, the notation $\tilde{\cdot}$ is used to denote the value of variable at the coordination game or consumption stage of period t in the case of a run or partial run on one or more banks.

When faced with a run, a bank first disposes of its liquid assets M_{bt}, reducing them to $\tilde{M}_{bt} = \max\{0, M_{bt} - (1 - H_{bt})D_{bt}\}$, which can be done without the bank suffering any loss. If this is insufficient to repay depositors, physical capital is liquidated down to $A_{bt} + (M_{bt} - \tilde{M}_{bt}) - (1 - H_{bt})D_{bt}$. This is feasible given that net worth is positive and there is convertibility between physical capital and goods, and there is no adjustment cost suffered by the bank unless physical capital needs to be reduced below λA_{bt}.

However, liquidating bank b's physical capital below λA_{bt} (equivalent to $H_{bt} < F_{bt}$) causes the bank to fail ($\Phi_{bt} = 1$) and its net worth is lost, which acts as a capital adjustment cost. If the bank fails, any remaining depositors are repaid by liquidating further physical capital. In general, the amount of deposits still held by households at the end of the coordination game is $\tilde{D}_{bt} = (1 - \Phi_{bt})H_{bt}D_{bt}$, where this formula applies both to surviving and failing banks. The bank's remaining physical capital is $\tilde{A}_{bt} = A_{bt} - (D_{bt} - \tilde{D}_{bt}) + (M_{bt} - \tilde{M}_{bt}) - \Phi_{bt}N_{bt}$, which implies its post-run balance sheet is $\tilde{A}_{bt} + \tilde{M}_{bt} = \tilde{D}_{bt} + \tilde{N}_{bt}$ with net worth $\tilde{N}_{bt} = (1 - \Phi_{bt})N_{bt}$. For a bank b that fails, capital adjustment costs $\Phi_{bt}N_{bt}$ wipe out all remaining assets ($\tilde{A}_{bt} = 0$) and net worth ($\tilde{N}_{bt} = 0$).

Runs that cause banks to dispose of illiquid assets affect the level of investment $\tilde{I}_{bt}$ and the future capital stock $\tilde{K}_{b,t+1} = X_{t+1}\tilde{A}_{bt}$. Capital adjustment costs are a resource cost that are accounted for by including them as part of investment $\tilde{I}_{bt} = \tilde{A}_{bt} - (1-\delta)K_{bt} + \Phi_{bt}N_{bt}$. A bank that fails at date t ceases to operate from date $t+1$. Any depositors who were still holding at the point of bank failure incur recovery costs in the bankruptcy process, and these resource costs are paid at the beginning of period $t+1$. Formally, these costs are treated like a capital adjustment cost and accounted for as part of investment expenditure, that is, for a bank b with $\Phi_{bt} = 1$, depositors' costs appear as $\tilde{I}_{b,t+1} = \theta\Phi_{bt}H_{bt}D_{bt}$, which replaces the investment equation for banks that have previously failed.

Households. Based on past decisions and outcomes, household h begins period t with deposits including accrued interest $(1+j_{b,t-1})(1-\Phi_{b,t-1})D_{b,t-1}H_{bh,t-1}$ that were held at surviving banks b. The household pays at date t a cost $\theta\Phi_{b,t-1}D_{b,t-1}H_{bh,t-1}$ incurred in recovering funds on deposit at a failing bank b in period $t-1$.

Household h's choices at the competitive-markets stage of period t determine a level of spending power S_{ht} carried into the subsequent banking and consumption stages that is not directly held in government-issued liquid assets M_{ht}:

$$S_{ht} = w_t L_{ht} + \Pi_t + G_t - T_t + \int_0^1 \{(1+j_{b,t-1})(1-\Phi_{b,t-1}) - \theta\Phi_{b,t-1}\} D_{b,t-1}H_{bh,t-1}\,\mathrm{d}b \\ - (1+\rho_{t-1})B_{h,t-1} + B_{ht} + (1+i_{t-1})\tilde{M}_{h,t-1} - M_{ht}, \quad (59)$$

where $\tilde{M}_{h,t-1}$ allows for holdings of liquid assets to change depending on outcomes at the banking stage. During the coordination game, households simultaneously make their deposit holding decisions $H_{bht} \in \{0,1\}$, which collectively determine any bank failures $\Phi_{bt} \in \{0,1\}$.

