The macroeconomic implications of the Gen-AI economy

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Abstract

We study the potential impact of the generative artificial intelligence (Gen-AI) revolution on the US economy through the lens of a multi-sector model in which we explicitly model the role of Gen-AI services in customer base management. In our model with carefully calibrated input-output linkages and the size of the Gen-AI sector, we find large spillovers of the Gen-AI productivity gains into the overall economy. A 10% increase in productivity in the Gen-AI sector over a 10 year horizon implies a 6% increase in aggregate GDP, despite the AI sector representing only 14% of the overall economy. That shock also implies a significant reallocation of labor away from the AI sector and into non-AI sectors. We decompose these effects into parts coming from the input-output structure and customer base management and find that they each contribute equally to the rise in GDP. In the absence of either channels, real GDP essentially does not respond to the increase in productivity in the AI sector.

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> Accelerated computing and generative AI have hit the tipping point. Demand is surging worldwide across companies, industries and nations.
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> Jensen Huang, Nvidia CEO and founder

1 Introduction

In the wake of the 21st century, the world has witnessed an unprecedented surge in technological advancements, with Artificial Intelligence (AI) standing as one of the potentially most transformative innovations of our time. As AI systems continue to evolve and permeate various facets of economic activity, it becomes increasingly important to understand the implications of this technology for the macroeconomy. The integration of AI into industries spanning from healthcare to manufacturing, finance, and beyond gives rise to changes in the economic landscape that are critical to analyze and quantify. While the impact of AI through automation and substitution of tasks in a production process has received significant attention in the literature (Aghion et al. (2018), Acemoglu and Restrepo (2018a), Acemoglu et al. (2022)), much less is known about how AI impacts the economy through data collection, analysis and distribution for the purpose of sales and customer base management.

This paper aims to fill this gap by employing a quantitative multi-sector model, which explicitly incorporates the impact of AI on customer build up, acquisition and retention across sectors. This choice is motivated by several industry trends, contending that harnessing generative AI enhances the efficiency of customer service. For example, Brynjolfsson et al. (2023) finds that the use of an AI tool leads to an increase of almost 14% in the productivity of customer support agents in a Fortune 500 software company. A 2023 report published by the Boston Consulting Group (Bamberger et al., 2023) suggests that the adoption of generative AI could potentially lead to a substantial increase in productivity within customer service operations, ranging from 30% to 50%. Furthermore, recent surveys conducted by Mckinsey & Company (Chui et al., 2023) reveal that organizations are increasingly utilizing generative AI in areas such as marketing and sales, product and service development, and service operations. Notably, the survey indicates that 77% of respondents in business, legal, and professional services sectors have experimented with generative AI tools since their introduction. Additionally, research from the International Monetary Fund (Melina et al., 2024) highlights that approximately 30% of employment in professional occupations in the

UK exhibits a high degree of exposure to generative AI technologies. Finally, Felten et al. (2023) identifies key sectors most impacted by this technological advancement, including legal services, investment activities, accounting, software publishing, and computer systems design.

In order to capture these forces formally, we set up a 3-sector model with an explicit input-output structure and frictions in building customer base via marketing expenditures, along the lines of Drozd and Nosal (2012). In the model, the Gen-AI-intensive sector produces marketing services, which are then used by all sectors to build their customer bases, which in turn determine the demand for their product. We model improvements in AI technology as a positive productivity shock in the gen-AI sector, which not only affects the cost of its services as an intermediate input into production, but the marketing cost as well. We calibrate the model's input-output structure using the 'use tables' of the BEA accounts. We map Sector 1, the gen-AI intensive sector, into eight NAICS 3-digit service industries most likely to be exposed to AI and playing a role in marketing activities. Sector 2 is mapped into more traditional service industries and Sector 3 is mapped into manufacturing industries. We then parameterize the customer base frictions to match marketing expenditure to sales ratio of 7% and wholesale markups of 10%.

Through the lens of our calibrated model, we study the impact of changes in the productivity of the gen-AI service sector on the allocation of inputs and aggregate economic activity. First, on business cycle frequency, a positive productivity shock in the gen-AI service sector leads to a shift in labor and capital away from gen-AI and towards manufacturing and other services. At the same time, it leads to an increase in aggregate output in all sectors, increased consumption and investment and a relatively modest impact on aggregate employment. Intuitively, improvements in gen-AI technology make it cheaper to build customer base in all sectors. Taking advantage of that improvement, however, requires higher production, which is achieved by the reallocation of capital and labor into Sectors 2 and 3 which did not get a positive productivity shock. As a result, output in the gen-AI sector goes up by less than the productivity shock, but output in the remaining sectors gets boosted. This spillover effect is driven by the marketing friction. In fact, in the version of the model without customer capital and just input-output linkages, aggregate output response is only 14% of the AI sector response on impact, while in the model with only customer capital the aggregate response is 90% of the AI sector response. Given that the gen-AI service sector size in our calibration is less than 15% of aggregate output, this shows a very powerful spillover effect.

Our main quantitative experiment simulates the effect of a permanent increase in gen-AI sector productivity, over the transition and across steady states. Specifically, we feed into the model a permanent productivity increase of 10% in the gen-AI sector, motivated by industry estimates.[1] Along the transition, the economy responds to this shock by increasing the use of the gen-AI intermediate good by all sectors in the economy. At the same time, employment drops significantly in the gen-AI service sector, settling 5% below its steady state 10 years after the shock, while the remaining sectors exhibit an increase of about 2% above their respective steady states. Capital accumulation drops temporarily in the gen-AI sector, while increasing immediately in the non-AI sectors. Marketing expenditures and search effort increases in all sectors, reflecting more reliance on the gen-AI sector's output. Finally, real output goes up by around 6% in *all* sectors, even though the fundamental productivity gains only hit the relatively small gen-AI sector.

Across steady states, a permanent 10% increase in productivity in the Gen-AI sector leads to an approximately 8% increase in aggregate GDP. About 90% of the change in GDP happens after 16 years of the initial shock. Since our quantitative model incorporates an input-output structure and a customer capital component, we are able to shut down each of those in turn to investigate the main forces behind the change. By solving versions of our model that isolate each of these elements, we find that both the input-output structure and customer capital friction contribute about half of to the rise in GDP. Specifically, a 10% increase in productivity in the Gen-AI service sector with only the input-output component results in a GDP increase of about 4%. In the absence of both channels, real GDP rises by a mere 0.9%.

Crucially, the input-output structure and customer search elements play a vital role in the reallocation of resources following the AI shock. In the baseline model, labor declines in the gen-AI service sector by 4%, while increasing in the other two sectors by around 2%. However, this strong labor relocation is solely driven by the customer capital friction and essentially disappears in the model with just input-output linkages. As for capital relocation, the input-output component and the customer capital component work in opposite directions. Input-output linkages push capital to increase roughly equally in all sectors, while customer search pushes capital to relocate away from the gen-AI services towards the other sectors, just like labor. In the case of capital, input-output linkages are stronger and capital increases in the baseline model across all sectors, but by a smaller amount in the gen-AI sector relative to the other sectors. We find that customer capital creates significant spillovers across sectors of the improvement in productivity in the service sector. Absent customer capital friction, most of the output increases are observed in the service sector (18%, versus 5% and 4% in the other 2 sectors), while in the baseline model the gains are more evenly distributed (11%,

[1]This exercise is motivated by a recent report by Goldman Sachs(Hatzius et al., 2023), which highlights the potential impact of generative AI of 7% over 10 years on the world economy. The 10% increase in gen-AI productivity comes close to generating that increase.

9% and 8%). As a result, the gain for aggregate GDP is much higher in the baseline model (9%) relative to a model with just input-output linkages (4%).

Our work relates to several strands of literature. First, our paper is connected to the literature on technological progress. Change (1990), Aghion and Howitt (1992), and Kogan et al. (2017) argue the important role of technological change in economic growth. Babina et al. (2024) empirically analyze AI-related technologies driven growth concentrates among larger firms through product innovation. We complement these analyses by focusing on the AI applications that pertain to overcoming frictions in customer acquisition and retention in the final and intermediate goods markets.

Second, our paper draws from the growing literature exploring the macroeconomic implications of AI. In the context of automation, Acemoglu and Restrepo (2018b) proposed task-based production technology and discussed the labor substitution effect. Acemoglu et al. (2022) show that AI affects the composition of occupations within AI-exposed firms. From the viewpoint of labor complement, Kanazawa et al. (2022) and Noy and Zhang (2023) show that an AI system improves workers' productivity and leads to a narrowing of the productivity gap between workers by benefiting the low-skilled more. Pizzinelli et al. (2023) examines the impact of AI on labor markets using cross-country variation. While existing literature on AI often investigates the implications on the labor market, we assess the effect of AI via customer acquisition efficiency, and show there is potential for large spillovers to other sectors and aggregate GDP.

