A Theory of Bank Balance Sheets*

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We present a theory of banking as the provision of means of payments–money, where the means of payments, to be used as such, must be stable in value. Trade requires the transfer of assets as payment, but due to information asymmetries, some risky assets are poor means of payment: they are illiquid and do not circulate. Banks swap those assets for their liabilities which can be designed to circulate although fully backed by assets that do not. This has implications for banks' balance sheets. Liquid assets, that could circulate on their own, are brought to the banks' balance sheet to enhance liquidity creation. Hence, liquid and illiquid assets are complementary factors for liquidity creation. We study banks' asset choice and liability design and implement it with standard instruments, such as saving deposits, time deposits, and bank loans. We argue that the optimal asset and liability composition resembles bank balance sheets in practice.

*All errors are ours.

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1 Introduction

In its essence, banking is about the swap of assets. When banks make loans or purchase securities, they acquire assets. In exchange, counterparts receive deposits, a bank liability. An evident distinction between the two sides of banks' balance sheets is that deposits are money: deposits circulate as a medium of exchange. A borrower granted loans, for example, obtains deposits she can use to make payments to others who, in turn, can subsequently use those same funds. A classic view of banking is that swapping assets is not mere accounting. Rather, money creation by banks is essential for economic activity.

The essential role of banks in creating money was first translated into formal modeling by Gorton and Pennacchi (1990). The idea, which is motivated by the issuance of private bank notes during the Free Banking era, is that economic trade is frictional due to adverse selection. Banks, however, can mitigate those frictions through the design of securities for their clients. Security design problems, e.g., those in DeMarzo and Duffie (1999) and Biais and Mariotti (2005), consist of structuring corporate liabilities fully backed by assets prone to adverse selection in ways that mitigate this problem. While security design is now a cornerstone of corporate finance, as a theory of money and banking, the theory is incomplete in three important dimensions. The goal of this paper is to complete the Gorton and Pennacchi (1990) security-design approach to money and banking along those three dimensions.

The first missing element from the security-design approach to money and banking is a notion of competition. Banks play a role as technical advisers or underwriters once a relationship is established, but they don't compete in a market for assets and liabilities. As a result, we lack predictions regarding the quantities and prices of the exchange of assets and liabilities. The problem is not trivial: banks must not only compete as underwriters of borrowers to their funding, but also to guarantee the use of their liabilities as a medium of exchange.

The second missing element from the security-design approach to money and banking is the coexistence of bank deposits and public money. As a result, we lack predictions regarding how the co-existence of deposits and outside money affects the security design function. It is unclear whether a competing medium of exchange constrains security design and how the safety of outside money enhances this process.

The third missing element from the security-design approach to money and banking is a notion of monetary circulation within the banking system. By the mid-1870s, for example, banks formed clearinghouses that issued common circulating notes. This institution remains alive today. In practice, when deposits circulate, banks are committed to absorbing each others' liabilities, something that does not occur with any other form of corporate liability. Yet, issuing a joint liability is likely prone to moral hazard. Because joint liability and circulation are missing from the security-design approach to money and banking, we lack an understanding of how limits to deposit circulation and bank integration interfere with their security design function.

Completing the theory along these dimensions is important to enhance our understanding of money and banking. For example, throughout the paper, we connect the predictions of the theory with the evolution of banking in the United States. It is also important for normative reasons. Along the paper, we connect the predictions of our theory with classical debates that have shaped monetary and financial regulation to this day. The paper proceeds in filling this gaps, one section at a time.

Core Environment. We embrace the Gorton and Pennacchi (1990)-view starting from the premise that many assets are illiquid due to asymmetric information. Banks purchase these illiquid assets in exchange for liabilities. Concretely, we consider a three-period two-state economy with an arbitrary distribution of assets in positive supply. Assets differ in terms of risk exposures providing a broad interpretation: some assets can be thought of as being cash, fixed-income securities, equities, mortgages, consumer or business loans. There is a continuum of producers with production opportunities who own assets, and a continuum of workers who supply the input for production, and labor. Producers and workers are matched bilaterally and anonymously. As in the literature following Lagos and Wright (2005), both producers and workers lack commitment. This precludes the use of credit in labor relations. Instead, producers must use assets as a means of payment. As in Rocheteau (2011), we assume that producers receive private information about their asset payoffs before production. This creates an asymmetric information problem, and implies that some risky assets may fail to be used for trade directly.

A key aspect of our theory is that to be used as a medium of exchange, assets must be sufficiently stable in value. In particular, high- and low-payoff states must not differ beyond a *liquidity coefficient*. To resolve the illiquidity of many assets in private exchanges, assets can be sold to banks in exchange for bank liabilities, before the arrival of private information. Following the spirit of the security design literature, banks can design liabilities with different payoff structures. These liabilities will circulate if their payoffs do not differ beyond the *liquidity coefficient*. Unlike the security design literature, we study equilibria in the market where bank assets and liabilities are traded freely.[1]

A Market for security design. When security-design takes place in a competitive market, we show that absent other frictions, banks will issue a unique liquid liability in exchange for all the assets in the economy. By contrast, what changes across assets with different liquidity properties is the price at which the bank liability is exchanged for them. Our theory also predicts that banks will not only purchase illiquid assets in exchange for liquid liabilities but also that they will purchase assets for which adverse selection is hardly a problem, as occurs in practice. That is, the purchase of liquid is a complementary input for the creation of liquidity out of illiquid assets.

Although we consider a market that involves trading of securities, as in Arrow-Debreu economies, the outcome is realistic. We show that, it can be implemented using a combination of checking deposits issued by banks and used for bilateral trades and over-collateralized non-recourse loans that implement the purchase of securities.[2]

Regarding the liquidity premia, assets that are liquid can may trade at a price above their expected value. We show that there are two fundamental securities that price all assets, just as in Arrow-Debreu economies: a fundamental liquid and illiquid securities. All assets are convex combination of these securities.

Liquidity premia arise depending on aggregate conditions. If the aggregate portfolio

[1]To explain the liability design problem, consider the simplest case first: when the banker makes an offer to a single producer holding some illiquid asset. The offer is a swap of the illiquid asset for a liquid liability. The banker profits from this swap because it issues a liability with a lower average payoff than the asset. In agreeing to this swap, the producer loses payoffs, but he gains liquidity. Namely, the banker can structure the state-contingent payoffs of the liability to make it safer than the underlying asset.

[2]The banker gives the producer a given quantity of deposits, the loan size, and the producer promises to repay a given quantity of consumption good, the face value, leaving his tree to the banker as collateral. The loan is over-collateralized, in that the face value is lower than the payoff of the tree in the high state. By making the loan over-collateralized, the banker effectively gives the producers an illiquid liability: an option to repurchase the tree at a strike price equal to the loan's face value. We further show that our two-state economy naturally extends to the continuum case.

of securities is liquid, there are no liquidity premia. If the aggregate portfolio is illiquid, then the distribution of asset endowments between banks and non-banks agents matters. For example, if banks are sufficiently wealthy, even if the non-bank sector holds illiquid securities, liquidity premia will not be present.

Private money creation is constrained efficient even if banks issue a unstable liability. The benefit of deviating from perfect stability is that the amount of perfectly-safe means of payments that can be issued is limited. This limit can be quite stringent and cause a large welfare loss in the form of unrealized mutually-beneficial transactions. This result underscores the potential welfare cost of regulations on the riskiness of bank deposits such as narrow banking proposals.[3]

Competition with other forms of money. To study the co-existence of private and public money, the second missing piece in the security-design approach to money and banking, we extend the core environment. Motivated by the historical co-existence of gold and banknotes as medium of exchange,we add geographical separation and a special "safe asset" with the assumption that bank liabilities are only used at the bank's location while the safe asset can be used everywhere. In equilibrium, the safe asset can be used by the producer to trade, but it can also be deposited in the banks in exchange for bank liabilities. We find that as bank liabilities are more widely accepted, the safe asset ends up in the vault of the bank; a more advanced banking system features higher usage of inside money relative to outside money. Perhaps surprisingly, we find that the value of the safe asset need not be decreasing in the acceptability of the bank liabilities, as one would expect if the safe asset and the bank liabilities are viewed as substitute means of payments. In the case where bank liabilities are scarce, for lack of assets that can back their issuance, the value of the safe asset can increase because it is an input into the provision of bank liabilities.

Limits to Deposit Circulation and Bank Integration. The problem of recognizability, that leads to a meaningful tension between private money issuance and public money is resolved by the emergence of joint liability, as in the historical accounts of private clearing houses. To study joint liability, the third missing piece in the security-design approach to

[3]For a more recent example, see proposed regulations for stablecoins see, for instance, the Bank of England's Discussion Paper: Regulatory regime for systemic payment systems using stablecoins and related service providers.

money and banking, we study a version of the model with a moral hazard constraint on the banks in a meaningful way. We assume the bank cannot abscond their assets but can overissue liabilities to acquire goods. This results in an incentive constraint that limits the value of the bank's liabilities. This extension leads to two distinct types of liquidity premia. On one hand, there is the liquidity premium due to aggregate conditions that we find even without moral hazard. This liquidity premium is present in the liquid reserve assets that the bank holds. On the other hand, there is a liquidity premium on the bank liabilities even after accounting for the premium on the bank's reserve assets. This premium is due to the bank's incentive constraint which prevents the bank from eliminating it.

The Monopoly Bank. Finally, we also examine the case of a monopolist bank. Market power is motivated by the idea that frictions in the issuance of joint liability are a force toward bank integration which will naturally lead to a market-power force. In this case, we study a monopolist bank with an optimal asset choice problem and a convex intermediation cost to acquire different types of assets.

The optimal liability design in this case is as follows: the monopoly banker will issue as many liquid liabilities as possible out of a given asset. It extracts the liquidity premia as rents. When private sector's asset pool is very risky, the liquidity the banker can create is insufficient to compensate the producer for surrendering his asset. In that case, the banker has to compensate those producers by issuing additional liabilities that do not circulate. This shows that market power is a force that induces inefficient liquidity creation to extract rents.

This extension highlights a tension between resolving moral-hazard problems by integrating banks vis-a-vis market power, that limits liquidity creation.

Bank Balance Sheets. Clearly, many factors that may affect balance sheet composition in practice are left out of our theory, such as banking regulation, maturity composition, or expertise. Yet, we argue that our theory goes in the direction of explaining qualitative observations about bank balance sheets along two dimensions: the asset and liability composition, and the holding shares by asset classes, out of the total supply outstanding.

We use data corresponding to all Chartered Depositary Institutions in the United States for two particular quarters and total supply outstanding for several asset classes. The data is obtained from the Flow of Funds, for the last quarters of 1985 and 2005.[4]

Figure 1 reveals that banks' liabilities are, for the most part, made up of checking deposits, time deposits and equity. Notice that checking deposits correspond to over 50-60% of the liabilities, while time deposits only represent 10-20%. Hence, checking deposits are much larger than time deposits. In the context of our model, this corresponds to the finding that bankers strive to create liquid liabilities rather than illiquid ones. Interbank liabilities are also important, but we argue that they would arise in our model if we introduced heterogeneous banks.

Figure 1 shows the asset composition. It reveals that over 70% of the assets on the left-side of the balance sheets are loans. The rest are tradable assets, such as Treasuries, Corporate Bonds, or Equities. In the context of our model, this corresponds to the observation that the banker finds it optimal to hold more illiquid assets than illiquid ones. Figure 2 shows the *fraction* of the outstanding supply held by banks, by asset classes. It shows, as suggested by the model, that banks hold a larger fraction of the total supply of illiquid than of liquid assets. Within liquid assets, banks hold a larger fraction of the supply of safer assets (fixed income) and a much smaller faction of the supply of riskier assets (equities)

Literature Review. Because we focus on the design of means of payment, our work is related to the money and payment literature following Lagos and Wright (2005). Berentsen, Camera, and Waller (2007) have studied the role of banks in helping agents insure against preference shocks: banks reallocate idle balance from agents who want to consume, towards agents who do not. In contrast, our model does not generate any such demand for insurance. Another closely related paper is Rocheteau (2011) who study bilateral trades with multiple assets in the presence of asymmetric information. We complement Rocheteau's work by adding a security design problem. We find that agents do not trade with assets directly. Instead, they find it optimal to deposit these assets in banks, and they trade by exchanging bank liabilities.

Farhi and Tirole (2015) consider the trade-off between tranching and bundling an asset in a model of bilateral asset trade with asymmetric information, with a focus on endogenous information acquisition. Tranching means re-structuring the payoff of some underlying as-

[4]We choose the same quarter because it is a simple way to control for seasonality. We focus on years that pre-date the 2007-2009 financial crisis, since monetary policy has changed dramatically since then, with substantial impact on banks' behavior.

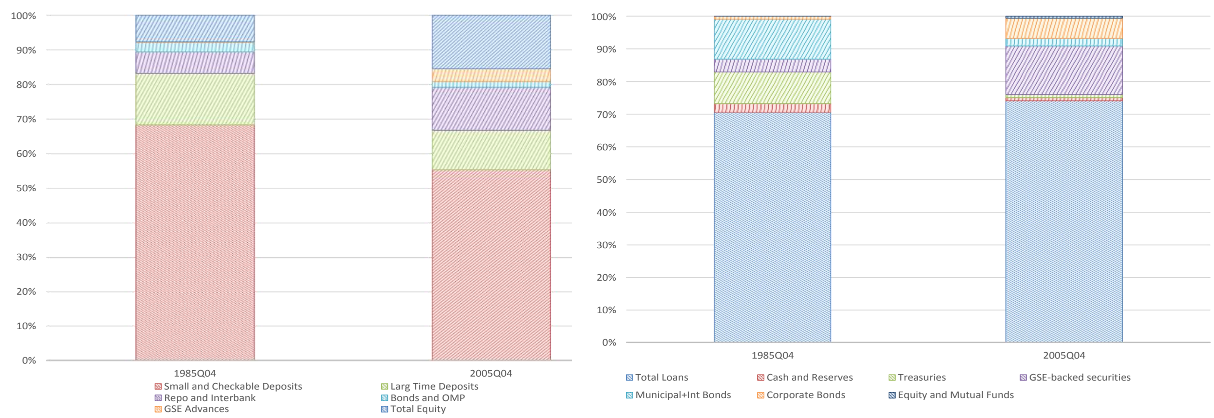


Figure 1: Typical asset and liability composition.