These decisions and outcomes for banks affect the consumption households can receive at the end of period t. Those choosing not to hold bank b's deposits can consume an extra amount D_{bt}. Those holding deposits at failing banks can recover them through the bankruptcy process by paying a per-unit cost θ at the beginning of the next period. After any withdrawals or bank failures, households may hold both goods and liquid assets, so these two markets are re-opened, though all other markets are closed at the final stage of period t.

In general, it is not automatic that liquid assets and goods would continue to exchange at the same one-for-one relative price from the first stage of period t, so here Λ_t specifies the market value of a unit of liquid assets in terms of goods at the final stage of the period. For liquid assets to have their defining characteristic, it must be shown that $\Lambda_t = 1$ in equilibrium, which will require the government to act in such a way that its liabilities M_t are indeed perfectly liquid.

Denoting household h's consumption for a general outcome of the coordination game by $\tilde{C}_{ht}$:

$$\tilde{C}_{ht} = S_{ht} - \tilde{T}_t - \int_0^1 (1-\Phi_{bt})H_{bht}D_{bt}\,\mathrm{d}b + \Lambda_t\left(M_{ht} - \tilde{M}_{ht}\right), \quad (60)$$

where $\tilde{M}_{ht}$ is the amount of liquid assets held at the end of period and carried into period $t+1$, and $\tilde{T}_t$ denotes a net lump-sum tax levied on all households after the coordination game.

Households' objective function is (18) and they are subject to (59) and (60) as constraints. The current-value Lagrangian multiplier on (59) is μ_{ht}, and the outcomecontingent multiplier on (60) is $\tilde{\mu}_{ht}$. The first-order conditions with respect to B_{ht}, L_{ht}, S_{ht} (chosen at the competitive-markets stage at the beginning of the period), and $\tilde{C}_{ht}$ (determined at the end of the period) are $\mu_{ht} = \beta(1+\rho_t)\mathbb{E}_{ht}[\mu_{h,t+1}]$, $w_t\mu_{ht} = \chi L_t^{1/\psi}$, $\mu_{ht} = \mathbb{E}_{ht}[\tilde{\mu}_{ht}]$, and $\tilde{\mu}_{ht} = \tilde{C}_{ht}^{-1/\sigma}$. Note that $\tilde{\mu}_{ht}$ and $\tilde{C}_{ht}$ are in general random variables because household h's information set does not perfectly predict outcomes in the coordination game and hence consumption.

Since there is a continuum of banks, the impact of any individual b on the constraint (60) is small, so households act as if risk neutral in respect of their strategies in the coordination game. The coordination game is also played simultaneously across all banks, so current payoffs are valued using the expected Lagrangian multiplier $\mu_{ht} = \mathbb{E}_{ht}[\tilde{\mu}_{ht}]$. Household h therefore discounts expected future payoffs using a discount factor $\beta\mathbb{E}_{ht}[\mu_{h,t+1}]/\mu_{ht}$, which is equal to $1/(1+\rho_t)$ using the B_{ht} first-order condition. The discount factor is thus common to all households, as supposed in the analysis of the coordination game, and is derived from the yield ρ_t on a risk-free but illiquid bond.

Government and market clearing. The government does not have access to any special production or storage technology allowing it directly to create an asset that is a liquid store of value. There is no reason why the value of a unit of M_t has to be the same at the first and final stages of period t, for example, if liquid assets held by banks were distributed to satisfy withdrawal demands. However, the government is able to adjust the supply of liquid assets to accommodate any changes in demand by varying its fiscal policy appropriately. The government's flow budget constraint at the final stage of period t (when only markets for goods and liquid assets are open) is $\tilde{T}_t = \Lambda_t\left(M_t - \tilde{M}_t\right)$, where $\tilde{M}_t$ is the end-of-period supply of liquid bonds and $\tilde{T}_t$ is a lump-sum tax levied on all households at the final stage.