Third, we contribute to the literature on the role of customer capital. Most of papers such as Kleshchelski and Vincent (2009), Drozd and Nosal (2012), Gourio and Rudanko (2014), Paciello et al. (2019), Roldan-Blanco and Gilbukh (2021), and Rudanko (2022) concentrate on the implications for firm price setting with long-term customer relationships in a product market with search friction. Morlacco and Zeke (2021) study the role of customer capital in the industry concentration and market power under the low-interest environment. The present paper develops the customer search framework of Drozd and Nosal (2012) to characterize the service industry and the focus on the economic dynamics through the special role of AI differentiates our work from the existing studies.

The paper is organized as follows. The model is outlined in Section 2. In Section 3, we explain our calibration strategy and discuss impulse responses. Section 4 contains our main quantitative results of dynamic responses to growth in the Gen-AI sector's productivity.

2 Model

We build a multi-sector general equilibrium model with customer capital friction. At the lowest level of aggregation, there are intermediate good producers in each sector j. They sell their output to retailers, who then re-sell it to households as final goods, to other producers as intermediate inputs. Sector 1 in the model is the gen-AI sector, whose output is additionally re-sold by retailers as marketing and search inputs. Trade between retailers and intermediate good producers is subject to a customer acquisition friction in the spirit of Drozd and Nosal (2012). Figure 1 provides a graphical representation of the flow of goods in the model.

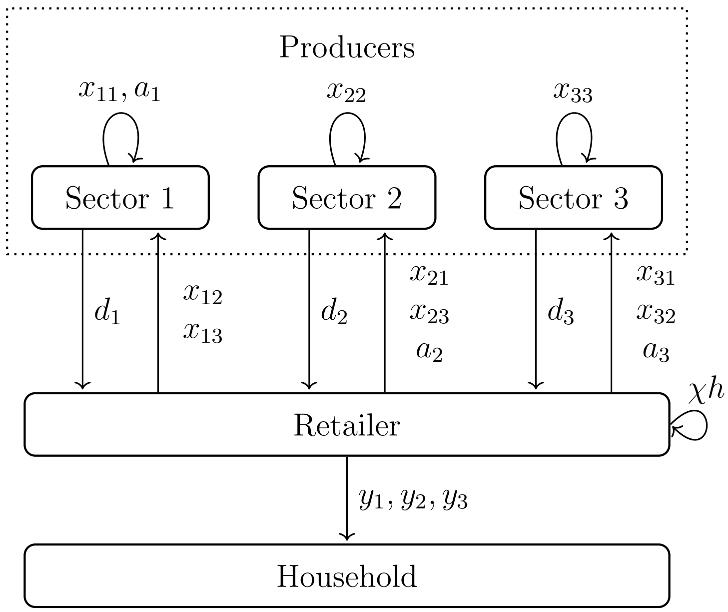

Figure 1: Overview of Model Structure: the flow of goods from sellers to purchasers

2.1 Intermediate Producers

We assume that intermediate producers are organized in three sectors, denoted by j, and mapped later in the calibration to US industries, separated by their potential exposure to AI. A unit measure of identical competitive producers operate in each sector. Each producer has access to a constant returns to scale production function $z_j F(k_j, l_j, \{x_{jm}\})$ that uses capital k_j, labor l_j, and intermediate inputs x_{jm} produced by each sector m, and is subject to a sector-specific stochastic technology shock z_j which follows an exogenous AR(1) process:

$$\ln z_{jt} = (1 - \rho_z^j) \ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt} \tag{1}$$

where $\rho_z^j \in [0, 1]$ is a persistence parameter.

Since the production function is assumed to be constant returns to scale, we can summarize the production process by an sectoral marginal cost v_j. Specifically, given factor prices w, r, sectoral output prices p_j and productivity shock z_j, the marginal cost, equal to per unit cost, is given by:[2]

$$v_{jt} \equiv \min_{k,l,x} \left\{ w_t l_{jt} + r_t k_{jt} + v_{jt} x_{jjt} + \sum_{m \neq j} p_{mt} x_{jmt} \mid z_{jt} F(k,l,x) = 1 \right\}$$

2.1.1 List of customers and Market shares

Intermediate producers do not directly trade with households, but instead sell their product to a sector of retailers, who then resell the goods to the households or other firms (as intermediate goods) in a competitive market. Trade between producers and retailers is subject to a friction, which we model as search and matching. Specifically, to match with retailers, each representative good j producer has access to an explicitly formulated marketing technology and accumulates a form of capital labeled marketing capital, m_j. We assume that each match with a retailer is long lasting and is subject to an exogenous separation rate $\delta_H \in [0, 1]$, and that marketing capital helps create new matches for a producer. Formally, each producer's customer base H_{jt} evolves as follows

$$H_{jt} = (1 - \delta_H) H_{jt-1} + \frac{m_j}{\sum_j \bar{m}_j} h_t \tag{2}$$

where $\bar{m}_j$ denotes the average levels of marketing capital of sector j producers and h_t the measure of searching retailers. Note that $\bar{m}_j = m_j$ in equilibrium. We assume that in each match in a producer's customer base, one unit of the good can be traded per period. Thus, sales of a given producer, denoted by d_{jt}, cannot exceed the size of the customer list H_{jt}:

$$d_{jt} \leq H_{jt}.$$

2.1.2 Marketing capital

As per equation (2), each sector j producer accumulates marketing capital m_j to attract searching retailers h_t by choosing the level of marketing input a_{jt}. Given last period's level of marketing capital m_{jt-1} and the current level of marketing input a_{jt}, current period

[2] For detailed derivations of v_{jt}, see Appendix A.2.

marketing capital m_{jt} is given by

$$m_{jt} = (1-\delta_m^j)m_{jt-1} + a_{jt} - \frac{\psi_j}{2}m_{jt-1}\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)^2,$$

where $\delta_m^j \in [0,1]$ denotes the depreciation rate of marketing capital in sector j and $\psi_j \in [0,\infty)$ denotes the market expansion friction parameter.

2.1.3 Profit Maximization

The instantaneous profit function Π_j of a producer in sector j is determined by the difference between the profit from sales and the total cost of marketing:

$$\Pi_{ijt} = \begin{cases} (q_{ijt} - v_{ijt})d_{ijt} - v_{ijt}a_{ijt} & (j=1) \\ (q_{ijt} - v_{ijt})d_{ijt} - p_{1t}a_{ijt} & (j \neq 1) \end{cases}$$

where q_j is the wholesale price, determined by bargaining with the retailers and described in detail later in this section. Note that Sector 1 faces the cost of marketing input equal to own sectoral marginal cost, since marketing is its own output.[3]

The maximization problem is given by

$$\max_{a_{jt},m_{jt},d_{jt},H_{jt}} \mathbb{E}_t\left[\sum_{k=0}\Omega_{t,t+k}\Pi_{jt+k}\right]$$

$$\text{s.t. } m_{jt} = (1-\delta_m^j)m_{jt-1} + a_{ijt} - \frac{\psi_j}{2}m_{jt-1}\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)^2$$

$$H_{jt} = (1-\delta_H)H_{jt-1} + \frac{m_{jt}}{\sum_j \bar{m}_{jt}}h_t$$

$$d_{jt} \leq H_{jt}$$

where $\Omega_{t,t+k}$ denotes the discount factor defined by $\beta^k u_c(c_{t+k}, l_{t+k})/u_c(c_t, l_t)$ derived from the household's problem.

2.2 Retailers

Atomless retailers purchase goods from each sector's producers and resell them in a local competitive market. In each period, there is a mass of retailers already matched with sector

[3] This assumption means that purchases of marketing inputs for Sector 1 producers are not subject to the search friction, as those producers use their own product for that. However, if Sector 1 producers needed to buy marketing input from retailers, just like the other sectors' producers, the results would not change in a significant way.

j producers, H_j, and a mass of new entrants h (searching retailers). A new entrant, upon paying the search cost χp_1 (in goods produced by Sector 1), meets with probability π_j a producer from sector j. Each entrant takes this probability as given, but in equilibrium it is determined by the marketing capital levels accumulated by the producers, according to

$$\pi_{jt} = \frac{m_{jt}}{\sum_j \bar{m}_{jt}}$$

where, as before, $\bar{m}_j$ denotes the average levels of marketing capital of sector j producers.

2.2.1 Bargaining and Wholesale Prices

We assume that each retailer bargains with the producer over the wholesalve price, q_j, to split the total surplus from a given match. This surplus is split in consistency with Nash bargaining solution with continual renegotiation. Specifically, the value of the intermediate producer in sector j, W_j, and the value for the retailer matched with a producer in sector j, J_j, are defined by

$$\begin{aligned} W_{jt} &= \max\{0, q_{jt} - v_{jt}\} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1} W_{jt+1}], \\ J_{jt} &= \max\{0, p_{jt} - q_{jt}\} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1} J_{jt+1}], \end{aligned}$$

where q_{jt} is the wholesale price and p_{jt} is the retail price of sector j output.