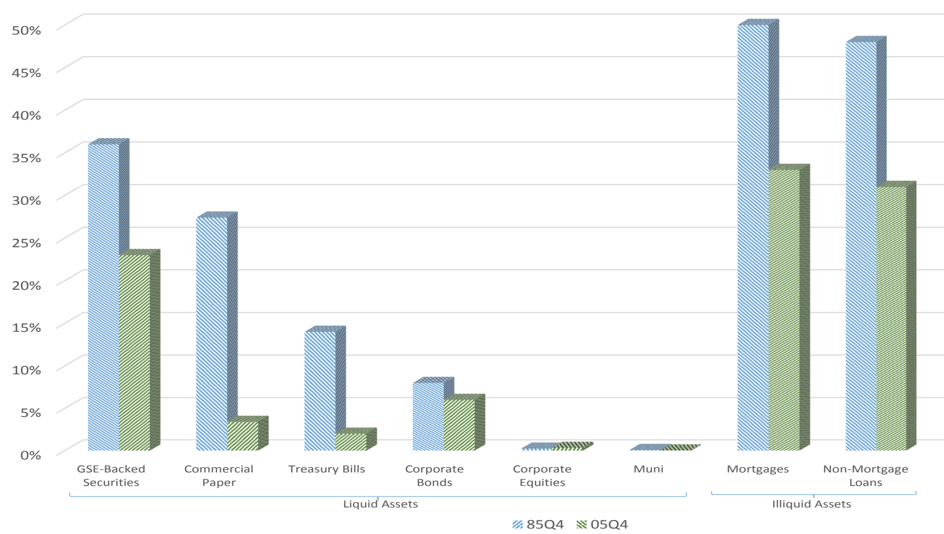


Figure 2: Asset composition relative to aggregate supply outstanding, by asset class.

set to create two assets for bilateral trade, a safe asset and a maximally risky asset. Bundling means keeping the underlying asset for bilateral trade. They derive conditions under which bundling leads to more trade than tranching. In contrast, we introduce multiple assets and study optimal asset choice and liability design. In that context, there is no binary choice between bundling and tranching: instead, the optimal balance sheet involves a combination of both. Bankers bundle multiple assets on the left side of their balance sheet, and find the optimal way to tranche the bundle to induce trade.

Our work complements the recent paper of Dang, Gorton, Holmström, and Ordonez (2017). They show that banks are optimally opaque about their balance sheet and design debt-like liabilities which reduce agent's incentive to acquire private information. They show that banks seek to hold safe assets to design securities minimizing incentives to acquire private information. We develop a different model in which we abstract from bank opacity and private information acquisition. This leads to a different liability design problem, and a different demand for safe assets. We derive predictions for the optimal design of a bank's balance sheet, in particular for banks' optimal asset holdings at all points of the liquidity and safety spectrum.

Finally, our model is related to the vast security design literature in corporate finance, such as DeMarzo and Duffie (1999) and Biais and Mariotti (2005), just to name a few. Our contribution relative to this literature is to study the design of means of payments backed by an optimally chosen portfolio of assets.

2 Model

2.1 The economic environment

There are three dates, $t \in \{0, 1, 2\}$, and two states, $\omega \in \{\ell, h\}$, with probability $\pi(\omega)$. The assumption of two states is without loss of generality.

Assets. There is a continuum of risky assets, "trees", in positive supply. Trees are heterogeneous and feature state-dependent payoffs that are realized at $t = 2$. Trees are indexed by a vector $R \in \mathbb{R}^2_+$ where the first and second coordinates correspond to the payoff of R in the low, $\omega = \ell$, and high, $\omega = h$, states respectively. We denote the payoff of tree R in state

ω as $R(\omega)$. The payoffs, which we call fruit, are always in consumption units. Without loss of generality, we assume that summing across all trees, high-state payoffs exceed low-state, although individually some trees may feature greater low-state payoffs.

Trees represent broad asset classes, such as cash, bonds, fixed-income securities, or equity. Trees are divisible, and property rights over entire trees can be traded.

Agents. Three types of agents populate the economy: a continuum of producers, a continuum of workers, and a continuum of bankers. All agents are risk neutral and only enjoy consumption at $t = 2$.

At $t = 0$, producers are endowed with tree portfolios. At $t = 1$, each producer operates a linear production technology, whereby q labor units yield ρq output units at $t = 2$, regardless of the state. Labor is supplied by workers at a marginal cost, normalized to one. We assume that the producers and workers lack commitment which precludes the use of credit in bilateral trades, even if the tree can be used as collateral.[5] Lack of commitment induces the use of assets as means of payment as is common in the monetary literature, e.g. Lagos and Wright (2005).

Bankers are also endowed with a portfolio of trees but can neither access the production technology nor supply labor. Instead, unlike producers and workers, a banker can commit to making future state-contingent payments. As a result, the banker is in a unique position to issue liabilities. We can argue that bankers are special because they can commit or, equivalently, have the legal skills to write enforceable contracts.

Importantly, producers have private information about the realization of ω at $t = 1$.

Three stages. The timing of the model is as follows:

- $t = 0$: centralized exchange. Bankers and producers exchange trees for state-contingent securities issued by the banker in a centralized exchange. In doing so, bankers design which liabilities to issue in exchange for trees.

- $t = 1$: bilateral trade. Producers learn the aggregate state, ω, and are bilaterally and anonymously matched with workers. The worker does not know the state. Labor is traded in exchange for trees or banker liabilities.

[5] Using the tree as collateral for state-contingent promises requires a commitment by the worker to return the tree in all states

- $t = 2$: payoff, settlement, and consumption. Output is produced, all trees pay off, contracts with bankers are settled, and all agents consume.

Assumptions. Throughout, we assume: (i) $\rho > 1$ which means that there are always gains from trade between the producer and the worker; (ii) $\pi(h)\rho < 1$ and $\pi(\ell)\rho < 1$, which ensure that some assets suffer from a lemon problem in bilateral trade, in either state. As will become clear, this creates gains from trade between the producers and banks.

2.2 Monetary Exchange

In this section, we study the bilateral trade between the producer and the worker in which the producer makes payment using an arbitrary state-contingent security. The security is either a tree purchased by the producer in a competitive market or a liability issued by the banker backed by trees. This trade is a key building block: it determines the value of using alternative means of payment to purchase labor. Afterward, we study the exchange between bankers and producers of trees for securities.

Workers compete by offering, for each security $D(\omega)$, a menu of quantities of labor input q in exchange for a quantity $n \in [0, 1]$ of the security. Trading is not exclusive; the producer is not restricted to hiring only one worker. Once the producer learns the aggregate state, he chooses among the workers' offers subject to his holdings of the security.[6]

In the unique equilibrium of this trading game, the workers post linear price schedules for each security. Specifically, securities with riskier payoffs are priced at their lowest value, while securities with safer payoffs are priced at their expected value.

Given a security $D(\omega)$, let $q(\omega), n(\omega)$ be the trade chosen by the producer in state ω. Then, the ex-ante value of the security for the producer is given by

$$\mathbb{U}\left[D\right] \equiv \mathbb{E}\left[\rho q(\omega) + \left[1 - n(\omega)\right] D(\omega)\right]. \tag{1}$$

Liquidity Classification. The following proposition characterizes the marginal value of the security:

[6]This is the kind of environment studied of Attar, Mariotti, and Salanié (2011). There are alternative ways of modeling the bilateral trade, for instance with signaling via retention as in DeMarzo and Duffie (1999) and applying the intuitive criterion. These alternatives do not affect most of our results. Moreover, non-exclusivity is a natural assumption to make in the context of assets that are media of exchange.

Proposition 1 *For a producer, the ex-ante value of a security.* $\mathbb{U}[D]$*, the contracted labor,* $q(\omega)$*, and the pattern of trade,* $n(\omega)$*, will depend on the security's payoffs according to the following classification:*

- *Liquid securities:*

$$\textit{if } D(h) \in [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]: \quad n(h) = n(\ell) = 1 \quad \textit{and} \quad \mathbb{U}[D] = \mathbb{U}^{\ell}[D] := \rho \mathbb{E}[D].$$

- *Illiquid securities:*

$$\textit{If } D(h) > \psi_H \cdot D(\ell): \quad n(h) = 0, n(\ell) = 1 \quad \textit{and} \quad \mathbb{U}[D] = \mathbb{U}^{i,h}[D] := \pi(h)D(h) + \pi(\ell)\rho D(\ell),$$
$$\textit{if } D(h) < \psi_L \cdot D(\ell): \quad n(h) = 1, n(\ell) = 0 \quad \textit{and} \quad \mathbb{U}[D] = \mathbb{U}^{i,\ell}[D] := \pi(h)\rho D(h) + \pi(\ell)D(\ell).$$

Thus, the ex-ante value of an illiquid security as

$$\mathbb{U}^{i}[D] := \begin{cases} \mathbb{U}^{i,h}[D], \textit{ for } D(h) > D(\ell) \\ \mathbb{U}^{i,\ell}[D], \textit{ for } D(h) < D(\ell) \end{cases}$$

The proposition shows that securities are segmented into liquid and illiquid securities according to a set of *liquidity coefficients*,

$$\psi_H \equiv \frac{\rho\pi(\ell)}{1-\rho\pi(h)} > 1, \qquad \psi_L \equiv \frac{1-\rho\pi(\ell)}{\rho\pi(h)} < 1.$$

Proposition 1 classifies securities into illiquid and liquid securities depending on whether they satisfy a liquidity condition: $D(h) \in [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]$. The liquidity condition implies that the state contingent payoffs $\{D(\ell), D(h)\}$ must fall in the cone blue vertical lines in Figure 3.

Illiquid securities violate the liquidity condition and fall either in the top or bottom regions, marked with red horizontal lines in Figure 3. Bilateral trade breaks down for such payoffs, and $n(h) = 0$. The security provides payoffs but does not provide liquidity services. The reason for this breakdown is private information: after learning the state, the producer's value of holding on to the security is larger than the value of purchasing the worker's labor at a pooling price, $D(h) > \rho \cdot \mathbb{E}[D] \Longleftrightarrow D(h) > \psi_H \cdot D(\ell)$, a classic lemons market condition. The lack of bilateral trade with these securities implies that $\mathbb{U}[D]$ is below $\rho \cdot \mathbb{E}[D]$, the value obtained when trade is efficient.

Liquid securities satisfy the liquidity condition and are always traded for labor, $n(\omega) = 1$. For liquid securities, the optimal bilateral trade is pooling: the producer sells its security in exchange for labor at the expected value $\mathbb{E}[D]$ in both states. The security price is lower than its underlying value in the high state. Hence, the producer can only purchase $\mathbb{E}[D]$ labor units which is less than the true value of the security, $D(h)$.

In summary, securities are liquid if their payoffs do not differ too much across states. Geometrically, their payoffs must fall within a cone of $\mathbb{R}^2_+$ that dictates their "information sensitivity", that is, their use in the bilateral exchange after producers learn the state.

Naturally, we can think of $\mathbb{U}[D]$ as an indirect utility associated with holding a security D. Figure 3 plots an indifference curve associated with $\mathbb{U}[D]$, with respect to the payoffs $\{D(\ell), D(h)\}$. The characterization shows that $\mathbb{U}[D]$ is increasing and concave in payoffs as long as the security is liquid, when $D(h) \in [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]$. In Figure 3, this corresponds to the green region marked with horizontal lines and the blue region marked with vertical lines. When $D(h) \notin [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]$, trade breaks down and $\mathbb{U}[D]$ jumps down discretely. Hence, the indifference condition features a discontinuity at the points at which the security becomes illiquid. At those points, the indifference curve shifts to higher payoffs that compensate the producer for lacking a liquidity service as a medium of exchange.

Without asymmetric information, the producer would be indifferent between securities with the same expected payoffs, and, in that case, his indifference curves would be flat. With private information, his indifference curve features discontinuities precisely at the boundaries of the cone that defines liquid securities. The discontinuities indicate that the producer prefers a security that pays slightly less in expectation but satisfies the liquidity condition—less in the high state in the case of the upper boundary and vice versa. Indeed, if he could commit ex-ante, the producer would want to reduce the payoff of his security to exploit the gains from trade. Structuring state-contingent payoffs ex-ante to avoid this commitment problem is the essence of security design.

So far, we have been silent about the actual securities producers use as a medium of exchange. The following section describes an environment where securities are claims on banks fully backed by trees. Thus, banks will provide a security design service.

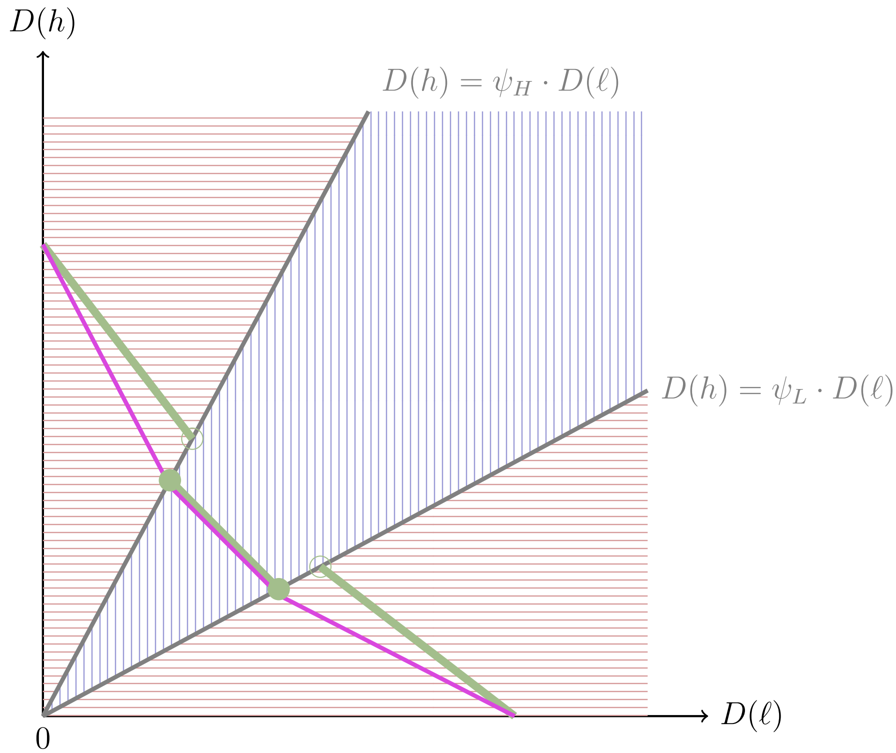


Figure 3: In green, the producer's indifference curve for an individual security. The indifference curve for a portfolio is depicted in purple. It coincides with the individual indifference curve in the liquid region.

The role of banks: An example. As a prelude to the following section, we describe how re-structuring the property rights over a securities payoff could enable its circulation: A security for which $D(h) > \psi_H \cdot D(\ell)$, does not circulate because their payoff in the high state is too large. If someone could dismantle the security into two securities. One that pays $\{0, D(h) - \psi_H \cdot D(\ell)\}$ and one that pays $\{D(\ell), \psi_H \cdot D(\ell)\}$. Whereas the first security cannot circulate, the second one can. Producers cannot restructure payoffs in our framework, but a bank can. Indeed, the bank has the ability to dismantle the assets from a single security, creating a security that circulates and one that doesn't, to maximize the liquidity potential of securities. It turns out that the bank does not have to dismantle a security in isolation. It can create liabilities backed by entire portfolios of trees. The next section is devoted to understanding this process: what trees are brought to the bank, what securities are issued, and at what prices.

Information Sensitivity and Liquidity. In this model, an asset can be used as means of payments if it is relatively safe. Dang, Gorton, Holmström, and Ordonez (2017) describe the defining characteristic of such an asset as information (in)sensitivity in the presence of information acquisition. These ideas share more than a passing resemblance. Indeed, if we consider a richer setting in which the investor can acquire information about the state of the world, the investor's belief distribution over payoffs will lie in the liquid region for any liquid security. Therefore, any of our liquid securities are information insensitive in the sense of Dang, Gorton, Holmström, and Ordonez (2017).