The market for liquid bonds clears where $\tilde{M}_{ht}$ and $\tilde{M}_{bt}$ summed over households $h \in [0,1]$ and banks $b \in [0,1]$ equals the final supply of bonds $\tilde{M}_t$. Assume the government sets the tax $\tilde{T}_t$ equal to the quantity of liquid assets disbursed by banks faced with runs:

$$\tilde{T}_t = \int_0^1 (M_{bt} - \tilde{M}_{bt})\,\mathrm{d}b. \tag{61}$$

Suppose that households do not want to hold liquid assets at either the beginning or end of period t, that is, $M_{ht} = 0$ and $\tilde{M}_{ht} = 0$. Given (61) and the government's budget constraint, the market for liquid assets at the end of period t clears only if $\Lambda_t = 1$. At the earlier, competitive-markets stage, market clearing requires $i_t \leq \rho_t$, and under that condition, $M_{ht} = 0$ is optimal. Finally, $\tilde{M}_{ht} = 0$ is also optimal if $(1+i_t)/(1+\rho_t) \leq \tilde{\mu}_{ht}/\mu_{ht}$. Budget constraints and the other market-clearing conditions imply goods-market equilibrium $\tilde{C}_t + \tilde{I}_t = Y_t$ holds, and from this, $\tilde{M}_{ht} = 0$ is confirmed for a range of bank-run outcomes where $\tilde{I}_t$ does not fall too far given that $\mu_{ht} = \mathbb{E}_t[\tilde{\mu}_{ht}]$.

E Dividend policy and net worth

This section studies banks' dividend policies. Bank b's objective in choosing the path of dividends $\{\Pi_{bt}\}$ is to maximize the present discounted value of dividends $\Pi_{bt} + V_{bt}$, where V_{bt} from (23) is the present value of future dividends discounted using the representative-household stochastic discount factor P_{ts}. Assuming net worth is positive, the bank wants to satisfy the no-run condition (26) at all dates. The other constraints are its balance-sheet identity (25), the evolution of pre-dividend (and pre-bonus) net worth $E_{bt} = (1+R_t)A_{b,t-1} + (1+i_{t-1})M_{b,t-1} - (1+j_{b,t-1})D_{b,t-1}$, post-dividend-and-bonus net worth $N_{bt} = E_{bt} - \Pi_{bt} - G_{bt}$, and the amount of net worth $G_{bt} = \max\{\gamma E_{bt}/(1+\gamma) - \Pi_{bt}, 0\}$ diverted as bonuses.

Start the analysis supposing that the bank's balance-sheet choices at date $t-1$ result in positive net worth E_{bt} with probability one. Note that if $\Pi_{bt} < \gamma E_{bt}/(1+\gamma)$ then $N_{bt} = E_{bt}/(1+\gamma)$, which is independent of Π_{bt}. It follows that current dividends Π_{bt} can be increased without affecting the path of future dividends, so banks always want to satisfy $\Pi_{bt} \geq \gamma E_{bt}/(1+\gamma)$, in which case $G_{bt} = 0$ and post-dividend net worth $N_{bt} = E_{bt} - \Pi_{bt}$ satisfies (24). This can be expressed as the minimum dividend constraint $\Pi_{bt} \geq \gamma N_{bt}$ from (15).

Consider the present discounted value of future dividends V_{bt} from (23) starting from an initial level of net worth N_{bt} after setting the current dividend Π_{bt}. The Lagrangian for maximizing V_{bt} with respect to dividend policy $\{\Pi_{bs}\}_{s=t+1}^{\infty}$ subject to the sequence of minimum-dividend constraints from $t+1$ onwards is

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty} P_{ts}\left\{\Pi_{bs} + \zeta_{bs}\left(\Pi_{bs} - \gamma N_{bs}\right)\right\}\right], \tag{62}$$

where ζ_{bs} is the state-contingent Lagrangian multiplier on the date-s minimum dividend constraint, expressed in current-value terms, which means it is multiplied by the stochastic discount factor P_{ts} without loss of generality. Other constraints on the bank and optimal choices of other variables such as M_{bs} and D_{bs} are accounted for directly when evaluating (62).