Given the above value functions and the bargaining power $\theta \in [0,1]$, at each date and state, the wholesale price is a solution to the Nash bargaining problem:

$$q^*_{jt} = \underset{q}{\operatorname{argmax}}\, J^{\theta}_{jt} W^{1-\theta}_{jt}$$

Under continual renegotiation, the wholesale price allocates θ fraction of the total instantaneous trade surplus to the producer and fraction $1-\theta$ to the retailer:

$$q_{jt} = \theta p_{jt} + (1-\theta) v_{jt}. \tag{3}$$

2.2.2 Free Entry Condition

Free entry into the retail sector governs the measures of searching retailers. Specifically, it states that the expected surplus for the retailer from matching is equal to the search cost incurred to obtain a match opportunity:

$$\sum_j \pi_{jt} J_{jt} \leq \chi p_{1t}.$$

The condition holds with equality whenever $h > 0$.[4]

2.3 Households

In each period, a unit measure of identical, infinitely lived households choose the level of consumption c, investment in physical capital i, labor supply l, purchases of sectoral goods y_j, and purchases one-period uncontingent bonds b_{t+1} to maximize the expected discounted lifetime utility

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

where u is strictly concave and satisfies the Inada conditions, and $\beta \in (0,1)$. The preferences over sectoral goods is determined by a CES aggregator G:

$$G(\{y_j\}) = \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where $\gamma > 0$ denotes the elasticity of substitution and $\omega_j \in [0,1]$ determines the share of expenditure on good j, satisfying $\sum_j \omega_j = 1$.

Households combine sectoral goods y_j through the above aggregator into a composite good which they use for consumption and investment, according to

$$c_t + i_t = G(\{y_j\}).$$

Physical capital follows the standard law of motion with adjustment cost:

$$k_t = (1-\delta)k_{t-1} + i_t - \phi(i_t, k_{t-1})$$

where $\delta \in (0,1)$ denotes the depreciation of physical capital, ϕ is the adjustment cost function.

Finally, the budget constraint is given by

$$\sum_j p_{jt} y_{jt} + b_{t+1} = R_{t-1} b_t + w_t l_t + r_t k_{t-1} + \Pi_t,$$

where p_j denotes the real retail price of good j, w the real wage, R the real (gross) risk free rate, r^K the real return on capital, Π the real profit from firms.

[4] The search cost χ is assumed uniformly bounded away from zero.

2.4 Market Clearing

The aggregate resource constraints are given by

$$z_{jt}F(k_{jt}, l_{jt}, \{x_{jmt}\}) = \begin{cases} y_{jt} + \sum_m x_{mjt} + \sum_m a_{mt} + \chi h_t & (j = 1) \\ y_{jt} + \sum_m x_{mjt} & (j \neq 1) \end{cases}$$

$$d_{jt} = \begin{cases} y_{jt} + \sum_{m \neq j} x_{mjt} + \sum_{m \neq j} a_{mt} + \chi h_t & (j = 1) \\ y_{jt} + \sum_{m \neq j} x_{mjt} & (j \neq 1) \end{cases}$$

Labor and capital markets clearing is

$$\sum_j l_{jt} = l_t, \quad \sum_j k_{jt} = k_{t-1}$$

where, following Ngai and Pissarides (2007), we assume the perfect labor and capital mobility across sectors.

3 Calibration

In this section, we explain how we pick functional forms and parameter values for our calibration.

3.1 Functional Forms

We set the utility function to be CRRA:

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \xi \frac{l_t^{1+\eta}}{1+\eta}$$

where ξ denotes the relative disutility from labor which determines the steady state value of l. The production technology for each sector is given by

$$F_{jt} = F(k_{jt}, l_{jt}, \{x_{jmt}\}) = k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j} \tag{4}$$

Finally, capital adjustment cost function takes the form

$$\phi(i_t, k_{t-1}) = \frac{\phi}{2} k_{t-1} \left(\frac{i_t}{k_{t-1}} - \delta \right)^2.$$

where α_m^j are the factor shares.

3.2 Parameter values calibrated independently

Standard parameters Consider first the parameters that can be selected independently from all other parameters. We set one period in the model corresponding to one year, which gives the discount factor $\beta = 0.9$ and capital depreciation rate $\delta = 0.1$. Following standard estimates in previous literature, we set the relative risk aversion $\sigma = 1.0$ and the inverse of Frisch labor supply elasticity $\eta = 2.0$. Finally, we arbitrarily set the sectoral elasticity of substitution $\gamma = 1.1$ and the the match separation rate $\delta_H = 1$.[5]

Table 1: Independently calibrated parameters

Parameter	Symbol	Value
Discount factor	β	0.9
Relative Risk Aversion	σ	1.0
Inverse of Frisch labor supply elasticity	η	2.0
Elasticity of Substitution	γ	1.1
Bargaining power	θ	0.5
Physical Capital Depreciation	δ	0.1
Customer list destruction rate	δ_H	1.0

Factor and sectoral shares Our goal is to map the three sectors in our model into a highly AI-intensive service sector (Sector 1), traditional service sector (Sector 2) and the rest (Sector 3). With that in mind, we classify 3-digit NAICS industries into the three sectors in our model as follows. In the *Baseline* calibration, we classify sector 1 as including 8 industries: (i) Publishing industries, except internet (includes software) (NAICS 511), (ii) Motion picture and sound recording industries (NAICS 512), (iii) Broadcasting and telecommunication (NAICS 515, 517), (iv) Data processing, internet publishing, and other information services (NAICS 518, 519), (v) Computer systems design and related services (NAICS 5415), (vi) Miscellaneous professional, scientific, and technical services (NAICS 5412-5414,5416-5419), (vii) Management of companies and enterprises (NAICS 55), and (viii) Administrative and support services (NAICS 561). Under the *Baseline* calibration, the gen-AI sector accounts for 14.2% of aggregate GDP. We also consider a much more restrictive calibration, which we call *Conservative*, where we classify Sector 1 as including: (i) Data processing, internet publishing, and other information services (NAICS 518, 519), (ii) Computer systems design and related services (NAICS 5415), (iii) Management of companies

[5] The sectoral elasticity of substitution falls broadly within the broad range of estimates in the literature (see (Ostry and Reinhart, 1992), (Stockman and Tesar, 1995)).

and enterprises (NAICS 55), and (iv) Administrative and support services (NAICS 561). Under this calibration, the gen-AI sector accounts for 9.8% of aggregate GDP. Sector 2 includes the remaining service sectors. Educational instruction, arts, design, and sports fall within this sector. Finally, Sector 3 includes the rest. Prime examples are manufacturing and construction. We report detailed industry classification into the three sectors in Table 9.

We compute the value of factor shares in each sector j using the 'use tables' of the input-output accounts constructed by the BEA. The use table shows how commodities are utilized by different sectors both as intermediate inputs and final goods. We calculate the payment values from sectors categorized by us as sector i to those categorized as sector j, where $i, j \in \{1, 2, 3\}$, which gives intermediate input expenditures of each sector. Next, we take the total compensation of employees and the gross operating surplus to stand in for labor and capital income, respectively. Finally, to get the shares α_j^i in equation (4), we normalize each of those expenditure components by their sum. Finally, we compute the averages of these normalized values over the period spanning from 2007 to 2018, in accordance with (Chui et al., 2023).

Table 2: Input share α_m^j based on the classification in Table 9.

	Sector 1	Sector 2	Sector 3
α_1^j	0.20	0.10	0.07
α_2^j	0.08	0.18	0.07
α_3^j	0.11	0.10	0.39
Labor α_L^j	0.39	0.23	0.29
Capital α_k^j	0.23	0.39	0.18

For sectoral consumption, we use sectoral quarterly consumption from the National Income and Product Accounts (NIPA). To match these sectoral consumption expenditures from NIPA classification to the IO account classification, we use the PCE Bridge Table[6] provided by the BEA. To allocate the NIPA components spending into the IO classification spending amount, we re-classify Purchasers' Value by Commodity Code and sum up the Purchasers' Value along our classification (Sectors 1-3) and then calculate the shares. We use the averages of these shares over 2007 to 2018.

Table 3: Spending share ω_j based on the classification in Table 9.

	Sector 1	Sector 2	Sector 3
ω_j	0.053	0.279	0.668

[6]We use the 73 commodities composition table to apply for each year, 2007-2018.

3.3 Parameter values calibrated jointly

The remaining parameters are selected jointly to align the model's predictions with a set of empirical moments. The labor disutility parameter ξ, the search cost χ, and the marketing capital depreciation δ_m^j are chosen to target steady state averages: unit labor supply as normalization, real GDP-weighted average gross wholesale markup[7] of 10%, and marketing expenditure to sales ratio[8] of 7%.

The second group of jointly determined parameters includes the persistence and volatility of the productivity process in each sector, ρ_j and σ_j, and the marketing capital adjustment cost parameters, ψ_j. To discipline the choice of these parameters, we use the simulated method of moments to match the volatility ratio of the producer price index (PPI) to the price index of personal consumption (PCE) in each sector, as well as the persistence and standard deviation of the solow residual in each sector.

To compute these targets, we use the BEA supply table to create the weighted average PPI, which we then use to create the sectoral PPI. The industry's supply to the outside of the industry is used as weight in accordance with the BLS[9] industry PPI weighting method.[10] Using the PCE Bridge Table provided by the BEA, we calculate the weighted average PCE for sectoral PCE, using the purchaser's value as the weight. Next, we use the HP-filter for log price indices with $\lambda = 100$ to calculate the sectoral price volatility. Finally, we compute the cycle components' standard deviation. As for the model-generated moments, we use the standard deviation of the % deviation from steady state of the corresponding variables.