Consider a variation on the environment with information variables. Let the state space be $\Theta \times \Omega = \{a, b\} \times \{\ell, h\}$. The state variable $\theta \in \{a, b\}$ is an information variable (i.e., a signal), while the state variable ω is a payoff-relevant variable. In other words, the payoff of a tree $R((\theta, \omega)) = R(\omega)$ as defined before, while ω and θ are not necessarily independently distributed. Assume that at $t = 1$, only the producer learns the value of θ and no one learns the value of ω.

It is trivial to see that if θ is perfectly informative about ω, the environment is identical to our model. Moreover, the solution of our model applies directly with some change of variables: $\pi(\ell), \pi(h)$ must be changed for $\Pr(\theta = a), \Pr(\theta = b)$ and $R(\ell), R(h)$ must be changed for $\mathbb{E}[R(\omega)|\theta = a] =: R(a)$ and $\mathbb{E}[R(\omega)|\theta = b] =: R(b)$. We wish to show that a liquid asset in the original model would be liquid in the information-expanded environment.

Let $D(\omega)$ be a liquid security and, without loss of generality, let $D(a) \geq D(b)$. Since $D(a)$ is an expectation over $D(h)$ and $D(\ell)$, then $D(a) \leq \max\{D(\ell), D(h)\}$. Since the security is liquid $\max\{D(\ell), D(h)\} \leq \rho\mathbb{E}[D(\omega)]$. By the law of iterated expectations, $\mathbb{E}[D(\omega)] = \mathbb{E}[D(\theta)]$. Therefore, $D(a) \leq \rho\mathbb{E}[D(a)]$ and the security must be liquid in the new information-expanded environment.

3 Competitive liability design

We now study the exchange of trees for bank liabilities. Producers can sell their trees for bank liabilities, which they can hold to maturity or use in trade. Bank liabilities are also indexed by some $D \in \mathbb{R}_+^2$, with the same interpretation: the first and second coordinates denote $t = 2$ payoffs in the low state and high states. From the previous section, we know that bank liabilities are liquid if they satisfy the liquidity condition: $D(h) \in [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]$ and are illiquid otherwise. We denote prices in terms of goods.

Producer and banker problems. Throughout the paper, we express asset portfolios as measures on $\mathbb{R}^2$—we use the terms portfolio and measures interchangeably. At $t = 0$, the producer and banker enter the period with respective portfolios μ_0^e and μ_0^b, respectively. They exchange trees for bank liabilities in a centralized exchange. The producer sells his assets and chooses portfolios of illiquid and liquid bank liabilities and trees, $\{\mu_1^i, \mu_1^\ell\}$. In turn, the banker buys a portfolio of trees μ_1^+ and issues a portfolio of liabilities μ_1^-. A tree R and bank liability D are traded at at prices $P(R)$ and $P(D)$, respectively.

At this stage, the producer solves:

Problem 1 (Producer Problem)**.**

$$\max_{\{\mu_1^i, \mu_1^\ell\} \geq 0} \int \mathbb{U}^\ell [D] \, d\mu_1^\ell(D) + \int \mathbb{U}^i [D] \, d\mu_1^i(D) \tag{2}$$

subject to:

$$\int P(D) \, d\mu_1^i(D) + \int P(D) \, d\mu_1^\ell(D) \leq \int P(R) \, d\mu_0^e(R). \tag{3}$$

Equation (12) is the producer's budget constraint: The right-hand side is the value of his initial endowment, calculated by integrating the portfolio measure against the price function $P(R)$. These funds are used to purchase a portfolio of illiquid liabilities, μ_1^i, and liquid liabilities μ_1^ℓ. His objective is to maximize the expected payoffs. The illiquid portfolio stays with the producer until maturity. Thus, the expected benefit of holding those securities is lower than that of holding liquid securities: $\mathbb{U}^i [D] < \mathbb{U}^\ell [D]$.

The lack of commitment on the producer's side is encoded in the assumption that he cannot issue securities. This is implicit in that measures, the portfolios, are positive. That assumption means the producer cannot trade using a security he does not own.

The banker's problem is similar to the producers's, except for two important distinctions. First, the banker lacks trading opportunities with workers. Second, the banker can issue liabilities, which allows him to restructure (securitize) trees. However, the banker can issue liabilities but subject to a limited liability constraint.

Problem 2 (Banker problem)**.**

$$\max_{\left\{\mu_1^+,\mu_1^-,c(\omega)\right\}\geq 0} \mathbb{E}\left[c\right] \tag{4}$$

subject to:

$$\int P\left(D\right) d\mu_1^+(D) \leq \int P\left(D\right) d\mu_1^-(D) + \int P\left(R\right) d\mu_0^b(R). \tag{5}$$

$$\forall\omega, \quad c\left(\omega\right) + \int D\left(\omega\right) d\mu_1^-(D) \leq \int D\left(\omega\right) d\mu_1^+(D). \tag{6}$$

At $t=0$, the banker enters with net worth $\int P\left(R\right) d\mu_0^b(R)$, and raises funds by issuing a portfolio of bank liabilities, $\mu_1^-(D)$. Among these, some liabilities are liquid, and some are illiquid, depending on their payoffs. The budget constraint (20) says that these funds are used to purchase a new menu of trees, $d\mu_1^+(D)$, which will include some trees or possibly other bankers' liabilities. Equation (21) is a budget constraint at $t=2$ that holds in each state ω. With the payoffs he collects, the banker pays his liabilities and consumes $c\left(\omega\right)$. Importantly, $c\left(\omega\right)\geq 0$ means the banker cannot produce goods. This assumption is important to guarantee that resources are not brought to some states artificially to enhance overall liquidity. This condition is also a limited liability constraint.

Competitive Equilibrium Next, we define a competitive equilibrium.

Definition 1. A competitive equilibrium is a price $P:\mathbb{R}_+^2\rightarrow\mathbb{R}_+$ and portfolios for producers and bankers $\left\{\mu_1^i\left(\cdot\right),\mu_1^\ell\left(\cdot\right)\right\}$ and $\left\{\mu_1^+\left(\cdot\right),\mu_1^-\left(\cdot\right)\right\}$ which satisfy two conditions,

1. The producer and banker portfolios are solutions to their problems.

2. The market for every security clears: For all Borel sets $B\in\mathbb{R}_+^2$

$$\int_B d\mu_1^i(D)+\int_B d\mu_1^\ell(D)+\int_B d\mu_1^+(D)\leq\int_B d\mu_1^-(D)+\int_B d\mu_0^e(D)+\int_B d\mu_0^b(D). \tag{7}$$

The definition is standard, although the market-clearing condition merits discussion. The right-hand side is the supply of securities. The condition says that the supply of security characterized by the vector D is either a tree—part of the banker's and producer's endowments—or issued by a bank. These securities must be held by either banks or producers as part of the liquid or illiquid portfolios.

3.1 Characterization

Notation. It is convenient to introduce notation for two Arrow-Debreu securities—henceforth, A-D securities—that span the space of securities: $e^{\ell} \equiv \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $e^{h} \equiv \begin{bmatrix} 0 & 1 \end{bmatrix}$. As is known, any security can be expressed as a linear combination of the two A-D securities. A special security is the normalized security at the liquidity boundary. We call this the *marginally liquid security*, which pays 1 unit of consumption in the low state and ψ_H in the high state:[7] $e^{\psi_H} = \begin{bmatrix} 1 & \psi_H \end{bmatrix} = e^{\ell} + \psi_H \cdot e^{h}$. Finally, another special security is the perfectly safe security, which pays 1 unit of consumption in each state: $e^{s} = \begin{bmatrix} 1 & 1 \end{bmatrix} = e^{\ell} + e^{h}$.

It is also convenient to normalize the price of the security that pays one consumption unit in the high state, $P\left(e^{h}\right) = \pi$. To further simplify the notation, we denote by $q \equiv P\left(e^{\ell}\right)$, the price of the A-D security.

Both of the producer's and banker's portfolios imply an overall position in terms of consumption good in the low and high state. We denote the producer's position as $(\mathbf{D}(\ell), \mathbf{D}(h))$ and the banker's position as $(\mathbf{C}(\ell), \mathbf{C}(h))$. Specifically,

$$\mathbf{D}(\ell) = \int D(\ell) d\left(\mu_1^i + \mu_1^{\ell}\right), \qquad \mathbf{D}(h) = \int D(h) d\left(\mu_1^i + \mu_1^{\ell}\right),$$

$$\mathbf{C}(\ell) = \int D(\ell) d\left(\mu_1^+ - \mu_1^-\right), \qquad \mathbf{C}(h) = \int D(h) d\left(\mu_1^+ - \mu_1^-\right).$$

Finally, we denote by $N(\ell)$ and $N(h)$ the aggregate resources in the low and high state,

[7] Similarly, we can define a marginally liquid security that pays more in the low state. That security would pay 1 unit of consumption in the low state and ψ_L in the high state. Since we assume that the aggregate payoffs will be larger in the high state, this security will not be relevant in equilibrium.

respectively,

$$N(\ell) \equiv \int R(\ell)\left(d\mu_0^e + d\mu_0^b\right) \quad \text{and} \quad N(h) \equiv \int R(h)\left(d\mu_0^e + d\mu_0^b\right). \tag{8}$$

Notice that the market-clearing condition can be written in terms of the overall positions $(\mathbf{D}(\ell), \mathbf{D}(h))$ and $(\mathbf{C}(\ell), \mathbf{C}(h))$. This follows directly from equation (7) specializing the Borel set to $\mathbb{R}^2$:

$$\mathbf{D}(\ell) + \mathbf{C}(\ell) = N(\ell), \tag{9}$$

$$\mathbf{D}(h) + \mathbf{C}(h) = N(h). \tag{10}$$

With the notation in hand, we now characterize the individual problems.

Characterization of Individual Problems. We begin with an important equilibrium property of security prices: no-arbitrage.

Proposition 2 (Non-Arbitrage pricing) *In any equilibrium, prices satisfy a non-arbitrage property: any security is priced at its replication cost (in the Arrow-Debreu basis):*

$$P(D) = qD(\ell) + \pi D(h).$$

The proposition establishes that equilibrium prices are arbitrage free and it follows from the banker's problem. If this property didn't hold, the banker could always increase its consumption in some state, contradicting the market-clearing condition. As a result, the banker has nothing to gain from the creation of securities. The following result is immediate:

Proposition 3 *Given non-arbitrage prices, the banker's problem can be solved as if the banker chose directly its aggregate portfolio* $(\mathbf{C}(\ell), \mathbf{C}(h))$. *In particular, the banker solves*

$$\max_{\{\mathbf{C}(\ell), \mathbf{C}(h)\} \geq 0} (1 - \pi)\mathbf{C}(\ell) + \pi \mathbf{C}(h)$$

subject to a single budget constraint:

$$q\mathbf{C}(\ell)+\pi\mathbf{C}(h)=n^b\equiv\int P\left(R\right)d\mu_0^b=qN^b(\ell)+\pi N^b(h).$$

This result implies that we can solve the problem as if the banker decides only to purchase A-D securities. This proposition is convenient to characterize the banker's optimal consumption as a function of the relative price of the two A-D security prices.

Clearly, the marginal rate of substitution for the banker is constant and equal to $\pi/(1-\pi)$. The banker's optimal overall portfolio is a corner solution; unless $q=1-\pi$, in which case the banker is indifferent. In summary,

Proposition 4 *The banker's optimal overall portfolio is given by*

- *Underpriced liquidity ($(1-\pi)>q$)*

$$\mathbf{C}(\ell)=N^b(\ell)+\frac{\pi}{q}N^b(h),\mathbf{C}(h)=0.$$

- *Fairly priced liquidity ($(1-\pi)=q$)*

$$q\mathbf{C}(\ell)+\pi\mathbf{C}(h)=qN^b(\ell)+\pi N^b(h).$$

- *Overpriced liquidity ($(1-\pi)<q$)*

$$\mathbf{C}(\ell)=0,\mathbf{C}(h)=N^b(h)+\frac{q}{\pi}N^b(\ell).$$

Next, we study a modified version of the producer's problem, one where we are explicit about prices. As with the banker, we denote by

$$n^e\equiv\int P\left(R\right)d\mu_0^e=qN^e(\ell)+\pi N^e(h),$$

the wealth of the producer. Also, we define Λ^ℓ the set of liquid securities, i.e, those with $\psi_H\cdot D\left(\ell\right)\geq D\left(h\right)\geq\psi_L D\left(\ell\right)$. We denote the set of the remaining, illiquid securities as Λ^I. By definition, $\mu_1^i\left(D\right)=0$ for any $D\notin\Lambda^I$ and $\mu_1^\ell\left(D\right)=0$ for any $D\notin\Lambda^\ell$.

The producer's indifference curve for a given security $D = (D(\ell), D(h))$ is represented in green in the following figure (Figure 3).

Given the overall resources of the producer $(N^e(\ell), N^e(h))$, we can find the producer's optimal security by finding the indifference curve that is tangent to his budget line. Notice that, for any price, choosing an illiquid security $D \in \Lambda^I$ other than the extreme ones (those with zero payoff in one state) is suboptimal. Indeed, if q is high enough, the producer is better off holding only the maximally illiquid security $D = (0, 1)$; otherwise, the producer is better off with a liquid security. In particular, the producer will be indifferent between the maximally illiquid security in the high state and the "marginally liquid" security when the price is $q^\star = (1-\pi)\psi_H$.[8] Similarly, the producer will be indifferent between the maximally illiquid security in the low state and the "marginally liquid security" in the low state, when the price is $q_\star = (1 - \pi)\psi_L$.

Following the reasoning above, we can rewrite the producer's indirect utility over portfolios as

$$\tilde{\mathbb{U}}(\mathbf{D}) = \begin{cases} q^\star \mathbf{D}(\ell) + \pi \mathbf{D}(h), & \text{if } \mathbf{D} \in \Lambda^I, \mathbf{D}(h) > \mathbf{D}(\ell) \\ \rho\left[(1-\pi)\mathbf{D}(\ell) + \pi \mathbf{D}(h)\right], & \text{if } \mathbf{D} \in \Lambda^\ell \\ q_\star \mathbf{D}(\ell) + \pi \mathbf{D}(h), & \text{if } \mathbf{D} \in \Lambda^I, \mathbf{D}(h) < \mathbf{D}(\ell) \end{cases}$$

Then, the producer's problem can be rewritten as

Problem 3. Producers solve:

$$\max_{\{\mathbf{D}(\ell), \mathbf{D}(h)\} \geq 0} \tilde{\mathbb{U}}(\mathbf{D}) \quad \text{s.t.} \quad q\mathbf{D}(\ell) + \pi \mathbf{D}(h) \leq q N^e(\ell) + \pi N^e(h).$$

This rewriting says that we can treat the producer as if his decision were only about his overall portfolio. Of course, when the solution involves $\mathbf{D} \in \Lambda^I$, this portfolio will be composed only of the maximally illiquid and the "marginally liquid" security.