By adding and subtracting terms, the Lagrangian (62) is equivalent to

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty} P_{ts}\left\{(1+\zeta_{bs})\left(\Pi_{bs} + N_{bs}\right) - \left(1 + (1+\gamma)\zeta_{bs}\right) N_{bs}\right\}\right].$$

Defining variables Ω_{bt} and Ψ_{bt} as follows

$$\Omega_{bt} = \frac{\mathbb{E}_t\left[\Psi_{b,t+1}\left(N_{b,t+1} + \Pi_{b,t+1}\right)\right]}{\mathbb{E}_t\left[\Psi_{b,t+1}\right]}, \quad \text{where } \Psi_{b,t} = P_{t-1,t}(1+\zeta_{bt}), \tag{63}$$

and noting that $P_{t,s+1} = P_{ts}P_{s,s+1}$ (see 23), the law of iterated expectations implies the Lagrangian (62) can be rewritten as follows in terms of these new variables:

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t}^{\infty} P_{ts}\Psi_{b,s+1}\Omega_{bs} - \sum_{s=t+1}^{\infty} P_{ts}(1 + (1+\gamma)\zeta_{bs})N_{bs}\right]. \tag{64}$$

Conjecturing that Ψ_{bt} is the same for all banks b, it follows from the definition (63) that Ω_{bt} is the same as the static objective function (31) considered in section 4 with a common stochastic discount factor $\Psi_t = \Psi_{bt}$ used to calculated the risk-adjusted expected return $r_t = \mathbb{E}_t[\Psi_{t+1}R_{t+1}]/\mathbb{E}_t[\Psi_{t+1}]$.

The problem of maximizing Ω_{bt} conditional on net worth N_{bt}, taking the bank's dividend policy as given, has been studied in section 4. Optimizing over M_{bt} and D_{bt} subject to the balance sheet (25), the no-run condition (26), and accounting for the impact on future net worth (24), results in the maximized value of Ω_{bt} from (42) that is linear in initial net worth N_{bt} and with a common coefficient $1 + (r_t - \lambda i_t)/(1-\lambda)$ for all banks. Substituting into (64) yields a Lagrangian that depends only on the path of the bank's net worth:

$$\Upsilon_{bt} = \mathbb{E}_t\left[\sum_{s=t}^{\infty} P_{ts}\Psi_{s+1}\left(1 + \frac{r_s - \lambda i_s}{1-\lambda}\right)N_{bs} - \sum_{s=t+1}^{\infty} P_{ts}(1 + (1+\gamma)\zeta_{bs})N_{bs}\right], \tag{65}$$

which also uses the conjecture $\Psi_{bt} = \Psi_t$. The bank then determines the path of net worth $\{N_{bs}\}_{s=t+1}^{\infty}$ by choosing dividends $\{\Pi_{bs}\}_{s=t+1}^{\infty}$. The first-order conditions are $\partial\Upsilon_{bt}/\partial N_{bs} = 0$ for $s = t+1, t+2, \ldots$ in all states of the world, and the associated Kuhn-Tucker conditions $\zeta_{bs} \geq 0$, $\Pi_{bs} - \gamma N_{bs} \geq 0$, and $\zeta_{bs}(\Pi_{bs} - \gamma N_{bs}) = 0$. From (65), the first-order conditions are

$$1 + (1+\gamma)\zeta_{bs} = \left(1 + \frac{r_s - \lambda i_s}{1-\lambda}\right)\mathbb{E}_s[\Psi_{s+1}]. \tag{66}$$

This shows that the Lagrangian multiplier ζ_{bs} is the same for all banks if Ψ_{s+1} and r_s are common to all banks. Referring to (63), that earlier conjecture is confirmed when the Lagrangian multipliers are the same for all b, the common value being denoted ζ_t for a date-t constraint in what follows.

Using the Kuhn-Tucker conditions satisfied by the solution of the constrained maximization problem, the Lagrangian (62) equals the present-value of future dividends V_{bt} from (23). By substituting the first-order conditions (66) into the expression for Υ_{bt} in (65) for $s = t+1, t+2, \ldots$, only one non-zero term in N_{bt} remains. This establishes that the present-discount value of future dividends V_{bt} is linear in net worth N_{bt}:

$$V_{bt} = v_t N_{bt}, \quad \text{where } v_t = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)\mathbb{E}_t[\Psi_{t+1}] \text{ and } \Psi_{t+1} = P_{t,t+1}(1 + \zeta_{t+1}). \tag{67}$$