Table 4: Price volatility based on the classification in Table 9

	Sector 1	Sector 2	Sector 3
σ_{PPI}	0.005	0.040	0.032
σ_{PCE}	0.005	0.011	0.012
$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	1.024	3.492	2.575

In order to generate targets for the productivity process, we use sectoral Solow residuals. The data come from the BEA-BLS Integrated Industry-level Production Accounts

[7]The average wholesale markup is computed as $(\text{rGDP}_1\mathcal{U}_1 + \text{rGDP}_2\mathcal{U}_2 + \text{rGDP}_3\mathcal{U}_3)/\text{rGDP}$, where $\mathcal{U}_j$ is sectoral markup.

[8]In the model, the marketing expenditure to sales ratio by sector is $\mathcal{M}_1 = (v_1 a_1)/(q_1 d_1)$ for $j = 1$ and $\mathcal{M}_j = (p_1 a_j)/(q_j d_j)$ for all other values of j. The calibration of average wholesale markup and marketing expenditure ratio follows closely Drozd and Nosal (2012).

[9]https://www.bls.gov/ppi/faqs/questions-and-answers.htm

[10]Note that we use the equally-weighted average index on sub-indices when the corresponding index is unavailable. Also, we treat the weights of indices that are unavailable until the middle of the period as zero and compute the index using the available indices only.

(KLEMS).[11] We create the weighted average TFP using the value added ratio as weights in order to construct the annual sectoral productivity. Next, we use the cubic-detrended log series to estimate the AR(1) model, yielding the persistence parameter ρ_i and the volatility parameter σ_i.[12]

Table 5: Parameters of productivity process based on the classification in Table 9.

	ρ_i			σ_i		
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
KLEMS	0.535	0.574	0.694	0.012	0.008	0.006

The corresponding Solow residual in the model is given by rGDP_{jt}/F_{jt}, where

$$\text{rGDP}_{jt} = \begin{cases} p_{10}(y_{1t} + \sum_j a_{jt}) & (j = 1) \\ p_{j0} y_{jt} & (j \neq 1) \end{cases} \tag{5}$$

Finally, we use the volatility of investment relative to the volatility of GDP as a target for the capital adjustment cost ϕ. The jointly calibrated parameters are reported in Table 6. Appendix K reports details of the *Conservative* calibration of the model.

Table 6: Jointly calibrated parameters.

Parameter	Symbol	Value	Target		Model	
Physical Capital Adjustment cost	ϕ	0.969	$\sigma_i/\sigma_{\text{GDP}}$	2.830	$\sigma_i/\sigma_{\text{rGDP}}$	2.554
Marketing Capital Adjustment cost	ψ_1	35.000	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	1.024	$\sigma_{q_1}/\sigma_{p_1}$	1.222
Marketing Capital Adjustment cost	ψ_2	30.000	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	3.492	$\sigma_{q_2}/\sigma_{p_2}$	2.708
Marketing Capital Adjustment cost	ψ_3	34.999	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	2.575	$\sigma_{q_3}/\sigma_{p_3}$	3.873
Persistence of productivity in Sector 1	ρ_1	0.501	KLEMS	0.535	rGDP_1/F_1	0.486
Persistence of productivity in Sector 2	ρ_2	0.598	KLEMS	0.574	rGDP_2/F_2	0.610
Persistence of productivity in Sector 3	ρ_3	0.401	KLEMS	0.694	rGDP_3/F_3	0.412
Standard deviation of productivity in Sector 1	σ_1	0.007	KLEMS	0.012	rGDP_1/F_1	0.016
Standard deviation of productivity in Sector 2	σ_2	0.007	KLEMS	0.008	rGDP_2/F_2	0.007
Standard deviation of productivity in Sector 3	σ_3	0.007	KLEMS	0.006	rGDP_3/F_3	0.007
Parameter of labor disutility	ξ	0.6781	Normalized steady state labor supply	1		
Search cost	χ	0.1150	Gross wholesale markup	10%		
Marketing Capital Depreciation	δ_m^1	0.2027	Marketing expenditure to Sales ratio	7%		
Marketing Capital Depreciation	δ_m^2	0.2663	Marketing expenditure to Sales ratio	7%		
Marketing Capital Depreciation	δ_m^3	0.2590	Marketing expenditure to Sales ratio	7%		

4 Quantitative Results

In this section, we discuss the quantitative predictions of our model. We conduct two quantitative exercises. First, we discuss responses of our economy to a productivity shock in

[11]Web Page Link: BLS (clickable). File Link: BLS (clickable)

[12]Alternatively, we implement (i) Hamilton Filter, (ii) HP filter, (iii) Linear Trend for removing a trend. The Hamilton Filter and HP filter give lower persistent parameters than a cubic trend. See Appendix H.

Sector 1, in order to shed light on the mechanism by which that shock spills over to the overall economy. Then, in our main quantitative exercise we analyze the implications of a permanent increase in gen-AI productivity of 10%. We discuss both the transition path to the new steady state as well as steady state comparisons.

4.1 A transitory Gen-AI shock

Before diving into the quantitative predictions of a permanent increase of the gen-AI sector's productivity, below, we shed light on the role of the key elements of our model by analyzing impulse responses to a sectoral productivity shock in Sector 1 (the gen-AI sector) in the parameterized baseline model.

Figure 2 shows the responses of key variables to a 1% positive productivity shock in Sector 1. The productivity shock reduces the marginal cost of Sector 1 (v_1), resulting in a reduction of the wholesale price of that sector's good (q_1). This in turn results in a drop of good 1's retail price, p_1, in accordance with the bargaining equation (3). Since the now cheaper Sector 1 output is used for marketing activities in all sectors and in the retail sector, marketing activity increases (a_js and h) across all sectors. Crucially, the unaffected Sectors 2 and 3 find one of their essential inputs cheaper (marketing), and in response increase labor, capital and intermediate good use, resulting in an increase in output in all sectors. As a result, the aggregate output expands by almost 0.5%, despite Sector 1 accounting for only 14% of total output. In terms of input use, this expansion of aggregate output comes with a muted response in aggregate labor and an increase in investment, accompanied by a reallocation of labor and capital away from Sector 1 into Sectors 2 and 3. The reallocation of labor and capital is specific to Sector 1 shock under the marketing friction. The friction makes marketing, produced by Sector 1, and an essential input in sales, cheaper. As a result, more sales can be achieved by using the now cheaper marketing input by all sectors, but it requires more production. In Sector 1, increased production is implied by the productivity shock, but in Sectors 2 and 3, it requires hiring more inputs. This is different from the standard impulse response to the productivity shock under just input-output structure, which would imply increased factor use in all sectors due to the spillovers. This can be clearly seen in Figure 3, which shows the impulse response in an economy without the marketing friction. Absent marketing friction, factor use increases in all sectors by similar magnitudes and the amount of spillover is small: Sector 1 productivity shock mostly affects Sector 1 output, which goes up by 1.5%, while aggregate output increases by less than 0.25%. Additionally, the input-output structure of the model interacts with the marketing frictions implying higher spillovers. In the baseline model, a 1% positive shock in Sector 1 implies an almost

0.5% increase in aggregate real output, and has large impact on output in Sectors 2 and 3, of almost 0.5% in each (Figure 2). By comparison, in the model without the input-output structure, the effect is still roughly evenly distributed across sectors, but smaller at around 0.2-0.25% (Figure 4).