The producer will only hold liquid securities if the price $q \in (q_\star, q^\star)$, only extreme illiquid securities if $q > q^\star$ or $q < q_\star$. If $q = q^\star$, the optimal portfolio consists of a mix of the maximally illiquid security and the "marginally liquid" security. For the sake of clarity,

[8] Consider the canonical maximally illiquid security $(0, 1)$. The price of this security and its payoff are equal to π. The marginally-liquid security $(1, \psi_H)$ has a price equal to $q + \psi_H \pi$ and a payoff equal to $\rho\left[(1-\pi) + \psi\pi\right] = \psi_H$. The producer will be willing to buy both securities only when $q = (1-\pi)\psi_H \equiv q^\star$.

we will restrict our exposition to the case where $q \geq 1-\pi$. Given the assumption that there are more payoffs in the high-state $N(h) \geq N(\ell)$, the equilibrium price will satisfy this condition.[9] The next proposition summarizes these results in terms of the specific securities held by the producer.

Proposition 5 *The optimal security holdings of the producer are as follows :*

- *No liquidity premium $(1-\pi) = q$:*

  ▷ $\mu_1^i(D) = 0$ *and any* $\mu_1^\ell(D)$ *such that* $n^e = \int p(D)\, d\mu_1^\ell(D)$ *is a solution.*

- *Positive liquidity premium $(1-\pi) < q$:*

  ▷ *Only*

  $$\mu_1^\ell\left(e^\psi\right) > 0 \text{ and/or } \mu_1^i\left(e^h\right) > 0.$$

  For all other securities, $\mu_1^\ell = \mu_1^i = 0$. In particular, we have the following sub-cases,

  ▷ *No illiquid assets, $q \in (1-\pi, q^\star)$ Only the marginally liquid security is held.*

  $$\mu_1^\ell\left(e^\psi\right) = \frac{n^e}{q+\pi\psi}.$$

  ▷ *Some illiquid assets, $q = q^\star$ Both the marginally liquid security and the maximally illiquid securities are held. In particular, and $\left\{\mu_1^\ell\left(e^\psi\right), \mu_1^i\left(e^h\right)\right\}$ that satisfies*

  $$(q+\pi\psi)\,\mu_1^\ell\left(e^\psi\right) + \pi\mu_1^i\left(e^h\right) = e$$

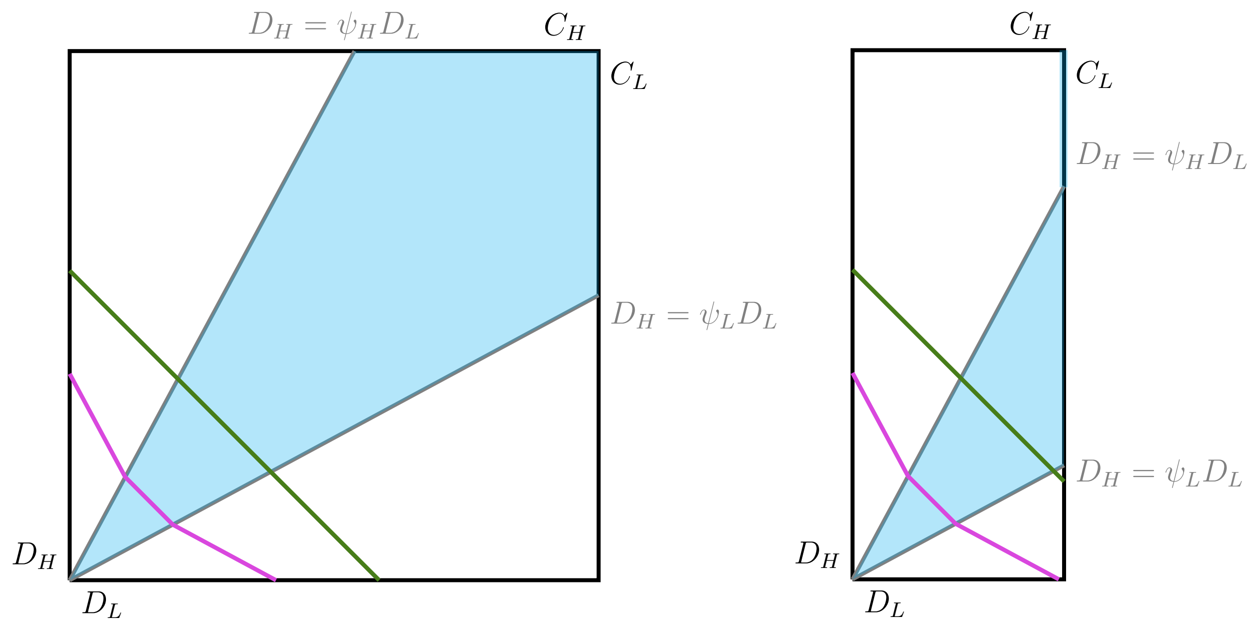
  is a solution.

  ▷ *Only illiquid assets, $q^\star < q$ Only the maximally illiquid security is held. In particular, it must solve,*

  $$\mu_1^i\left(e^h\right) = n^e/\pi.$$

[9]Given the symmetry of the environment, describing the case with $N(\ell) \geq N(h)$ would be superfluous.

Competitive Equilibrium. As we have shown, from the perspective of the overall portfolios, the environment is a two-agents, two-goods economy. We can represent this economy using Edgeworth Boxes. The following representation (Figure 4a) displays the producer and banker's indifference curves. The contract "curve" or set of Pareto efficient points is represented as the shaded blue area.



(a) Edgeworth box. Liquid Aggregate Portfolio, $N(h) \in (\psi_L N(\ell), \psi_H N(\ell))$.

(b) Edgeworth box. Illiquid Aggregate Portfolio, $N(h) \notin (\psi_L N(\ell), \psi_H N(\ell))$.

Figure 4: Edgeworth boxes. Liquid and Illiquid Aggregate Portfolios. Green line is a banker portfolio indifference curve. Purple line is an producer portfolio indifference curve. Shaded blue are is the set of all Pareto efficient points. Notice this set includes some points at the edge of the box.

Notice that at any point in the shaded blue area, the marginal rate of substitution of the banker and producer are equal. This corresponds to the liquid producer portfolios.

Importantly, the Edgeworth Box can look differently depending on the distribution of aggregate resources across states, $N(\ell), N(h)$. Figure 4b displays the Edgeworth Box in the case of scarce aggregate liquidity, when the aggregate portfolio is illiquid.

Notice that, in each case, the set of Pareto efficient allocations includes those where the producer only holds liquid securities. When the aggregate portfolio is illiquid, $N(h) > \psi_H N(\ell)$, this set also includes allocations where the producer holds the maximally illiquid security–those at the edge of the box.

Proposition 6 *If the aggregate portfolio is liquid, then the equilibrium price is* $q = 1 - \pi$ *(there is no liquidity premium or discount). Specifically,* $q > 1 - \pi$ *only if* $N(h) > \psi N(\ell)$.

We refer to the case where the aggregate portfolio is liquid as the "abundant liquidity case." We call the other case, the "scarce liquidity case." Intuitively, when liquidity is abundant, the banker and the producer can always trade at the actuarially fair prices and find a liquid portfolio for the producer. However, when liquidity is scarce and the producer's resources are large enough, the liquid portfolio that the producer would like to buy at the fair prices will not be feasible. Therefore, the price will have to include a premium.

When there is a premium or a discount, the equilibrium price will reflect the excess demand of the producer for the L-state and H-state asset, respectively. In particular, the equilibrium price takes the following form

Theorem 7 *There exists an equilibrium. The equilibrium price* q *takes values in* $[(1 - \pi), q^{\star}]$. *In particular, we have three cases*

- *Scarce Liquidity* $N(h) > \psi_H N(\ell)$*:*

$$q = \min \left\{ q^{\star}, \max \left\{ \pi \frac{N^e(h) - \psi_H N(\ell)}{N^b(\ell)}, 1 - \pi \right\} \right\}.$$

- *Abundant Liquidity* $N(h) \in [\psi_L N(\ell), \psi_H N(\ell)]$*:* $\quad q = 1 - \pi.$

The expressions for the prices are intuitive. The limits are given by the marginal rates of substitution of the producer. The intermediate expression in the case of scarce liquidity states that the relative price q/π should equal the ratio of the producer's excess holdings of the illiquid asset ($N^e(h) - \psi_H N(\ell)$) to the banker's excess holdings of the liquid asset ($N^b(\ell)$). This must be so since, at these intermediate prices, the producer only wants to hold the "marginally liquid" security and the banker will sell all of his liquid assets.

While scarce aggregate liquidity is necessary to have a premium on the L-state A-D security, it is not a sufficient condition. In Figure 5, we can see the set of endowments that would result in $q = 1 - \pi$: those such that the producer's desired portfolio at that price is feasible. Similarly, if the producer is very wealthy (i.e., the producer owns most of the assets), then the price must be $q^{\star}$ because that is the price at which the producer is willing to hold both illiquid and liquid securities. Figure 5 shows the determination of the price in the intermediate case.

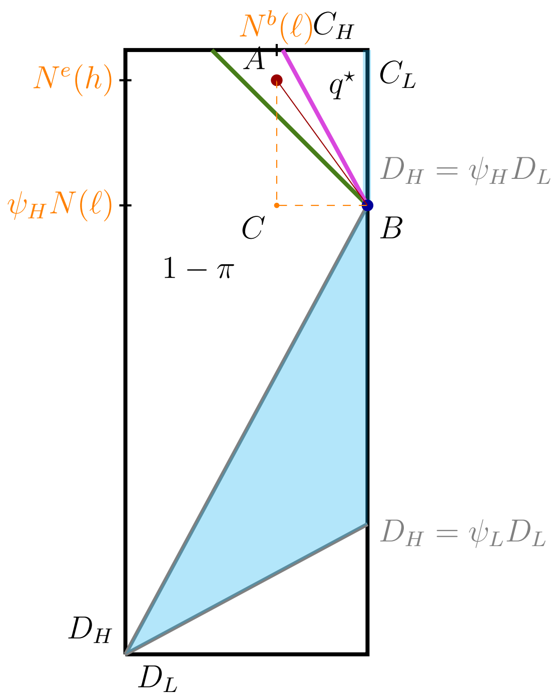


Figure 5: Edgeworth Box with Scarce Liquidity $N(h) > \psi_H N(\ell)$ - Example with $q \in (1-\pi, q^\star)$. Point A represents the initial endowment. Point B is the equilibrium allocation: all liquid assets are used to create the "marginally" liquid security, and the producer holds no illiquid securities.

In this example, the initial endowment will result in an intermediate price $q \in (1-\pi, q^\star)$. If the price were $q = 1-\pi$, the producer would like to buy more liquid securities than can be created. The equilibrium relative price q/π is the slope of the red line connecting points A and B. This slope is given by the ratio of the sides A to C and C to B. The first side is the producer's excess holdings of illiquid securities: $N^e(h) - \psi N(\ell)$. The second one is the endowment of liquid assets by the banker: $N^b(\ell)$.

Special Case: No banker wealth. In the extreme case where the bankers have no wealth, the equilibrium price will depend only on the aggregate liquidity. In particular, in each case, the endowment (and equilibrium allocation) will be at the top-right corner of each Edgeworth box. Therefore, the equilibrium price will be

$$q = \begin{cases} q_{\star}, & \text{if } N(h) < \psi_L N(\ell), \\ 1-\pi, & \text{if } N(h) \in [\psi_L N(\ell), \psi_H N(\ell)), \\ q^{\star}, & \text{if } N(h) > \psi_H N(\ell), \end{cases}$$

with the standard multiplicity at the kinks. Thus, without banker wealth, the equilibrium price reflects the aggregate liquidity: there is a premium if liquidity is scarce and actuarially fair prices if liquidity is abundant.

4 Safe Assets and Recognizability

We extend the model to study the role of safe assets (and outside money) along with its interaction with bank's provision of means of payments. To do this, we introduce geographical separation. There are two locations, each one containing a continuum of producers, workers, and bankers.

The match between producers and workers can happen within and across locations, with probabilities ν and $1-\nu$. We assume that the assets and bank liabilities from one location are not accepted by a worker from the other one. This assumption can be justified if the workers in one location cannot recognize or evaluate claims on the other one, so that in equilibrium they decide not to accept those payments.

In contrast to the other assets and bank liabilities, there is a *safe* asset which can be recognized everywhere. The safe asset, besides being universally recognized, has the same payoff in each state. At $t = 0$, the producers and bankers are endowed with some holdings of the safe asset.

The timing of the model is the same as before. In the competitive market at $t = 0$, the producers and bankers trade risky assets, bank liabilities, and safe assets. We denote the price of the safe asset by $\mathbf{P}$, the price of the A-D security paying in state ℓ by q, and we normalize the price of the A-D security paying in state h to be π.

4.1 The Bilateral Trading Problem

Given the recognizability of bank liabilities ν and the presence of safe assets, the value of bringing an asset to the bilateral trade will be different from the benchmark case. In particular, given a (non-safe) asset D, the value of that asset for the producer will be given by $\tilde{\mathbb{U}}[D] = \nu\mathbb{U}[D] + (1-\nu)\mathbb{E}[D]$. Effectively, this means that the return on the project is $\tilde{\rho} = \nu\rho + (1-\nu)$.

Proposition 8 *For a producer, the ex-ante value of a security.* $\mathbb{U}[D]$*, the contracted labor,* $q(\omega)$*, and the pattern of trade,* $n(\omega)$*, will depend on the security's payoffs according to the following classification:*

- *Liquid securities:*

$$\textit{if } D(h) \in [\psi_L \cdot D(\ell), \psi_H \cdot D(\ell)]: \quad n(h) = n(\ell) = 1 \quad \textit{and} \quad \mathbb{U}[D] = \mathbb{U}^{\ell}[D] := \rho\mathbb{E}[D].$$

- *Illiquid securities:*

$$\textit{If } D(h) > \psi_H \cdot D(\ell): \quad n(h) = 0, n(\ell) = 1 \quad \textit{and} \quad \mathbb{U}[D] = \mathbb{U}^{i,h}[D] := \pi(h)D(h) + \pi(\ell)\rho D(\ell),$$
$$\textit{if } D(h) < \psi_L \cdot D(\ell): \quad n(h) = 1, n(\ell) = 0 \quad \textit{and} \quad \mathbb{U}[D] = \mathbb{U}^{i,\ell}[D] := \pi(h)\rho D(h) + \pi(\ell)D(\ell).$$

Thus, the ex-ante value of an illiquid security as

$$\mathbb{U}^{i}[D] := \begin{cases} \mathbb{U}^{i,h}[D], \textit{ for } D(h) > D(\ell) \\ \mathbb{U}^{i,\ell}[D], \textit{ for } D(h) < D(\ell) \end{cases}$$

As before, the proposition shows that securities are segmented into liquid and illiquid securities according to a set of *liquidity coefficients* (ψ_L, ψ_H). These coefficients do not depend on the probability of a local match, ν, because once the match has been realized the return on the project for the producer is ρ.

The value of the safe asset is simply given by $\mathbb{U}[(1,1)] = \rho$, given that it is liquid and can be traded in both types of matches.

4.2 The Producer and Banker Problems

The problem of the agents in the competitive market is different because the value of assets and bank liabilities take into account the recognizability and because of the presence of the safe asset. Let N_s^e and N_s^b denote the producer and banker initial holdings of the safe asset; while S^e and S^b denote their final holdings of the safe asset. We write the problems of the producer and the banker next.