The coefficient v_t of net worth is the market-to-book ratio: the market value of a bank's future dividends divided by its ex-dividend net worth. This ratio is common to all banks. Comparison of v_t from (67) to the first-order conditions (66) shows that $v_s = 1+(1+\gamma)\zeta_s$ for $s \geq t+1$. From the Kuhn-Tucker conditions, the market-to-book ratio satisfies $v_s \geq 1$, and the date-s minimum-dividend constraint binds if $v_s > 1$. Considering the choice of the date-t dividend Π_{bt}, bank b wants to maximize $\Pi_{bt} + V_{bt} = (\Pi_{bt} + N_{bt}) + (v_t - 1)N_{bt}$, which uses (67). Since $\Pi_{bt} + N_{bt}$ depends on past decision (see 24), the minimum dividend constraint binds if $v_t > 1$.

It has been supposed that banks' choices do not lead to equity $N_{b,t+1}$ becoming negative for any realization of $Q_{b,t+1}$ in (43). Recall that recapitalization by the investment fund costs $1+\xi$ units of net worth in other banks for each unit of capital injected into an insolvent bank. First note that since $V_{bt} = v_t N_{bt}$ with $v_t \geq 1$ for solvent banks, this recapitalization reduces the present value of dividends the investment fund is able to distribute to households. Second, the leverage and liquidity choices of individual banks are not uniquely pinned down by the optimality condition that equalizes fragility. It follows that as long as aggregate bank net worth does not become negative in any state of the world, individual banks can choose leverage to ensure that net worth remains positive with probability one without having to take actions that reduce their present discounted value of dividends.

F Steady state

In this section, we analyse the long-run dynamics of the model by studying the steady state. The model's steady state is a constant sequence for prices and quantities that satisfies the model's equilibrium conditions. Along the steady state, the quantity of liquid assets M, i.e. the policy variable, is constant.

We look for a steady state with a strictly positive liquidity premium $\rho - i > 0$ and bank net worth $N > 0$. Combining equations (39) and (43), we obtain

$$q - \rho = \theta + 2\sqrt{\theta(\rho - i)} > 0. \tag{68}$$

Evaluating the formula for the banks' market-to-book ratio (45) in steady state, we obtain

$$v = \frac{\gamma(1+q)}{(1+\gamma)(1+\rho) - (1+q)} > 1, \tag{69}$$

which implies that the minimum dividend constraint is binding in steady state so that

$$\Pi = \gamma N. \tag{70}$$

Together with the law of motion for banks' net worth in (44), a binding minimum dividend constraint implies that in a steady state

$$q = \gamma \tag{71}$$

for $N > 0$.[70] First, we notice from (68) that a parametric restriction

$$\gamma > \rho + \theta \tag{72}$$

is necessary for (71) to be sustained with a strictly positive liquidity premium.[71] Under this restriction, we pin down the steady-state liquidity premium as

$$\rho - i = \frac{[\gamma - (\rho + \theta)]^2}{4\theta}. \tag{73}$$

This liquidity premium creates the right level of returns on bank net worth so that bank net worth is stable. Interestingly, it is independent of policy. We can determine the steady-state balance-sheet structure of banks with equations (25), (26) and (38) as

$$N = \left[1 - \lambda - \frac{\gamma - (\rho + \theta)}{2\theta}\left(\lambda + \frac{M}{K}\right)\right] K. \tag{74}$$

To have positive net worth in steady state, we need to restrict the equity friction with

$$\gamma \leq \rho + \theta \frac{2 - \lambda}{\lambda} \tag{75}$$

and policy with

$$M < \left[\frac{2\theta(1 - \lambda)}{\gamma - (\rho + \theta)} - \lambda\right] K. \tag{76}$$

An excessively strong equity friction makes it impossible to sustain positive net worth in steady state even with no liquidity. An excessively large supply of liquid assets rules out a fragile steady state with positive liquidity premium for any positive level of net worth.

The key finding that in the long run liquidity policy has no effect on the liquidity premium, and thus fragility, is due to the endogenous structure of banks' balance sheet. Increases in liquid-asset supply crowd out bank net worth in the long run to the point where fragility is unchanged.