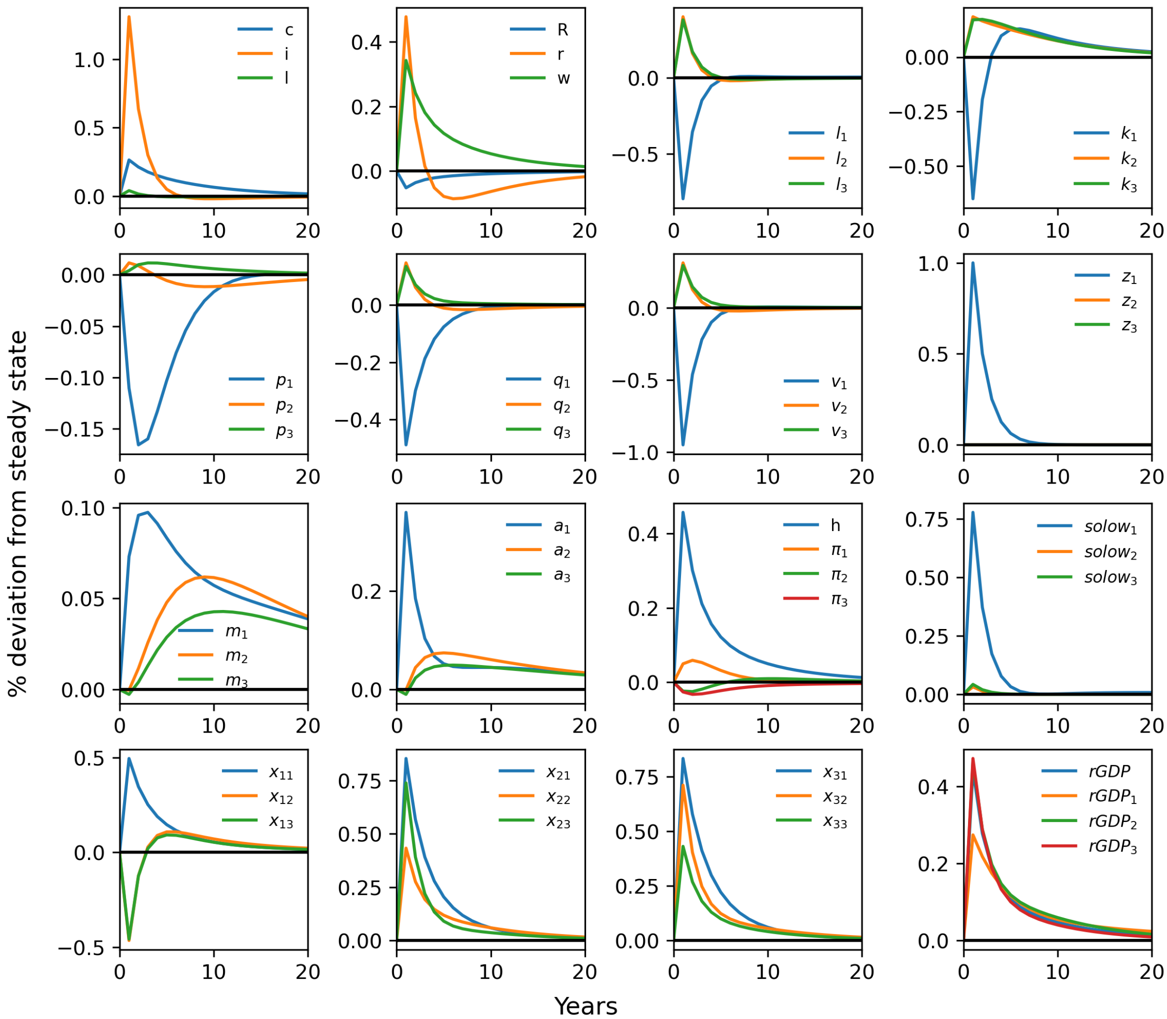

Figure 2: IRFs after a Gen-AI shock

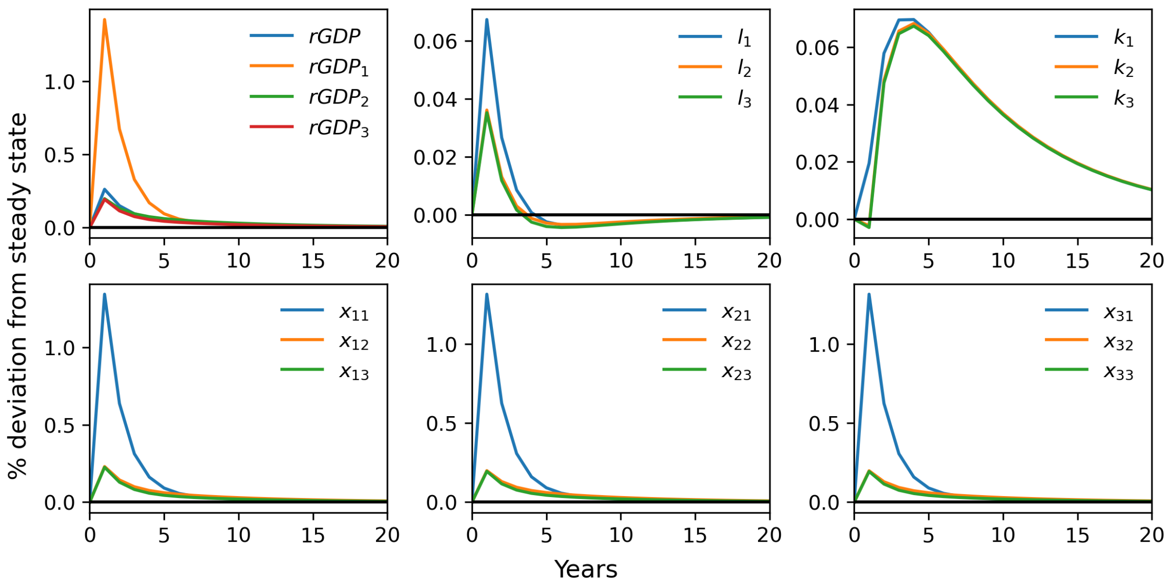

Figure 3: IRFs after a Gen-AI shock w/o Marketing Capital

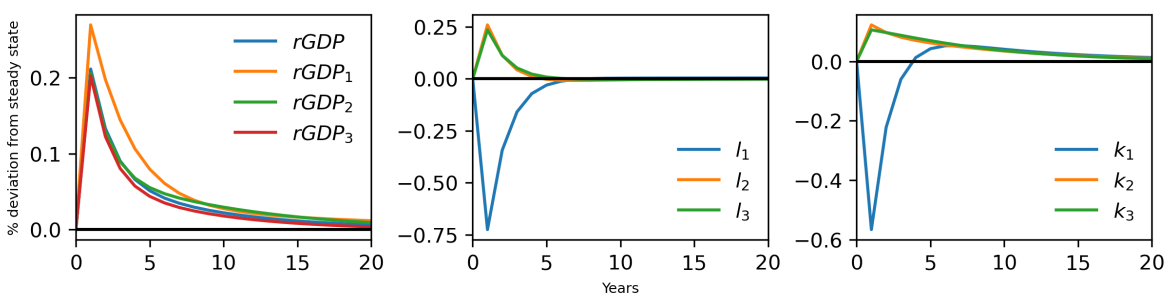

Figure 4: IRFs after a Gen-AI shock w/o IO

4.2 Permanent Productivity Increase in the gen-AI Sector

In this section, we study the effects of a permanent long-run increase of the productivity in the gen-AI sector (Sector 1) for the steady state and the transition path of the model economy. Formally, we increase the steady state productivity level in Sector 1, z_j^* in (1), by 10%. This number, while arbitrarily chosen, turns out to be aligned with a recent study by Goldman Sachs (2023), which makes the case that world GDP could go up by as much as 7% in the next decade as a result of Gen-AI progress and introduction. Our experiment predicts that at the 10 year horizon, real GDP increases by roughly 6%, close to that number. Figure 5 presents the transiton path of the model economy to the new steady state.

The top left panel shows the response of aggregate GDP and its components as per equation (5). Even though the main shock directly hits only Sector 1, most of the effect on aggregate GDP is accounted for by growth in Sectors 2 and 3 (orange and green bars). This is due to the fact that Sector 1 is only 14% of GDP in the initial steady state, and its share does not go up almost at all in the new steady state. This, combined with an increase in sectoral GDP across all sectors, seen in the bottom right panel, implies that most of the aggregate GDP growth due to gen-AI productivity increase works via spillovers to other sectors. The boom in aggregate GDP is accompanied by an investment boom (middle top panel), leading to a 6% increase in capital, together with a permanent reallocation of labor and temporary reallocation of capital from Sector 1 to Sectors 2 and 3 (bottom panels). The reallocation of labor together with large increases in sectoral GDP imply increases in labor productivity across sectors and in the aggregate, with Sectors 2 and 3 experiencing about 4% increase after 10 year while Sector 1 over 11% in the same time frame.

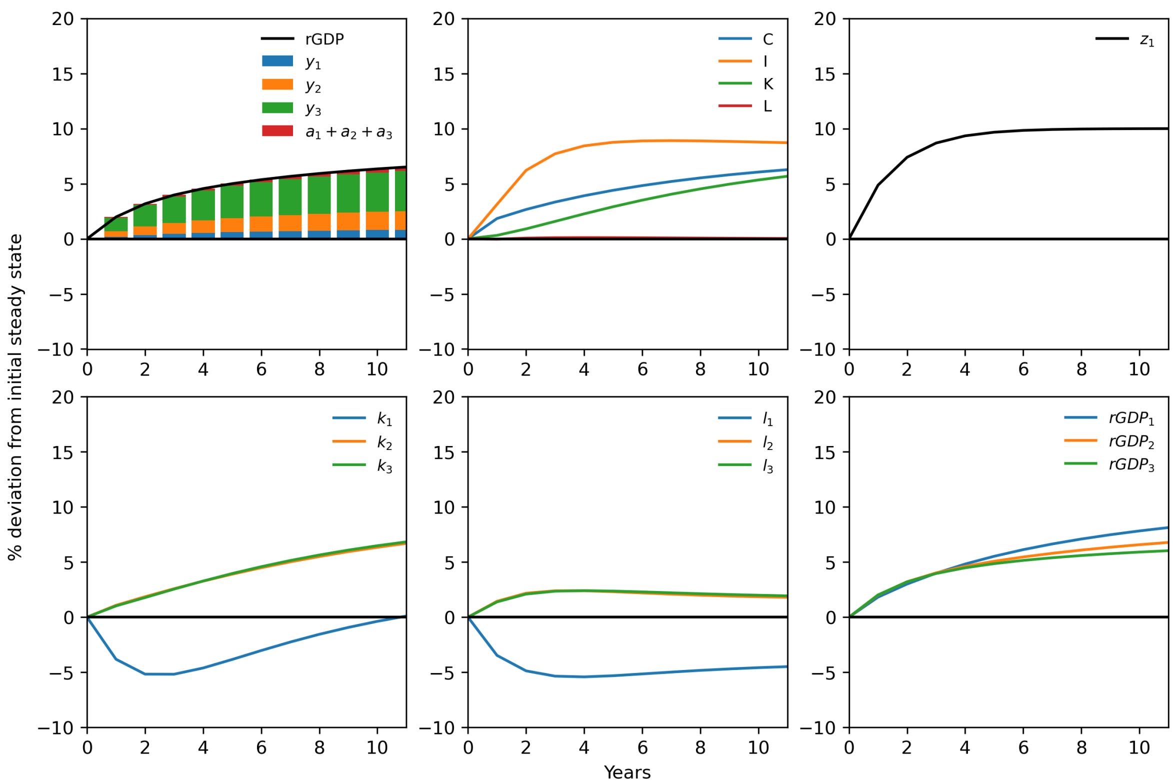

Figure 5: Transitional paths of 10% in Sector 1 productivity, Baseline calibration.