Problem 4 (Producer Problem)**.**

$$\max_{\left\{\mu_1^i,\mu_1^\ell,S^e\right\}\geq 0} \int \tilde{\mathbb{U}}^\ell \left[D\right] d\mu_1^\ell(D) + \int \tilde{\mathbb{U}}^i \left[D\right] d\mu_1^i(D) + \rho \cdot S^e \tag{11}$$

subject to:

$$\mathbf{P} \cdot S^e + \int P\left(D\right) d\mu_1^i(D) + \int P\left(D\right) d\mu_1^\ell(D) \leq \mathbf{P} \cdot N_s^e + \int P\left(R\right) d\mu_0^e(R). \tag{12}$$

Here, we have used the fact that the expected utility derived from a safe asset is ρ for the producer.

Problem 5 (Banker problem)**.**

$$\max_{\left\{\mu_1^+,\mu_1^-,S^b,c(\omega)\right\}\geq 0} \mathbb{E}\left[c\right] \tag{13}$$

subject to:

$$\mathbf{P} \cdot S^b + \int P\left(D\right) d\mu_1^+(D) \leq \mathbf{P} \cdot N_s^b + \int P\left(D\right) d\mu_1^-(D) + \int P\left(R\right) d\mu_0^b(R). \tag{14}$$

$$\forall\omega, \quad c\left(\omega\right) + \int D\left(\omega\right) d\mu_1^-(D) \leq \int D\left(\omega\right) d\mu_1^+(D) + S^b. \tag{15}$$

The resource constraint (15) shows that the bank can hold safe assets as backing for bank liabilities. However, the restriction on the bank holdings of safe assets to be nonnegative says that the bank cannot issue safe assets. The bank can still issue a liability with constant payoffs. The difference between these two is that the bank liability will be subject to the recognizability problem. The bank's ability to issue constant-payoff liabilities will create a lower bound for the price of the safe asset $\mathbf{P}$ at its replication cost $q+\pi$.

Lemma 9 *In equilibrium, the price of the safe asset* $\mathbf{P}$ *must be at least equal to* $q+\pi$*.*

For clarity, we include the definition of equilibrium with safe assets. The only differences are that the problems of the producer and banker include the safe asset and the new market for the safe asset must clear.

Definition 2. A competitive equilibrium is a price for regular assets $P:\mathbb{R}_+^2\to\mathbb{R}_+$, a price for the safe asset $\mathbf{P}$, and portfolios for producers $\left\{\mu_1^i\left(\cdot\right),\mu_1^\ell\left(\cdot\right),S^e\right\}$ and bankers $\left\{\mu_1^+\left(\cdot\right),\mu_1^-\left(\cdot\right),S^b\right\}$, including holdings of safe assets, which satisfy two conditions,

1. The producer and banker portfolios are solutions to their corresponding problems.

2. The market for every asset clears

$$\begin{aligned}\forall D\in\mathbb{R}_+^2,\quad \mu_1^i(D)+\mu_1^\ell(D)+\mu_1^+(D) &\leq \mu_1^-(D)+\mu_0^e(D)+\mu_0^b(D), &(16)\\ S^e+S^b &\leq N_s^e+N_s^b. &(17)\end{aligned}$$

The problems of the agents can be simplified as before. The producer and the banker can choose their overall portfolio of payoffs in each state with the addition of their holdings of safe assets.

4.3 Equilibrium Characterization

As in section 3.1, the equilibrium can be different depending on the aggregate liquidity. In the same way, we can distinguish three cases, when aggregate liquidity is scarce, normal, or abundant. As before, there can be a liquidity premium only in the first case and a liquidity discount only in the third case.

4.3.1 Abundant Liquidity

In the case of abundant liquidity, the price of the L-state A-D security will be $q=1-\pi$. The price of the safe asset will depend on its supply and the relative wealth of the producer. If the producer is relatively wealthy, he will end up holding bank liabilities and must therefore be indifferent between them and the safe asset. That means the price should be $\mathbf{P} = \rho/\tilde{\rho} > 1$. On the other hand, if the producer is relatively poor and he cannot afford to buy all of the safe assets, the bank will hold them instead. Then, the price of the safe asset must be $\mathbf{P} = 1$. The intermediate case occurs when the producer's wealth is neither too small or too large and he can buy all of the safe asset at some price $\mathbf{P} \in [1, \rho/\tilde{\rho}]$. In that case, the price of the safe asset would be given by

$$P_0 = \frac{\pi N^e(h) + (1-\pi)N^e(\ell)}{N_s^b}.$$

Thus, we have that if $N(h) \in [\psi_L N(\ell), \psi_H N(\ell)]$, the prices are $q = 1 - \pi$ and $\mathbf{P} = \min\{\rho/\tilde{\rho}, \max\{P_0, 1\}\}$. If $P_0 < 1$, the bank issues no liabilities and retains some but not all of its safe assets. If $P_0 > \rho/\tilde{\rho}$, the bank issues some liabilities and sells all of its safe assets. In the remaining case, the bank issues no liabilities and sells all of its safe assets. Naturally, we can interpret these cases as abundant, scarce, and normal supply of safe assets, respectively.

In summary,

Proposition 10 *In the abundant liquidity case, $N(h) \in [\psi_L N(\ell), \psi_H N(\ell)]$, the equilibrium prices are*

$$q = 1 - \pi, \text{ and } \mathbf{P} = \min\{\rho/\tilde{\rho}, \max\{P_0, 1\}\},$$

where

$$P_0 = \frac{\pi N^e(h) + (1-\pi)N^e(\ell)}{N_s^b}.$$

4.3.2 Scarce Liquidity

In the case of scarce liquidity, the price of the L-state A-D security will be $q \geq 1 - \pi$. As before, there is an upper bound on the price in this case, given by the point of indifference between the bank's marginally liquid liability and the illiquid liability, $q^\star = (1-\pi)\tilde{\psi_H}$. Note that this bound depends on the recognizability probability. In between these bounds,

$1-\pi$ and $q^\star$, there is a possibility for an intermediate price. In the benchmark case, this price was given by $\min\left\{q^\star, \max\left\{\pi\frac{N^e(h)-\psi N(\ell)}{N^b(\ell)}, 1-\pi\right\}\right\}$. With safe assets, this intermediate price will correspond to one out of three possible cases depending on who holds the safe assets: the producer, the banker, or both.

Proposition 11 *In the scarce liquidity case, $N(h) > \psi_H N(\ell)$, the equilibrium price is*

$$q = \min\{1-\pi, \max\{q^\star, \min\{q_{SE}, \max\{q_{SB}, \tilde{q}\}\}\},$$

where

$$\begin{aligned} q_{SB} &= \pi\frac{(N^e(h)+N_s^e)-\psi_H(N(\ell)+N_s)}{N^b(\ell)+N_s^b}, \\ q_{SE} &= \pi\frac{N^e(h)-\psi_H\left(N(\ell)+\frac{\rho}{\psi_H}N_s^b\right)}{N^b(\ell)+\frac{\rho}{\psi_H}N_s^b}, \\ \tilde{q} &= (1-\pi)+\frac{(\rho-\tilde{\rho})\bar{\psi_H}}{\psi_H-\rho}, \\ q^\star &= (1-\pi)\psi_H-\psi_H\frac{\rho-1}{\rho}(1-\nu). \end{aligned}$$

The equilibrium price of the safe asset is

$$\mathbf{P} = \begin{cases} \rho, & \text{if } q = q^\star \\ \frac{\rho}{\psi_H}(q+\pi\psi_H), & \text{if } q = q_{SE} \\ \pi + q, & \text{if } q = \tilde{q} \text{ or } q = q_{SB} \end{cases}$$

If the safe assets are held only by the producer, the price of the safe asset must be $\mathbf{P} = \frac{\rho}{\psi_H}(q+\pi\psi_H)$, while the price q must be given by $q_{SE} = \pi\frac{N^e(h)-\psi\left(N(\ell)+\frac{\rho}{\psi_H}N_s^b\right)}{N^b(\ell)+\frac{\rho}{\psi_H}N_s^b}$. If the safe assets are held by both agents, the price of the safe asset must be $\mathbf{P} = \pi + q$ and the price q must be $\tilde{q} = (1-\pi)+\frac{(\rho-\tilde{\rho})\bar{\psi}}{\tilde{\rho}\psi-\rho}$. Otherwise, if all the safe assets are held by the banker, the price of the safe asset must be $\mathbf{P} = \pi + q$ and the price q must be $q_{SB} = \pi\frac{(N^e(h)+N_s^e)-\psi(N(\ell)+N_s)}{N^b(\ell)+N_s^b}$.

Given the endowments and parameters, in equilibrium, only one of these cases can be relevant. If $q_{SE} < \tilde{q}$, then q_{SE} is the relevant price because the price of the safe asset is strictly greater than $q+\pi$ rationalizing the bank's zero holdings of safe assets. On the other hand, if $q_{SE} > \tilde{q}$, then the relevant price is no longer q_{SE}. If that were the price, then the safe asset price would be lower than its replication cost and the bank would optimally decide to hold the safe asset. In turn, the relevant price is the maximum between $\tilde{q}$ and q_{SB}. If $\tilde{q} > q_{SB}$, q_{SB} cannot be an equilibrium price because it would imply that the price of the safe asset is above its replication cost, contradicting the fact of banks holding the safe asset.

Thus, the equilibrium price with scarce liquidity is given by

$$q = \min\{1-\pi, \max\{q^\star, \min\{q_{SE}, \max\{q_{SB}, \tilde{q}\}\}\}.$$

Naturally, when the supply of safe assets is zero, the prices q_{SE} and q_{SB} are equal (and equal to the price in the benchmark case). The expression above readily reduces to the original one in section 3.1.

With this multitude of prices, it is useful to classify them into two groups. First, there are prices such as $q^\star, \tilde{q}$ which are based on indifference conditions and, as such, do not depend on endowments. These prices determine the indifference between the illiquid and marginally liquid liability, and the indifference between the safe asset and the marginally liquid liability (when the bank is willing to hold the safe asset), respectively. Second, there are prices such as q_{SE}, q_{SB} which are based on market-clearing conditions and clearly depend on endowments. These are the prices that would achieve market clearing if the producer uses the bank's marginally liquid liability along with and without safe assets, respectively.

It is instructive to understand the effect of recognizability on prices when safe assets are in zero supply. This corresponds to the benchmark case of the model. The first thing to notice is that when assets are not recognizable, $\nu = 0$, there can be no liquidity premium. If there were a liquidity premium for fully recognizable assets, $\nu = 1$, then the price of liquidity must increase steadily with ν up to a point. This happens because recognizability affects the upper bound on the value of bank liabilities. For low recognizability, this bound is relevant. At a sufficiently high recognizability, however, the price comes from budget constraints; the price reflects the relative endowments of the producer and the banker. This explains the pattern of the price q with respect to recognizability ν. The price initially

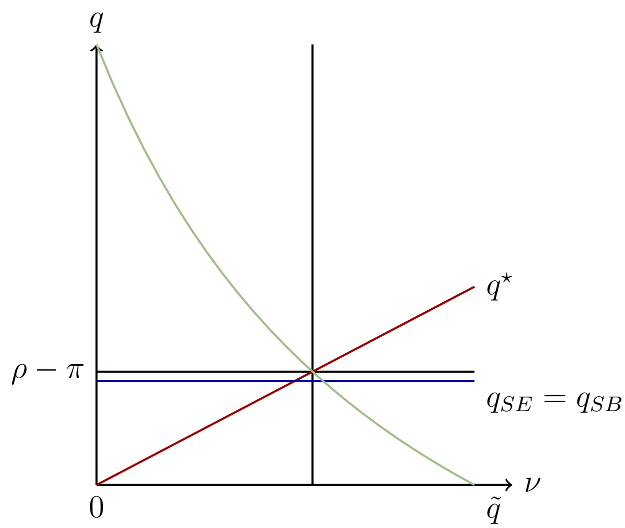
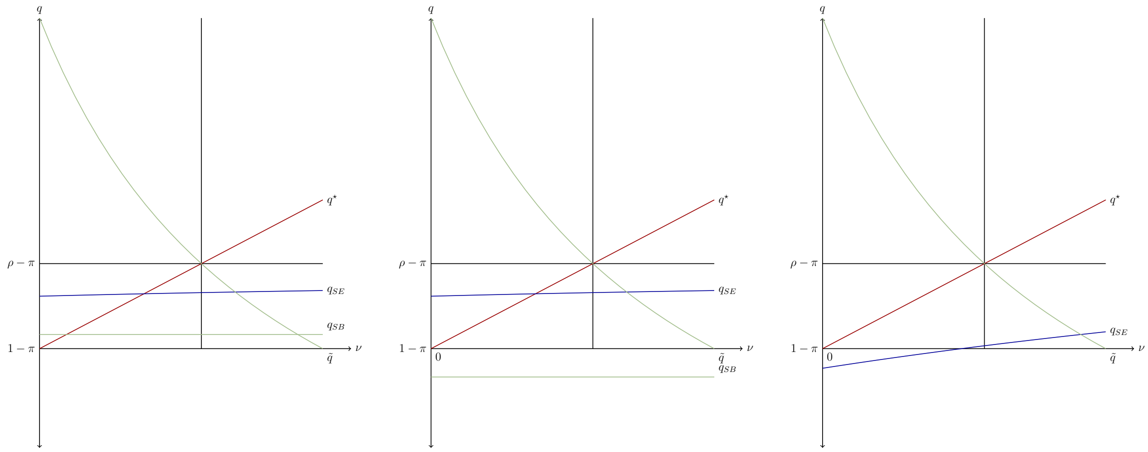


Figure 6: Prices - Scarce Liquidity without Safe Assets

increases, tracking $q^\star$, until it reaches $q_{SE} = q_{SB}$ and remains there for higher ν.

The price of the safe asset can still be obtained, even though the supply is zero. For low recognizability-i.e., when $q = q^\star$, the price of the safe asset must be at least ρ. There are two possibilities for the effect of recognizability on the price of the safe asset: the price might be monotonically increasing or decreasing on recognizability, depending on how scarce liquidity is. If liquidity is sufficiently scarce such that $q_{SE} = q_{SB} > \rho - \pi$, then the price of the safe asset will equal ρ until $\nu = \psi^{-1}$ and then, passing to the hands of the bank, be equal to $q+\pi$. The price of the safe asset will then continue increasing until $q = q_{SE}$ and remain constant thereafter. On the other hand, if liquidity is only moderately scarce such that $q_{SE} < \rho - \pi$, then the price of the safe asset will decrease from ρ when $q = q_{SE}$ so that there is indifference between the safe asset and the bank's marginally liquid liability. This decrease will continue until a level of recognizability such that $q_{SE} = \tilde{q}$, at that point the price of the safe asset will be constant at $q_{SE} + \pi$.

The reason for these different regimes is that the safe asset plays a dual role in this economy. On one hand, it is a means of payment by itself, which absent sufficient alternatives is enough to give it a value of ρ. On the other hand, it is a way for the bank to create more liabilities with a value of $q+\pi$. The first role is prevalent when recognizability is low, while the second role matters when recognizability is high. The effect of recognizability on the price of the safe asset therefore depends on whether the first or second role give the



(a) Prices - Scarce Liquidity with Scarce Safe Assets

(b) Prices - Scarce Liquidity with Normal Safe Assets

(c) Prices - Moderately Scarce Liquidity with Safe Assets

Figure 7: Scarce Liquidity with Safe Assets

safe asset a greater value.