As is standard in a real business cycle model, the steady-state risk-free rate is pinned down by the Euler equation in (20) as $\rho = (1 - \beta)/\beta$ and the steady-state level of

[70] The upper limit on M identified below rules out $N = 0$ and $q < \gamma$ in the steady state with strictly positive liquidity premium.

[71] If this is violated, then net worth grows up to the point where there is no fragility and the liquidity premium is zero.

capital is the unique strictly-positive solution to the system of equations given by

$$(1-\alpha)K^{\alpha(1+\frac{1}{\sigma})} = L^{\alpha+\frac{1}{\psi}}\left(ZL^{1-\alpha}-\delta\right)^{-\frac{1}{\sigma}} \tag{77}$$

and

$$K = \left(\frac{\alpha}{r-\delta}\right)^{\frac{1}{1-\alpha}} L. \tag{78}$$

G Solving the full macroeconomic model

The full macroeconomic model is represented by the following set of equations:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{79}$$

$$C_t + I_t = Y_t \tag{80}$$

$$(1-\alpha)\frac{Y_t}{L_t} = w_t \tag{81}$$

$$C_t^{\frac{1}{\sigma}} L_t^{\frac{1}{\psi}} = w_t \tag{82}$$

$$K_t = X_t A_{t-1} \tag{83}$$

$$A_t = (1-\delta)K_t + I_t \tag{84}$$

$$P_t = \beta\left(\frac{C_t}{C_{t-1}}\right)^{-\frac{1}{\sigma}} \tag{85}$$

$$\frac{1}{1+\rho_t} = \mathbb{E}_t P_{t+1} \tag{86}$$

$$R_t = \left(\alpha\frac{Y_t}{K_t} + 1 - \delta\right)\frac{K_t}{A_{t-1}} - 1 \tag{87}$$

$$r_t = \frac{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})R_{t+1}]}{\mathbb{E}_t P_{t+1}(1+\zeta_{t+1})} \tag{88}$$

$$A_t + M_t = D_t + N_t \tag{89}$$

$$N_t = \frac{1+Q_t}{1+\gamma} N_{t-1} \tag{90}$$

$$F_t = 1 - \lambda - \frac{\lambda N_t + (1-\lambda)M_t}{D_t} \tag{91}$$

$$Q_t = q_{t-1} + (R_t - r_{t-1})\frac{A_{t-1}}{N_{t-1}} \tag{92}$$

$$r_t = (1-\lambda)q_t + \lambda i_t \tag{93}$$

$$r_t - i_t = (1-\lambda)\left(\sqrt{\theta} + \sqrt{\rho_t - i_t}\right)^2 \tag{94}$$

$$j_t - \rho_t = \sqrt{\theta}\sqrt{\rho_t - i_t} \tag{95}$$

$$\rho_t - i_t = \theta\frac{F_t^2}{(1-F_t)^2} \tag{96}$$

$$V_t = \mathbb{E}_t[P_{t+1}(V_{t+1} + \Pi_{t+1})] \tag{97}$$

$$\Pi_t = \gamma N_t \tag{98}$$

The variables Z_t and X_t denote exogenous levels of TFP and capital quality. There is also an equation describing how the supply of liquidity M_t is determined, which could be exogenous or endogenous. In what follows, $\hat{}$ denotes the deviation of a variable from its steady-state value: a simple deviation for variables already measured as percentages, and log deviations for all other variables.

The production function (79) in log deviations is:

$$\hat{Y}_t = \hat{Z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t \tag{99}$$

Letting $\kappa = K/Y$ denote the steady-state capital-output ratio, since $I = \delta K$, the log linearization of the aggregate demand equation (80) is:

$$(1-\delta\kappa)\hat{C}_t + \delta\kappa\hat{I}_t = \hat{Y}_t \tag{100}$$

Labour demand (81) and supply (82) in log deviations are:

$$\hat{Y}_t - \hat{L}_t = \hat{w}_t \tag{101}$$

$$\frac{1}{\sigma}\hat{C}_t + \frac{1}{\psi}\hat{L}_t = \hat{w}_t \tag{102}$$

Since $X = 1$, the supply of capital to firms (83) and capital investment (84) have the following log-linear forms:

$$\hat{K}_t = \hat{X}_t + \hat{A}_{t-1} \tag{103}$$

$$\hat{A}_t = (1-\delta)\hat{K}_t + \delta\hat{I}_t \tag{104}$$

Using $P = \beta$ and $\rho = (1-\beta)/\beta$, the stochastic discount factor P_t from (85) and the implied risk-free rate (86) in terms of deviations from steady state:

$$\hat{P}_t = -\frac{1}{\sigma}(\hat{C}_t - \hat{C}_{t-1}) \tag{105}$$

$$\hat{\rho}_t = -(1+\rho)\mathbb{E}_t\hat{P}_{t+1} \tag{106}$$

Using $r = R = (\alpha/\kappa) - \delta$, the equations (87) and (88) for the ex-post and risk-adjusted expected returns on physical capital have the following approximate forms:

$$\hat{R}_t = (r+\delta)(\hat{Y}_t - \hat{K}_t) + (1+r)(\hat{K}_t - \hat{A}_{t-1}) \tag{107}$$

$$\hat{r}_t = \mathbb{E}_t\hat{R}_{t+1} \tag{108}$$

In terms of $n = N/(A+M)$ and $m = M/(A+M)$, the bank balance sheet (89) and accumulation of net worth (90) equations can be approximated as follows (noting $Q = \gamma$):

$$(1-m)\hat{A}_t + m\hat{M}_t = (1-n)\hat{D}_t + n\hat{N}_t \tag{109}$$

$$\hat{N}_t = \hat{N}_{t-1} + \frac{\hat{Q}_t}{1+\gamma} \tag{110}$$

The approximation of the equation for bank fragility (91) is:

$$\hat{F}_t = \frac{(\lambda n + (1-\lambda)m)(1-m)}{(1-n)^2}\hat{A}_t - \frac{(\lambda + (1-\lambda)m)n}{(1-n)^2}\hat{N}_t - \frac{((1-\lambda)(1-m)-n)m}{(1-n)^2}\hat{M}_t \tag{111}$$

Equations (92), (93) and (94) for the returns on bank assets and liabilities become:

$$\hat{Q}_t = \hat{q}_{t-1} + \frac{(1-m)}{n}(\hat{R}_t - \hat{r}_t) \tag{112}$$

$$\hat{r}_t = (1-\lambda)\hat{q}_t + \lambda\hat{i}_t \tag{113}$$

$$\hat{r}_t - \hat{i}_t = (1-\lambda)\left(1 + \frac{\sqrt{\theta}}{\sqrt{\rho - i}}\right)(\hat{\rho}_t - \hat{i}_t) \tag{114}$$

Equations (95) and (96) linking bank fragility, funding costs, and the liquidity premium have the following approximations:

$$\hat{j}_t - \hat{\rho}_t = \frac{1}{2}\frac{\sqrt{\theta}}{\sqrt{\rho - i}}(\hat{\rho}_t - \hat{i}_t) \tag{115}$$

$$\hat{\rho}_t - \hat{i}_t = 2\frac{\sqrt{\rho - i}}{\sqrt{\theta}}\left(\sqrt{\theta} + \sqrt{\rho - i}\right)^2 \hat{F}_t \tag{116}$$

The log linearization of the stock-market value equation (97) is:

$$\hat{V}_t = \frac{1}{1+\rho}\mathbb{E}_t\hat{V}_{t+1} + \frac{\rho}{1+\rho}\mathbb{E}_t\hat{N}_{t+1} - \frac{1}{1+\rho}\hat{\rho}_t \tag{117}$$

Together with these main equations, there are auxiliary equations defining other variables. The funding spread is $\hat{j}_t - \hat{\rho}_t$, the liquidity premium $\hat{\rho}_t - \hat{i}_t$, the credit spread $\hat{r}_t - \hat{i}_t$, the total size of banks' balance sheets $(1-m)\hat{A}_t + m\hat{M}_t$, the capitalization ratio $\hat{n}_t = n(1-n)(\hat{N}_t - \hat{D}_t)$, the liquidity ratio $\hat{m}_t = m(1-m)(\hat{M}_t - \hat{A}_t)$, and the market-to-book value ratio $\hat{v}_t = \hat{V}_t - \hat{N}_t$.