We observe that the transition to the higher GDP state involves no quantitatively significant change in aggregate labor. To understand this result, note that the return on capital in the steady state is fixed and given by $r = 1/\beta - 1 + \delta$, which is independent of the productivity level. In contrast, aggregate wages do depend on relative prices and hence on productivity in the gen-AI sector. These two forces combined result in higher capital in the new steady state, as it is cheaper, and no change in labor because it is more expensive.

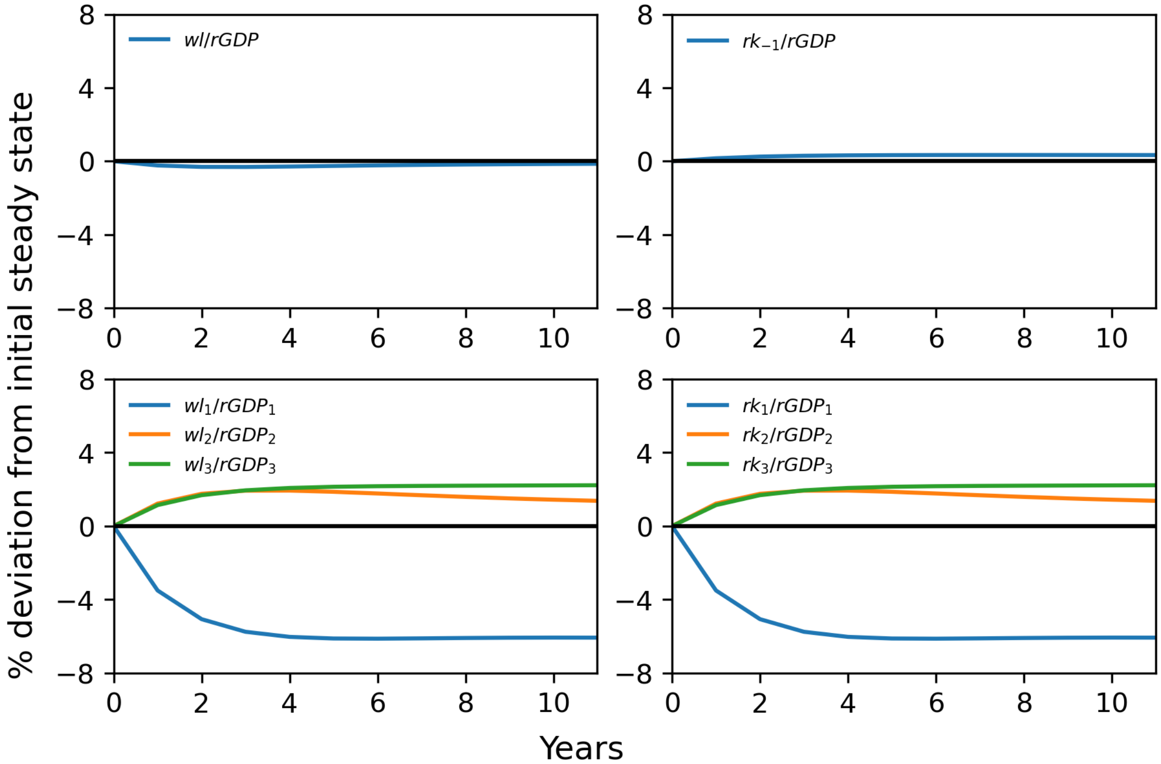

Figure 6: Labor and income shares during transitional path, Baseline calibration.

As illustrated in Figure 5, sectoral labor and capital exhibit complex dynamics, prompting an examination of their income share behavior during the transition period. Figure 6 presents these shares at both the sectoral and aggregate levels. At the sectoral level (bottom row), the AI sector experiences a notable decrease of approximately 6% in both labor and capital shares after a decade. This trend contrasts sharply with the other two sectors, where shares increase relative to their respective real GDP. The combined effect of these sectoral dynamics manifests at the aggregate level as a slight decline in the overall labor share (-0.3%), accompanied by a corresponding increase in the capital income share (0.5%).

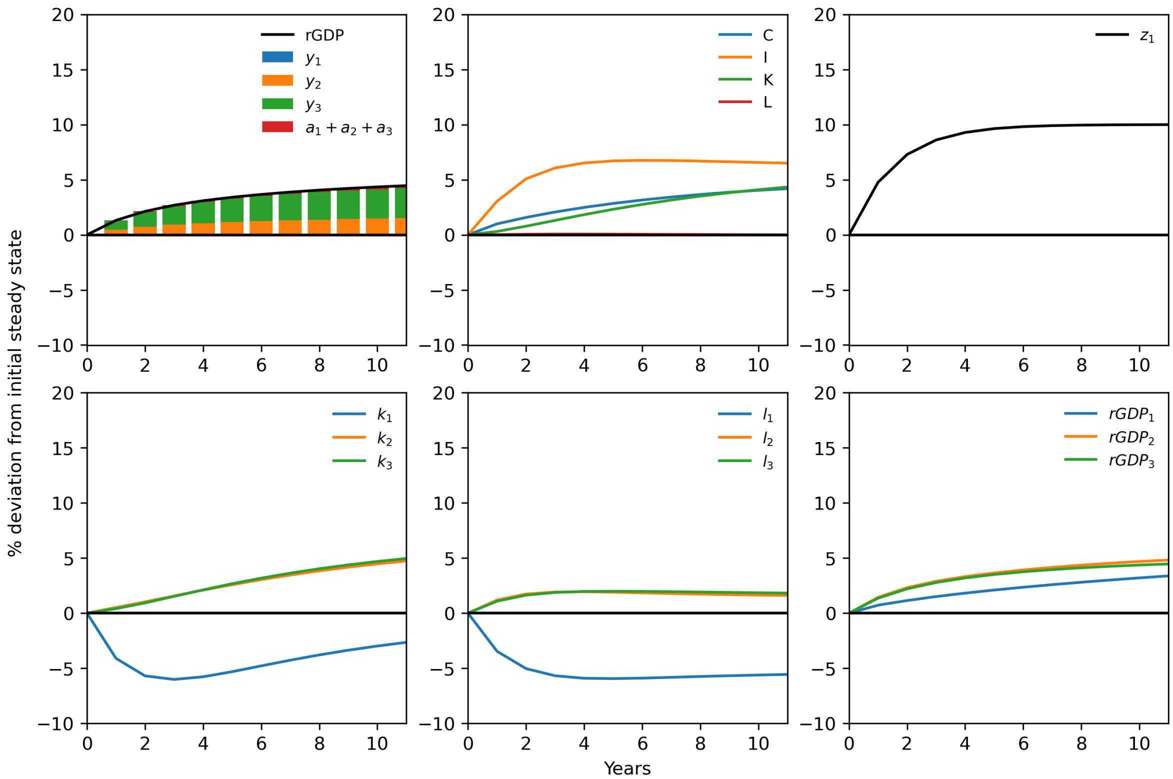

Figure 7: Transitional paths of 10% in Sector 1 productivity, Conservative calibration.

Figures 7 and 8 present the transition results for the *Conservative* calibration. The impact on aggregate GDP is smaller due to the smaller size of the gen-AI sector, but the spillover effects are still very strong, implying an almost 5% increase in GDP despite Sector 1's share being less than 10%.

As a robustness check to the magnitude of the effects, we use estimated uncertainty around the the productivity growth in the U.K. during the first industrial revolution from Bouscasse et al. (2021) as a guide to the potential bounds on the growth in gen-AI. Based on their results, we add 2 scenarios to our baseline exercise: we impose that GDP growth can be 50% higher or 50% lower than the benchmark experiment above. Figure 9 displays the resulting dynamic paths.

More generally, a pertinent question concerns the comparison of the 10% productivity boost in Sector 1 with historical epochs of marked productivity enhancements. Oxford Economics, in a recent report, highlighted the productivity leaps observed across various nations and eras. Notably, U.S. productivity witnessed a 20% increase from 1917 to 1927, a period that succeeded the advent of groundbreaking technologies such as electrification, the internal combustion engine, and the telephone in the late 19th century. Viewing from the lens of historical precedents, a 10% productivity surge in Sector 1 appears plausible.