When safe assets are in positive supply, there is a difference between the prices q_{SE} and q_{SB}. The difference comes from the fact that safe assets allow the bank to issue more liabilities. The price q_{SE} is increasing in recognizability because as bank liabilities are more recognizable, the price of the safe asset is lower, increasing the amount the producers can spend on acquiring bank liabilities. The equilibrium price is eventually decreasing once the safe asset holdings shift from the producer to the bank. Notice that this decrease depends on the quantity of safe assets.

It is interesting to remark on the absence of these effects in the normal liquidity case. With normal liquidity, the value to the producer of the safe asset is always greater than the value of a bank liability. The former leads to a payoff of ρ, while the latter leads to a value of $\tilde{\rho}$. Therefore, the producer will always be the marginal investor in the safe asset. While it is possible for the bank to hold part of the safe assets in equilibrium, for some endowment configurations, this can only happen in cases where the producer is poor enough that he cannot afford the full supply of safe assets and cannot afford to acquire any bank liabilities. Thus, in the normal liquidity case, the safe asset is only used as a means of payment, but it is not used to enhance the creation of bank liabilities.

5 Moral Hazard

We now add moral hazard to the banks' problem. Banks could issue an benefit from liquid deposits without backing them with assets. This amounts to a limit on the value of liquid securities issued by the bank; in particular, the expected value of liquid securities must be lower than the expected bank profits.

$$\int \mathbb{E}\,(D)\, d\left(\mu_1^+(D) - \mu_1^-(D)\right) \geq \int \mathbb{E}\,(D)\, d\mu_1^-(D)\mathbb{1}(\mathbf{D} \in \Lambda^L). \tag{18}$$

The bank's problem with moral hazard is therefore

Problem 6. Bankers solve:

$$\max_{\left\{\mu_1^+, \mu_1^-, c(\omega)\right\} \geq 0} \quad \mathbb{E}\,[c] \tag{19}$$

subject to:

$$\int P\,(D)\, d\mu_1^+(D) \leq \int P\,(D)\, d\mu_1^-(D) + \int P\,(R)\, d\mu_0^b(R). \tag{20}$$

$$\forall \omega, \quad c\,(\omega) + \int D\,(\omega)\, d\mu_1^-(D) \leq \int D\,(\omega)\, d\mu_1^+(D) \tag{21}$$

$$\int \mathbb{E}\,(D)\, d\left(\mu_1^+(D) - \mu_1^-(D)\right) \geq \int \mathbb{E}\,(D)\, d\mu_1^-(D)\mathbb{1}(\mathbf{D} \in \Lambda^L). \tag{22}$$

The producer's problem and the equilibrium definition remain unchanged.

Under moral hazard, the equilibrium prices will be different than in the benchmark model. The benchmark equilibrium prices required the activity of the banks which might be curtailed if the incentive constraint is binding. Specifically, the prices need not be fully linear in payoffs. The following example shows this.

Example 1. Consider an economy with only two illiquid assets: $(0, 1)$, $(1, 0)$. Both assets are held by the producer. In the benchmark case, prices would be linear and bank profits would be zero. Therefore, the bank's incentive constraint prohibits issuance of liquid securities. However, since prices are linear, the producer will either demand liquid securities or only one of the illiquid assets; in either case, there is no market clearing.

5.1 Conditions for benchmark allocation to be IC

Consider an economy as in the benchmark case, with endowments of assets for the banker and producer. As before, the initial portfolios of assets are summarized by $(N^b(\ell), N^b(h))$ and $(N^e(\ell), N^e(h))$ for the banker and producer respectively.

Abundant Liquidity. Recall that in the abundant liquidity case, the price is always $q = 1-\pi$ and the producer holds a liquid portfolio with expected payoff $(1-\pi)N^e(\ell)+\pi N^e(h)$.

The equilibrium outcome without moral hazard can fail to be an equilibrium with moral hazard for two reasons. First, the amount of liquid securities held by the producer in the equilibrium outcome might be so large that the bank would violate its incentive compatibility condition. A potential solution is to implement the outcome with a mix of liquid assets and liquid bank liabilities, since the producer is willing to purchase any liquid securities at the abundant-liquidity prices. However, this might lead us into the second reason. If the producer holds some liquid assets, the bank's portfolio might be too risky to support the issuance of s bank liabilities, even though liquidity is abundant. The issue is that the producer by holding liquid securities, which are safer than illiquid ones, can turn the bank's final portfolio illiquid.

While the general conditions for equivalence of equilibrium outcomes with and without moral hazard are difficult to summarize concisely, the reader might benefit from some sufficient conditions that sketch the contours of those conditions. One sufficient condition is for the value (expected payoff) of liquid assets to be greater than the value of the producer's assets. In this case, the producer can use his endowment to buy only liquid assets, the bank can issue no liabilities, and the incentive compatibility constraint is satisfied.

Another sufficient condition is for the banker to be wealthier than the producer. Since the banker's incentive constraint is relaxed by the banker's initial wealth, a sufficiently wealthy bank will be able to accommodate an producer's demands without incentive problems.

Finally, in the case that the bank is not wealthy enough to accommodate the producer's full demand for liquid securities, and liquid assets are not enough to satiate the producer's demand, the equivalence might still hold with the following two conditions. First, the bank must be wealthy enough to accommodate the producer's residual demand after acquiring the liquid assets. Second, the portfolio of illiquid assets must be liquid. In this case, the banker can meet the producer's demand simply by combining illiquid assets.

To recap, it is not necessarily the case that the outcomes with abundant liquidity in the benchmark setting are also equilibrium outcomes with moral hazard. Moral hazard imposes a limit on the bank's ability to create liquid liabilities which brings up considerations of the composition of the initial asset pool that were irrelevant in the benchmark case.

Let us denote $N^e := (1-\pi)N^e(\ell) + \pi N^e(h)$, $N^b := (1-\pi)N^b(\ell) + \pi N^b(h)$, the expected payoff of the initial endowments of the producer and banker, respectively. These are also the wealth of the producer and banker at the equilibrium prices with abundant liquidity. Similarly, denote N^{liq} as the expected payoff of all liquid assets, $\int \mathbb{E}[R]\,\mathrm{d}(\mu_0^e + \mu_0^b)(R)$ over the set of liquid assets $\{R : R(h)/R(\ell) \in [\psi_L, \psi_H]\}$.

The incentive compatibility condition of the bank can be written as

$$(1-k)D^b \leq N^b,$$

where D^b is the expected payoff of the banker's liquid liabilities. The producer's demand for liquid liabilities will equal his wealth and can be written as

$$D^b + D^a = N^e,$$

where D^a is the expected payoff of the producer's final holding of liquid assets. The maximum value that D^a can take is N^{liq} if the producer holds all liquid assets.

If the producer holds no liquid assets, then we can write the incentive constraint as

$$(1-k)N^e \leq N^b.$$

Therefore, the bank being wealthier than the producer is a sufficient condition since the assumption of abundant liquidity implies that the resource constraints will be satisfied as well.

The bank's incentive compatibility condition, when the producer holds all liquid assets, can be written as

$$(1-k)[N^e - N^{liq}] \leq N^b.$$

We can see that if the value of liquid assets is greater than the value of producer's initial assets, the bank will not need to issue liabilities and the incentive and resource constraints will be trivially satisfied.

If the value of liquid assets is lower than the value of the producer's initial assets, then the incentive constraint is nontrivial

$$(1-k)[N^e - N^{liq}] \leq N^b.$$

More importantly, the resource constraints will be

$$D^b(\ell) \leq N^{illiq}(\ell), D^b(h) \leq N^{illiq}(h),$$

where $N^{illiq}(\omega)$ is the aggregate payoff in state ω of the portfolio of illiquid assets. Since $D^b = N^e - N^{liq} \leq N - N^{liq} = N^{illiq}$, the only concern is the feasibility of issuing a bank liability that has the right ratio of payoffs i.e., $D^b(h) \in [\psi_L D^b(\ell), \psi_H D^b(\ell)]$. This is evidently guaranteed if $N^{illiq}(h) \in [\psi_L N^{illiq}(\ell), \psi_H N^{illiq}(\ell)]$.

Scarce Liquidity. With scarce liquidity, the benchmark equilibrium allocation gives the producer only the marginally-liquid security $(1, \psi_H)$. Assuming no mass of initial assets with such payoffs, these securities must consist only of bank liabilities. Thus, a sufficient condition is given by

$$(1-k)N^e \leq N^b.$$

5.2 General equilibrium with moral hazard

With moral hazard, we must consider the incentive compatibility condition of the banker. Solving for equilibrium outcomes is complicated due to the fact that the incentive constraint limits the issuance of bank liabilities and makes the composition of the initial pool of assets relevant, unlike the benchmark model where only the aggregate payoff of the portfolio of assets mattered for equilibrium outcomes. Moreover, for some initial assets, the incentive compatibility condition can make the resource constraint binding even though it was slack in the benchmark equilibrium.

We proceed by analyzing equilibrium outcomes when only the incentive constraint is binding and deriving conditions on initial assets for this to be the case. This case is interesting because it features a liquidity premium in the sense that all liquid assets and securities are more expensive than illiquid assets.

Incentive Constraint Binding, Resource Constraint Slack Notice first that any illiquid assets are priced by the bank. The incentive constraint affects the holding and issuance of an illiquid security in the same way, so in equilibrium the banker must be indifferent between issuing and holding an illiquid security. Since the resource constraint is slack, the price of the illiquid assets is proportional to their expected payoff.

Since the banker's incentive constraint is binding, the banker faces an extra (shadow) cost of issuing liquid securities over purchasing them. Given that the resource constraints don't bind, the banker is unwilling to hold any liquid securities and, given that the incentive constraint binds, the banker is willing to issue any liquid securities. By market clearing, the producer must hold all liquid securities, thus the price of liquid securities must be proportional to their expected payoff. However, since the banker must be willing to issue those liquid securities, their price relative to their expected payoff must be larger than that of the illiquid assets.

Thus, we see that the price of assets in this case must consist of two functions, one for illiquid assets and another for liquid assets. Normalizing, without loss of generality, the price of illiquid assets to be equal to their expected payoff, we can write the price of a liquid asset as its expected payoff times a liquidity premium which we call $(1 + \mu)$.

The reasoning above also solves for the allocations of liquid assets. The producer holds all the initial liquid assets, while the banker supplies enough liquid liabilities until the incentive constraint binds. The producer will not hold any illiquid assets under our assumption that the return on the project is large enough relative to the bank's incentive to deviate. Therefore, the banker must hold all the illiquid assets. In this way, the value of the bank's liquid liabilities is determined as the expected payoff of all illiquid assets divided by $2 - k$, the maximum amount of bank liquid liabilities that is incentive compatible.

To verify that this is an equilibrium, we must check that the resource constraints are satisfied. Since the bank is issuing only liquid liabilities and holds only illiquid assets, the resource constraint requires that the payoff of illiquid assets is enough to cover those bank liabilities. A sufficient condition is that the portfolio of illiquid assets is liquid, since the producer's demand for liquid securities must be weakly lower than the expected payoff of illiquid securities. Moreover, since the incentive constraint is binding and there is a liquidity premium, the demand for liquid securities is depressed, so the portfolio of illiquid assets need not be liquid, it need only be close to being liquid. Formally,

$$D^b(\ell) \le N^{illiq}(\ell),$$
$$D^b(h) \le N^{illiq}(h)$$

are the resource constraints and

$$(1-\pi)D^b(\ell) + \pi D^b(h) = \frac{N^{illiq}}{2-k}$$

is the value of the bank liabilities. Assuming that the portfolio of illiquid assets has higher payoffs in the high state, then this will be an equlbrium if

$$\frac{N^{illiq}(h)}{N^{illiq}(\ell)} =: \gamma^i \le (1-k)\frac{1-\pi}{\pi} + (2-k)\psi_H,$$

which is evidently satisfied if the portfolio of illiquid assets is liquid, $\gamma^i \le \psi_H$. The case where the portfolio of liquid assets has higher payoffs in the low state yields a similar expression

$$\frac{1}{\gamma^i} \le \frac{2-k}{\psi_L} + (1-k)\frac{\pi}{1-\pi},$$

which holds if $\gamma^i \ge \psi_L$.

Incentive and Resource Constraints Binding When the resource constraint binds along with the incentive constraint, the banker might be willing to acquire some liquid assets. Assume, without loss of generality, that the resource constraint in the low state binds. The banker thus assigns extra value to payoffs in the low state. This might induce the banker to buy liquid assets with payoffs in the low state at a price that the producer is unwilling to match. In this way, liquid assets will be separated into two sets, those with relatively higher payoffs in the high state held by the producer and the other ones held by the banker.

The pricing function for assets can be described as

$$P(R) = \begin{cases} (1+\mu)[(1-\pi)R_L + \pi R_H] & \text{for liquid securities with } R_H/R_L \geq z^*, \\ qR_L + \pi R_H & \text{otherwise,} \end{cases}$$

where $z^* \in [\psi_L, \psi_H]$.

In Figure 8, we plot this pricing function as isocost curves when the incentive constraint is binding. The isocost curves consists of all assets that are equal in price. Notice that the liquid securities held by the entrepreneur are relatively more expensive and priced differently from illiquid securities. In the benchmark case, the bank would have issued liquid securities to make profits; with moral hazard, the banker's issuance cannot close down this price differential.

For the following analysis, let us identify each asset by its ratio of payoff in the high state to that in the low state. As is familiar by now, assets with a ration in between ψ_L and ψ_H are liquid. If the banker buys all liquid assets with ratio $z \in [\psi_L, \psi_H]$ or lower, then the banker's resource constraints read

$$D^b(\ell) \leq N^{illiq}(\ell) + N_z^{liq}(\ell),$$
$$D^b(h) \leq N^{illiq}(h) + N_z^{liq}(h)$$

where $N_z^{liq}(\omega)$ is the low-state payoff of the portfolio of liquid assets with ratio of payoffs equal to z or lower. Since those assets have relatively higher payoffs in the low state, the portfolio of bank assets becomes more liquid with higher z.

Notice that since the producer is indifferent between all assets with ratio z or greater and the banker values payoffs in the low state more, the banker will issue only the marginally liquid security $(1, \psi_H)$.