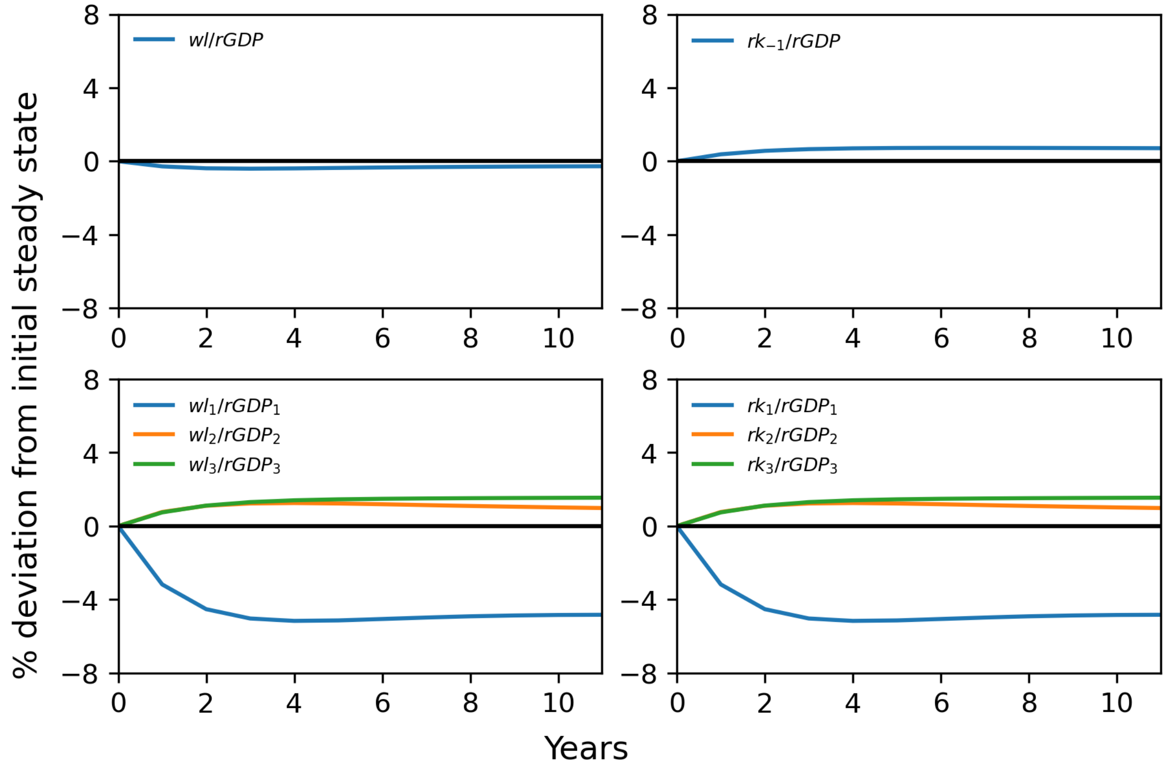

Figure 8: Labor and income shares during transitional path, Conservative calibration.

Nonetheless, it's critical to acknowledge that the technological innovations of the second industrial revolution took considerable time to manifest in tangible productivity gains. This historical gradualism stands in stark contrast to the expected rapid realization of productivity gains in our current projections, which assumes a more immediate impact.

4.3 Decomposing Long-Run Effects

Below, we decompose the long-run effects of an increase in gen-AI productivity into ones coming from the input-output structure of the model and the customer search friction. To this end, we report percentage changes between steady state values of variables for four versions of the model: (i) the *Benchmark*, (ii) the *No Production Network* model in which we shut down the input-output linkages, (iii) the *No Customer Search* model in which we shut off the search friction and finally (iv) the *No Production Network nor Customer Search* model in which we shut off both.

To compute steady states without production network, we set the intermediate goods input parameter $\alpha_j^m = 0$ and set α_k^j and α_l^j to be $\alpha_k^j = \alpha_l^j = 1$. To compute aggregate Solow residuals, we define the aggregate real GDP $\text{rGDP}_t = \sum_j \text{rGDP}_{jt}$ and the aggregate

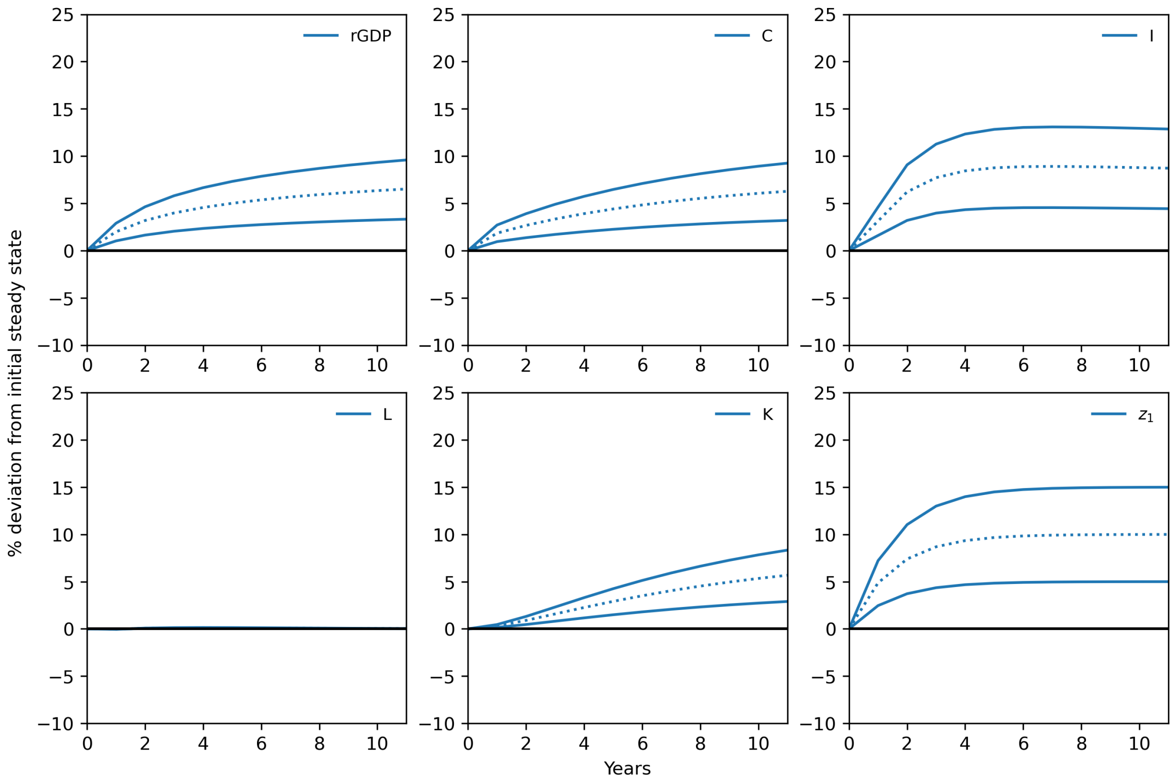

Figure 9: Simulation with 1.5 times higher / 0.5 times lower path than baseline

intermediate input $X_t = \sum_j \left(\frac{\text{rGDP}_{jt}}{\text{rGDP}_t} \sum_m x_{jmt} \right)$ and then assume the following relationship:

$$\text{rGDP}_t = \text{Solow}_t K_{t-1}^{\alpha_k} L_t^{\alpha_l} X_t^{\alpha_x}, \quad \alpha_i \in [0,1]$$

and then we estimate input share parameters by non-negative least squares using stochastic simulated data

$$\underset{\{\alpha_i\}}{\text{argmin}} \, ||\text{r}\hat{\text{GDP}}_t - (\alpha_0 + \alpha_1 \hat{K}_{t-1} + \alpha_2 \hat{L}_t + \alpha_3 \hat{X}_t)||_2^2, \quad \alpha_i \in [0,1]$$

Table 7 reports the results. In the long-run, a 10% increase in gen-AI sector productivity implies an almost 8% increase in aggregate GDP, driven by large spillovers across sectors: gen-AI sector GDP goes up by 12%, while for Sectors 2 and 3 the change is 8% and 6.7%, respectively. In line with the transition analysis, there is a reallocation of labor from Sector 1 to Sectors 2 and 3, and a relative reallocation of capital, although capital goes up in all sectors. Labor productivity goes up by more than 7% overall, again showing large spillover effects, with Sectors 2 and 3 exhibiting an increase of 6.5% and 5.1%, respectively.

Columns 3-5 of Table 7 provide a quantification of where the effects are coming from.

Looking at aggregate GDP, the contribution of the input-output structure and the marketing friction are roughly evenly split. Without the customer capital friction, or the input-output structure, the effect would be halved (columns 3 and 4). An important difference between the input-output effect and the customer capital effect is on the reallocation of factors. The customer capital friction implies a reallocation of both capital and labor from the gen-AI sector to the other two sectors, while in the input-output only model (column 4), these effects are not present. Finally, in the model without customer capital or the input-output structure, the effects of the increase in gen-AI productivity are much more proportional to the size of that sector, and imply a very small effect on aggregate GDP.