We can now write the banker's incentive constraint as

$$(2-k)D^b \leq N^{illiq} + N_z^{liq},$$

where N_z^{liq} is the expected value of all liquid assets with payoff ratio below z. Since we assume the incentive constraint is binding, this holds with equality. Recall the banker's resource constraint is

$$D^b(\ell) \leq N^{illiq}(\ell) + N_z^{liq}(\ell),$$
$$D^b(h) \leq N^{illiq}(h) + N_z^{liq}(h).$$

Since the banker is issuing the marginally liquid liability and the banker is resource constrained in the low state, it must be true that

$$(2-k)(1-\pi+\pi)[N^{illiq}(\ell) + N_z^{liq}(\ell)] = N^{illiq} + N_z^{liq}.$$

This equation determines the cutoff liquid asset z. To see this, let γ_z be the ratio of payoff in the high state to that in the low state for the banker's final assets: the illiquid assets along with the liquid assets with payoff ratio below z. Then, we can write the previous conditions as

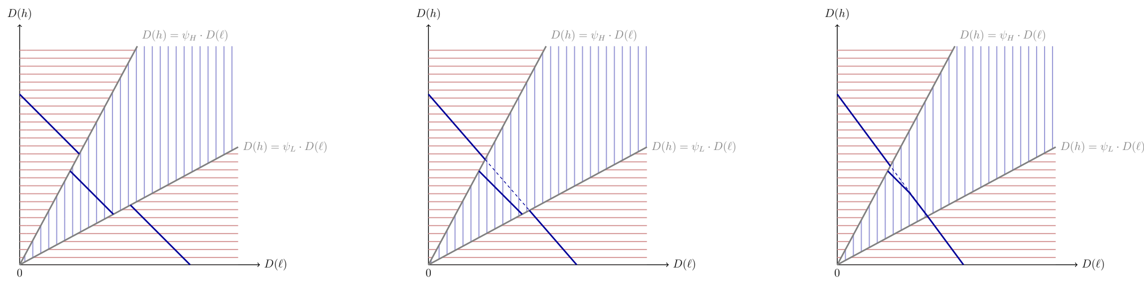
$$\gamma_{z^*} = (1-k)\frac{1-\pi}{\pi} + (2-k)\psi_H.$$

As established above, γ is decreasing in z because we assumed that the portfolio of illiquid assets was illiquid with a higher payoff in the high state. In other words, if the portfolio of illiquid assets was liquid or not illiquid enough, $\gamma_{\psi_L} < (1-k)\frac{1-\pi}{\pi} + (2-k)\psi_H$, then we would be in the previous case of a slack resource constraint.

For the resource constraint to be binding, the portfolio of illiquid assets must be illiquid enough. The banker's holdings of liquid assets then make his portfolio less illiquid so that the banker can issue more liquid liabilities. Since resources are scarce, the bank will do this only up to the point that give him enough resources to cover those liabilities; determining the cutoff asset $(1, z^*)$.

In this equilibrium, liquid assets are separated into two types: those used as means of payments by the producer and those used as reserve assets by the banker. The reserve assets are ones with higher payoffs in the low-payoff state. One interpretation of this is that these assets are those with negative beta (Brunnermeier's good friend analogy).

The convenience yields vary across these liquid assets. The expected return rate, in



(a) Binding incentive constraint, slack resource constraint.

(b) Binding incentive and resource constraint. Bank holds no liquid assets.

(c) Binding incentive and resource constraint. Bank holds some liquid assets.

Figure 8: Isocost curves. Equilibrium asset prices with moral hazard.

terms of expected payoff, for a liquid security held by the producer is $(1 + \mu)^{-1}$. The expected return rate for a liquid asset held by the banker is necessarily higher than those for the producer, in particular the maximum return rate for the banker is $(1 + \mu z^*/\psi_L)^{-1}$. If we interpret the convenience yield as the difference to the 'fair' return on assets, 1, then we can see that both the producer and the banker pay convenience yields, but these have different origins. The producer pays convenience yields for securities that can be used as means of payment (checking accounts), while the banker pays convenience yields for securities (government bonds) that allow it to expand its issuance of means of payments.

6 The problem of a Monopolist

We now study the problem of a monopolist bank. In order to do so, we use some of the results from the previous sections where we characterized the preferences of the producer over different securities.

In particular, a monopolist bank faces a unit-continuum of identical producers. Each producer holds a single asset $D = (N^e(\ell), N^e(h))$. To keep the notation consistent with the previous sections, we denote the producer's asset portfolio as the measure μ_0^e, without loss of generality. The bank holds a portfolio of assets μ_0^b. The banker makes a take-it-or-leave-it offer to each producer: the producer, if he accepts, obtains a portfolio of securities $\{\mu_1^i, \mu_1^\ell\}$ in exchange for the producer's assets μ_0^e. The producer then decides whether to accept or reject this offer. The bank is constrained in terms of the securities it can issue. In particular, the bank's constraint is

$$\int D(\omega)d(\mu_1^i + \mu_1^\ell) \leq \int D(\omega)d(\mu_0^e + \mu_0^b),\ \text{for}\ \omega \in \{\ell, h\}.$$

That is, the bank must be able to honor the payments of the securities using either the producer's or its own assets.

Before proceeding to the analysis, we highlight the differences between this setting and the competitive one studied in the previous section. First, in this setting, the bank can set the terms of the exchange. Second, the producer, were he not to participate, can only trade using his asset, as it is. The first point implies that the profit-maximizing bank will offer the producer a portfolio of securities which give him the same utility as his original asset. The second point implies that the producer's utility over his original asset is the utility defined over an individual security (see Proposition 1 or Problem 1), rather than the utility defined over a portfolio (as in Problem 3).

The gains from trade between the bank and the producer depend on the type of asset that the producer holds. In particular, there will be no gains from trade if the producer's asset is liquid. Notice that among the liquid portfolios, the marginal rates of substitution of the bank and the producer are both equal, with value $\pi/(1-\pi)$. Moreover, the section of the producer's indifference curve corresponding to illiquid portfolios lies behind the bank's initial indifference curve. That is, any illiquid portfolio that the producer would find acceptable would leave the bank worse off.

The counterpart of the previous statement is that there could be gains from trade if the producer's initial asset is illiquid or liquid-separating. In that case, the bank would like to offer a liquid-pooling portfolio that delivers the same utility. The only limitation to such an exchange comes from the available assets. Specifically, the liquid-pooling portfolios that would exhaust the gains from trade might not be feasible for lack of assets. We analyze the bank's problem for those types of assets next.

Producer's asset is illiquid. Now, we study the case in which the producer's asset has payoffs $(N^e(\ell), N^e(h))$ with $N^e(h) > \psi_H N^e(\ell)$. While the solution to the previous case required only a single security, the bank could in principle issue multiple securities. In fact, in the current case, if liquidity is scarce, the bank would like to give the

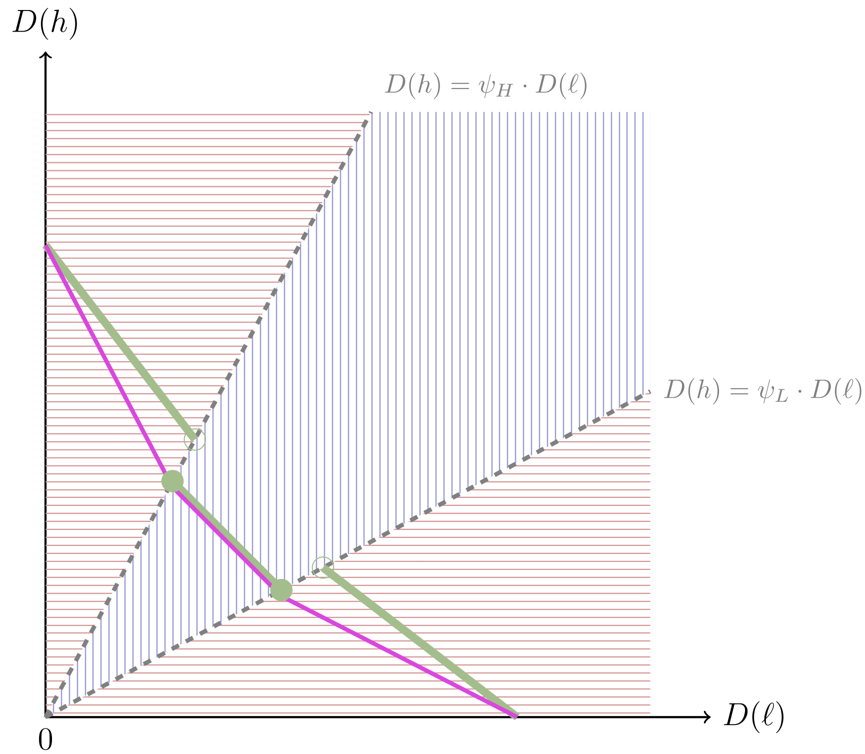


Figure 9: In green, the producer's indifference curve for an individual security. The indifference curve for a portfolio is given by joining the purple lines with the green line in the blue region.

producer two securities: the maximally illiquid security and the "marginally liquid" security. This ability of the bank implies that the bank can convexify the producer's indifference curve. In other words,, any asset $(N^e(\ell), N^e(h))$ can be split into two securities $(D(\ell)^A, D(h)^A)$ and $(D(\ell)^B, D(h)^B)$ with proportions λ and $1-\lambda$ such that $\lambda D^A + (1-\lambda)D^B = (N^e(\ell), N^e(h))$. In our analysis, this amounts to the producer evaluating his overall securities portfolio by the portfolio utility as in Problem 3.

The main difference between this case and the previous one lies on the discontinuity of the producer's security indifference curve. In particular, the bank can, via securitization, use this discontinuity to provide an acceptable portfolio to the producer at a much lower cost. In fact, for illiquid securities that are close to being liquid, the bank can create an equally-valuable "marginally liquid" security by *lowering* payoffs. This observation motivates the following classification of illiquid assets.

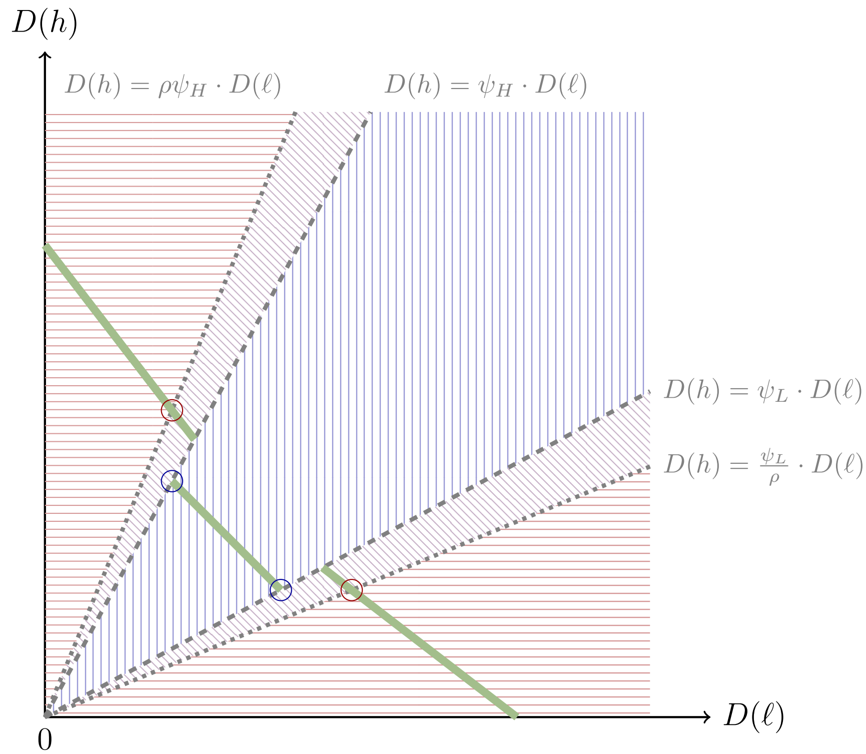


Figure 10: Assets are classified according to their ratio of payoffs $N^e(h)/N^e(\ell)$ as (red) fundamentally illiquid, (purple) structurally illiquid, (blue) liquid. The red circles are structurally illiquid assets; the producer is indifferent between them and their liquid tranche, the blue circles.

Definition 12 (Structurally and Fundamentally Illiquid Assets.) *An asset $(D(\ell), D(h))$ is structurally illiquid if*

- *It is illiquid, $D(h) > \psi_H D(\ell)$ or $D(h) < \psi_L D(\ell)$ but*
- *not illiquid enough, $D(h) \leq \rho\psi_H D(\ell)$ or $D(h) \geq \frac{\psi_L}{\rho} D(\ell)$.*

An asset is fundamentally illiquid if it is illiquid, but not structurally illiquid.

Proposition 13 *If the producer holds a structurally illiquid asset, the bank's solution consists of offering the "marginally liquid" security that makes the producer indifferent. This security is always feasible, even if the bank has no assets.*

In summary, a structurally illiquid asset does not require extra assets to be *structured* into a liquid asset that has the same value to the producer.

In contrast, the bank cannot do this with a fundamentally illiquid asset. In that case, the “marginally liquid” security that would make the producer indifferent requires the bank to combine the producer’s illiquid asset with other liquid assets. If there are not enough liquid assets, the best the bank can do is to offer the largest feasible amount of “marginally liquid” securities and make up the difference with the maximally illiquid security.

7 Conclusion

Our paper extends to the Gorton and Pennacchi (1990) security-design view of money and banking along three dimensions with historical bearing: competition for bank money, co-existence of private and public money, and joint liability. In each variation of the core model, there is a tension between the liquidity creation process and some agency friction.

Policies such as capital requirements, reserve requirements, central bank discounts, deposit insurance, last-resort lending, separation of banking activities, etc., are all about regulating the balance sheet of banks. Good policy prescriptions identify an externality that markets cannot correct. Extensions to our theory should speak to those externalities and the best way to correct them.

One potential route are relaxing the commitment assumption on the side of banks: A critical aspect of our model is that while producers have information about the assets on the banker's balance sheet, banks commit to issuing securities with known payoffs across states—even though states are unknown. A natural extension of our theory would allow the banker to deceive workers about their choice of assets. What regulatory mechanisms could be employed to guarantee incentives and what are the costs for liquidity creation? The answer to this question is left for future work.

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Appendix

A Detail of Figures 1 to 2

The data on asset and liability composition corresponds to US Chartered Depositary Institutions presented in Table L.110 of the Flow of Funds Tables. Figure1 is constructed as follows. We use several series for liabilities in Table L.110 to construct the 6 series of in Figure 1. For the category Small and Checkable Deposits we use the the sum of the data series checkable deposits and small time deposits. For Large Time Deposits we use the series for large time deposits. For REPO and Interbank we use the the sum of the series net interbank transactions, federal funds, and security repurchase agreements (net). For the category Bonds and OMP we use the sum of the series Corporate Bonds, Foreign Bonds and Open-Market Paper. The category GSE advances is the series for FHLB advances and and Sallie Mae loans. The category Equity is the sum of total financial assets minus total financial liabilities plus equity investment by bank holding companies (subcategory of miscellaneous liabilities) minus taxes payable minus unidentified miscellaneous liabilities.

Figure 1 presents seven categories. The category for Total Loans corresponds to the series for total loans. Cash and reserves is the sum of the series of cash assets, Fed Funds and reverse RP's with banks and other assets. Treasuries is the sum of the series treasury and agency securities. GSE-Backed Securities is the series for GSE-backed securities. Municipal+Int Bonds is the series for municipal bonds and foreign issued bonds. Corporate bonds is the series for corporate bonds. Equity and Mutual funds is the sum of the series equity shares and mutual fund shares.

Figure 2 presents seven asset classes. Total outstanding amounts and holdings by depositary institutions of each class is found in a different table of the flow of funds. We selected the following categories that are prevalent among the asset holdings of US Depositary Institutions. Data for the Commercial Paper category is found in the table for Open Market paper, table L.209. The category Treasury Bills corresponds to the series on Treasury securities from table L.210. The category GSE-backed is the series for securities Agency- and GSE-backed securities, table L.211. The category Muni in the figure corresponds to the table for Municipal Bonds, table L.212. The category Corporate Bonds is the series Table L.213 in the figure. The category for Total Loans corresponds to the series for total loans. Cash and reserves is the sum of the series of cash assets, Fed Funds, and reverse RP's with banks and other assets. Treasuries is the sum of the series treasury and agency securities. GSE-Backed Securities is the series for GSE-backed securities. The category Municipal+Int Bonds is the series for municipal bonds and foreign issued bonds. Corporate bonds is the series for corporate and foreign bonds, table L.213. The category Corporate securities corresponds to the series in table L.223. For the category Mortgages we use the series found in table L.217 corresponding to total mortgages. For the category Non-Mortgage loans, we take the sum of total loans in table L.214 and subtract the entries in table L.217 to get series for non-mortgage loans in outstanding amounts and held by depositary institutions.

B Omitted proofs

B.1 Equilibria in the Bilateral Trading Problem with Exclusivity

Let the producer's holdings in a meeting with a worker be $(D(\ell), D(h))$. A trading outcome consists of an amount of labor, q, and a share of the asset, n, for each state of the world $\omega \in \{\ell, h\}$. In this way, the trading outcome $\{(q(\ell), n(\ell)); (q(h), n(h))\}$ indicates that the producer would hire $q(\ell)$ units of labor and pay the worker with a share $n(\ell)$ of his asset in the low state.

A Perfect Bayesian equilibrium of this trading problem consists of a trading outcome and worker's beliefs as a function of the producer's offer such that the trading outcome is individually rational for both players, incentive compatible for the producer and that the worker's beliefs are consistent with Bayes rule for the equilibrium offers.

In particular, the individual rationality conditions are

$$n(\omega)\mathbb{E}[D(\omega)|n(\omega), q(\omega)] \geq q(\omega), \tag{B.1}$$
$$\rho \cdot q(\omega) + (1 - n(\omega)) \cdot D(\omega) \geq D(\omega), \tag{B.2}$$

for every state ω. The first condition is the worker's individual rationality constraint and the expectation is taken with respect to the worker's beliefs conditional on the offer. The second condition is the producer's individual rationality constraint.

The incentive compatibility conditions are

$$\rho \cdot q(\omega) + (1 - n(\omega)) \cdot D(\omega) \geq \rho \cdot q(\tilde{\omega}) + (1 - n(\tilde{\omega})) \cdot D(\omega), \tag{B.3}$$

for all ω and $\tilde{\omega}$.

B.1.1 Pooling Outcomes

In a pooling trading outcome, the outcomes are the same across states. Simplifying notation to $q = q(\omega)$ and $n = n(\omega)$, we can write the IR conditions as

$$n \cdot \mathbb{E}[D(\omega)] \geq q, \tag{B.4}$$
$$\rho \cdot q \geq n \cdot D(\omega). \tag{B.5}$$

In this case, the IC conditions are trivially satisfied.

Figure 11 plots the individual rationality constraints for two assets with the same expected value. In each case, the outcomes that are individually rational for the worker are below the red line. The outcomes above the blue line are individually rational for the producer in the high state. The ones above the green line are individually rational for the producer in the low state. A pooling outcome to be part of an equilibrium must satisfy all

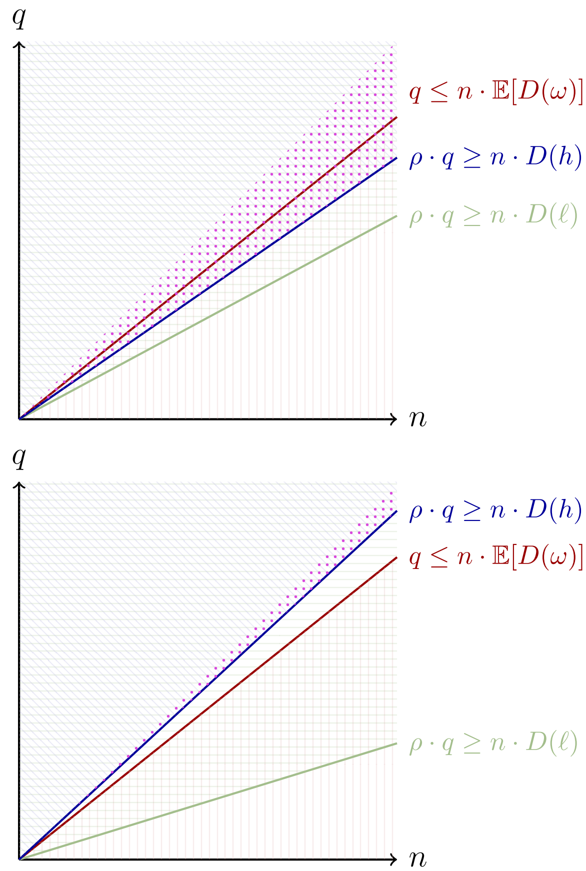

Figure 11: Pooling trading outcomes. Left: Liquid asset $D(h) \in [D(\ell), \psi_H \cdot D(\ell)]$. Right: Illiquid asset $D(h) \geq \psi_H \cdot D(\ell)$. In red, IR outcomes for the worker. In blue and green, IR outcomes for the producer in the high and low state, respectively. Notice that there are no IR pooling outcomes for the illiquid asset (right panel).

IR conditions.

The figure shows that the IR conditions can be satisfied when the asset's payoffs are relatively stable across states. However, if the asset pays much more in one of the states, there can be no equilibrium pooling outcomes. In particular, in the right panel, the asset's payoff is so much larger in the high state that the producer would not be willing to hire the worker in spite of the gains from trade.

It is then clear to see that the pooling outcome can be sustained only if the blue and green lines lie below the red line. Put differently, the asset must be such that

$$\rho \cdot \mathbb{E}[D(\omega)] \geq D(h) \text{ and } \rho \cdot \mathbb{E}[D(\omega)] \geq D(\ell).$$

$$\rho \cdot [\pi(h)D(h) + \pi(\ell)D(\ell)] \geq D(h) \text{ and } \rho \cdot [\pi(h)D(h) + \pi(\ell)D(\ell)] \geq D(\ell).$$

Notice that the condition will be trivially satisfied in state ω if $\rho \cdot \pi(\omega) > 1$ which motivated our choice of assumptions to the contrary in Section 2.1. Given these assumptions, we can solve for the *liquidity bounds*

$$\psi_H \equiv \frac{\rho\pi(\ell)}{1-\rho\pi(h)} > 1, \psi_L \equiv \frac{1-\rho\pi(\ell)}{\rho\pi(h)} < 1.$$

An asset can sustain a pooling trading outcome only if $D(h)/D(\ell) \in [\psi_L, \psi_H]$.

B.1.2 Separating Outcomes

In a separating trading outcome, the worker's beliefs must be accurate which gives rise to the following IR conditions.

$$n(\omega) \cdot D(\omega) \geq q(\omega), \tag{B.6}$$

$$\rho \cdot q(\omega) \geq n(\omega) \cdot D(\omega), \tag{B.7}$$

for all ω. The first condition is the IR condition for the worker. The second condition is the IR condition for the producers. With a separating outcome, the incentive compatibility conditions will be relevant.

Notice in the left panel of Figure 12 that the worker's IR constraint is now split into two pieces, one for each state. There are always individually rational separating outcomes, a natural consequence of the gains from trade.

Furthermore, we can see in the figure that the producer in the high state has no incentive to pretend to be in the low state. Any low-state allocation that is individually rational will be strictly worse for the high-state producer than the no-trade outcome $q = 0, n = 0$. Taking the low-state outcome to be $q(\ell) = D(\ell), n(\ell) = 1$, the IC constraint for the low-state producer is given by the translation of his IR constraint to the allocation point; represented

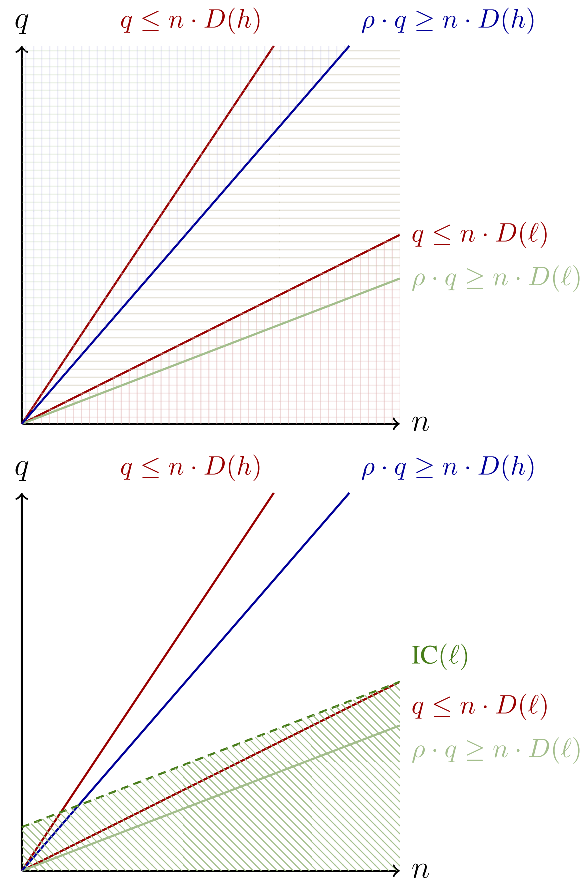

Figure 12: Separating outcomes. Left: shaded IR regions. Right: IC constraint for low-state producer given outcome $q(\ell) = D(\ell), n(\ell) = 1$.

as a dashed line in the right panel. An incentive-compatible trading outcome for the high state must therefore lie below this line; these outcomes are represented as the shaded area in green. Clearly, the producer in the high state cannot realize the full gains from trade with separation.

B.1.3 Equilibrium Selection

As is standard in asymmetric information problems, the analysis of the individual rationality and incentive compatibility constraints reduces the set of possible outcomes but does not select a unique outcome. Given the goals of the paper, we must establish a criterion for selecting the equilibrium outcomes.

In this section, we describe in detail and explain our preferred equilibrium selection, the one we use in the paper. We then show the results of using different criteria and how these results are qualitatively similar.

Preferred Selection: Non-exclusive trading. Our preferred selection consists of the pooling outcome whenever this is feasible and a separating outcome with no trade in the high-state and full trade in the low-state. This selection is the unique equilibrium of a bilateral trading game with nonexclusive competition as in Attar et al. (2011). The equilibrium strategies in this game are particularly simple: workers post linear price schedules at a unit price equal to either the value in the low state or the expected value depending on the distribution of values of the asset. Nonexclusive trading rules out signaling through retention because retention is not incentive compatible: an producer could hire another worker with the remaining share of his asset, securing a higher payoff regardless of its risk profile.

Alternative Selection: Signaling through retention. Assuming exclusive trading, the producer in the high state can credibly signal the high value of the asset by retaining a fraction. In the right panel of Figure 12, any outcome in the triangle formed by the worker's high-state IR constraint (upper red line), the producer's high-state IR constraint (blue line), and the IC constraint of the low-state producer (dashed green line) other than the no-trade outcome involves signaling through retention. The trading outcome that maximizes the gains from trade can be solved as

$$\rho \cdot n(h)D(h) - n(h) \cdot D(\ell) = (\rho - 1) \cdot D(\ell) \rightarrow n(h) = \frac{(\rho - 1) \cdot D(\ell)}{\rho \cdot D(h) - D(\ell)},$$

where we have used the fact that $q(h) = n(h)D(h)$. The level of retention is increasing in the ratio of payoffs $D(h)/D(\ell)$.

This means that the producer's value of holding an asset with payoffs $D(\ell), D(h)$ is

$$\mathbb{U}[D] = \rho \cdot \mathbb{E}[D(\omega)] - (\rho - 1) \left(\frac{\rho \cdot (D(h) - D(\ell))}{\rho \cdot D(h) - D(\ell)} \right) \pi(h)D(h).$$

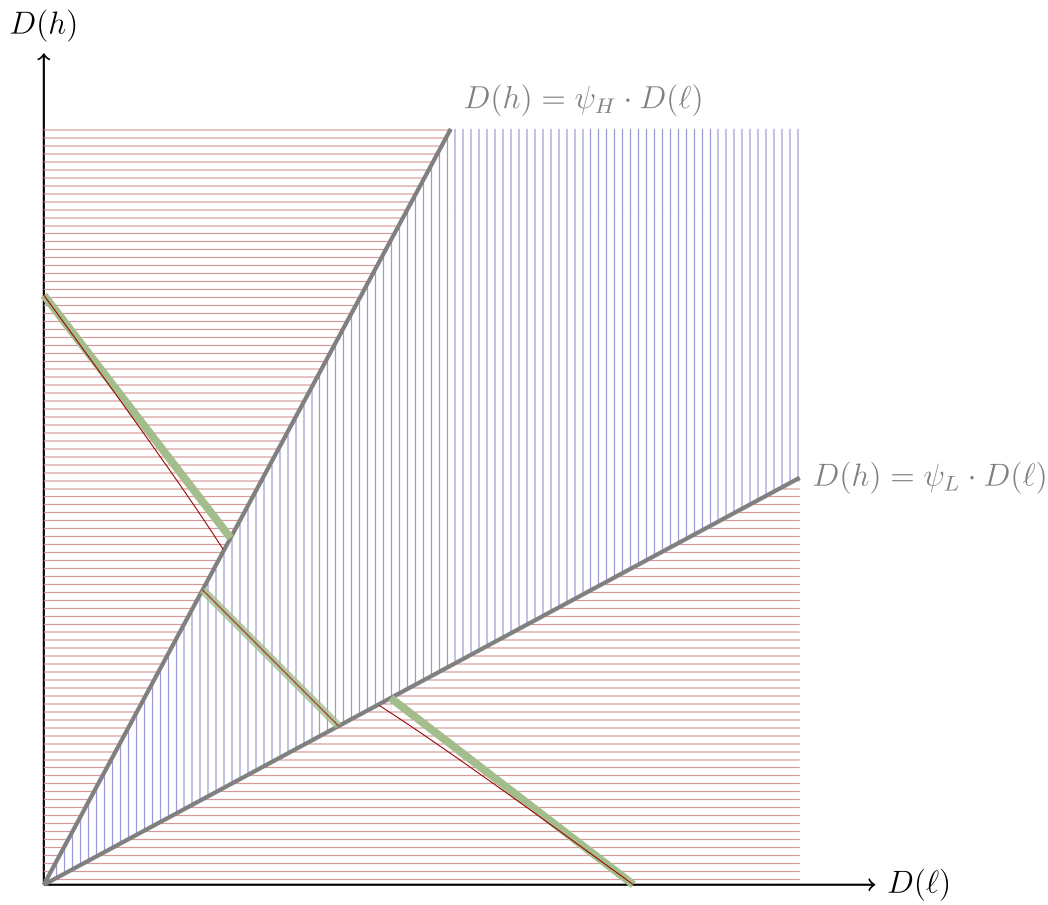

Figure 13: Indirect utility for the asset when producer can signal through retention. Utility is unchanged in the pooling region, but it is higher in the separating region.