Table 7: z_1 shock of 10% GDP increase: % deviation from initial steady state. Baseline calibration.

Variable	Benchmark	No Network	No Search	No Network & Search
$rGDP$	7.8	3.8	4.3	0.9
$rGDP_1$	12.2	7.3	16.1	11.4
$rGDP_2$	8.0	4.0	4.3	0.5
$rGDP_3$	6.7	3.0	3.4	0.3
c	7.5	3.6	4.3	0.9
i	8.1	4.2	4.3	0.9
l	0.0	0.0	0.0	0.0
k	8.1	4.2	4.3	0.9
w	7.6	3.7	4.3	0.9
l_1	-3.3	-5.2	0.3	0.9
l_2	1.4	1.6	0.0	0.0
l_3	1.4	1.5	-0.1	-0.1
k_1	4.0	-1.6	4.6	1.8
k_2	9.2	5.4	4.3	0.9
k_3	9.2	5.3	4.2	0.9
$rGDP/l$	7.7	3.7	4.3	0.9
$rGDP_1/l_1$	16.1	13.1	15.8	10.4
$rGDP_2/l_2$	6.5	2.3	4.3	0.6
$rGDP_3/l_3$	5.1	1.4	3.4	0.4

5 Conclusions

This paper uses a quantitative multi-sector model model with a calibrated input-output structure, which explicitly incorporates the impact of AI on customer build up, acquisition and retention across sectors, in order to explore the impact of AI on aggregate economic activity. Our findings provide a quantification of the so far unexplored channel by which gen-AI can improve customer acquisition and management. We find large spillover effects of productivity improvements in AI technology into all sectors in the economy, especially those for which customer base management and marketing activities are an important part of the production and sales process.

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Appendix

A Optimality conditions

This section flushes out the optimality conditions for the different actors in our model.

A.1 Households

From the households' cost minimization problem, we obtain

$$y_{jt} = \omega_j (p_{jt})^{-\gamma} G_t, \quad 1 = P_t = \left[\sum_j \omega_j (p_{jt})^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \quad c_t + i_t = \sum_j p_{jt} y_{jt}$$

Optimality requires the following FOCs:

$$\begin{aligned}
\frac{u_2(c_t, l_t)}{u_1(c_t, l_t)} &= -w_t \\
u_1(c_t, l_t) &= \beta R_t \mathbb{E}_t[u_1(c_{t+1}, l_{t+1})] \\
\frac{u_1(c_t, l_t)}{1 - \phi_1(i_t, k_{t-1})} &= \beta \mathbb{E}_t \left[\frac{u_1(c_{t+1}, l_{t+1})}{1 - \phi_1(i_{t+1}, k_t)} \left\{ 1 - \delta + r^K_{t+1}(1 - \phi_1(i_{t+1}, k_t)) - \phi_2(i_{t+1}, k_t) \right\} \right]
\end{aligned}$$

A.2 Producers

From the firm's cost minimization problem, it follows that

$$\min r^K_t k_{jt} + w_t l_{jt} + \sum_m p_{jmt} x_{jmt} \quad \text{s.t.} \quad Y_{jt} = z_{jt} k_{jt}^{\alpha^j_k} l_{jt}^{\alpha^j_l} \prod_m x_{jmt}^{\alpha^j_m}, \quad \alpha^j_k + \alpha^j_l + \sum_m \alpha^j_m = 1$$

the marginal cost v_{jt} under the above production technology is given by[13]

$$v_{jt} = \frac{1}{z_{jt}} \frac{(r^K_t)^{\alpha^j_k} (w_t)^{\alpha^j_l} v_{jt}^{\alpha^j_j} \prod_{m \neq j} p_{mt}^{\alpha^j_m}}{(\alpha^j_k)^{\alpha^j_k} (\alpha^j_l)^{\alpha^j_l} \prod_m (\alpha^j_m)^{\alpha^j_m}} = \left(\frac{1}{z_{jt} \varrho_j} (r^K_t)^{\alpha^j_k} (w_t)^{\alpha^j_l} \prod_{m \neq j} p_{mt}^{\alpha^j_m} \right)^{\frac{1}{1-\alpha^j_j}}$$

where $\varrho_j = (\alpha^j_k)^{\alpha^j_k} (\alpha^j_l)^{\alpha^j_l} \prod_m (\alpha^j_m)^{\alpha^j_m}$.
The implied factor demands are given by

$$\frac{k_{jt}}{l_{jt}} = \frac{a^j_k}{a^j_l} \frac{w_t}{r^K_t}, \quad \frac{x_{jmt}}{l_{jt}} = \begin{cases} \frac{\alpha^j_j}{\alpha^j_l} \frac{w_t}{v_{jt}} & (m = j) \\ \frac{\alpha^j_m}{\alpha^j_l} \frac{w_t}{p_{mt}} & (m \neq j) \end{cases}$$

[13] See Appendix for the detailed derivation.

Note that

$$k_{jt} = \frac{\alpha_k^j \tau_{jt}}{\varrho_j r_t^K} F_{jt} \tag{6}$$

$$l_{jt} = \frac{\alpha_l^j \tau_{jt}}{\varrho_j w_t} F_{jt} \tag{7}$$

$$x_{jmt} = \begin{cases} \frac{\alpha_j^j \tau_{jt}}{\varrho_j v_{jt}} F_{jt} & (m = j) \\ \frac{\alpha_m^j \tau_{jt}}{\varrho_j p_{mt}} F_{jt} & (m \neq j) \end{cases} \tag{8}$$

where

$$\tau_{jt} = (r_t^K)^{\alpha_k^j} (w_t)^{\alpha_l^j} (v_{jt})^{\alpha_j^j} \prod_{m \neq j} (p_{mt})^{\alpha_m^j}$$

As for the profit maximization, define the Lagrangian multipliers by λ_{jt}, μ_{jt}, ν_{jt}.

$$\begin{aligned} \mathbb{E}_t \Bigg[\sum_{k=0} \Omega_{t,t+k} \Bigg\{ \Pi_{jt+k} &+ \lambda_{jt} \left((1-\delta_m^j) m_{jt-1} + a_{jt} - \frac{\psi_j}{2} m_{jt-1} \left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j \right)^2 - m_{jt} \right) \\ &+ \mu_{jt} \left((1-\delta_H) H_{jt-1} + \frac{m_{jt}}{\sum_j \bar{m}_{jt}} h_t - H_{jt} \right) \\ &+ \nu_{jt} (H_{jt} - d_{jt}) \Bigg\} \Bigg] \end{aligned}$$

where

$$\Pi_{jt} = \begin{cases} (q_{jt} - v_{jt}) d_{jt} - v_{jt} A_{jt} a_{jt} & (j = 1) \\ (q_{jt} - v_{jt}) d_{jt} - p_{1t} A_{jt} a_{jt} & (j \neq 1) \end{cases}$$

FOCs yield

$$\begin{aligned} [a_{jt}]: \quad & \lambda_{jt} = \begin{cases} \frac{A_{jt}}{1-\psi_j \left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j \right)} v_{1t} & (j = 1) \\ \frac{A_{jt}}{1-\psi_j \left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j \right)} p_{1t} & (j \neq 1) \end{cases} \\ [d_{jt}]: \quad & \nu_{jt} = (q_{jt} - v_{jt}) \\ [H_{jt}]: \quad & \mu_{jt} = \nu_{jt} + (1-\delta_H) \mathbb{E}_t [\Omega_{t,t+1} \mu_{jt+1}] \end{aligned}$$

$$[m_{jt}]: \quad \lambda_{jt} = \frac{1}{\sum_j \bar{m}_{jt}} h_t \mu_{jt} + \mathbb{E}_t\left[\lambda_{jt+1}\Omega_{t,t+1}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right]$$

Note that $\mu_{jt} = W_{jt}$.

B Model Summary

B.1 Equations

There are 62 (=10+15+12+7+8+6+4) equations

B.1.1 Household (10(=3×1[y]+7) equations)

$$\xi \frac{l_t^\eta}{c_t^{-\sigma}} = w_t \tag{9}$$

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t[c_{t+1}^{-\sigma}] \tag{10}$$

$$\begin{aligned}\frac{c_t^{-\sigma}}{1-\phi\left(\frac{i_t}{k_{t-1}}-\delta\right)} = \beta\mathbb{E}_t\Bigg[\frac{c_{t+1}^{-\sigma}}{1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)}\Bigg\{1-\delta+r_{t+1}^K\left(1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\right) \\ + \phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\frac{i_{t+1}}{k_t}\Bigg\}\Bigg]\end{aligned} \tag{11}$$

$$\Omega_{t,t+1} = \beta\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \tag{12}$$

$$c_t + i_t = G_t \tag{13}$$

$$k_t = (1-\delta)k_{t-1} + i_t - \frac{\phi}{2}k_{t-1}\left(\frac{i_t}{k_{t-1}}-\delta\right)^2 \tag{14}$$

$$P_t = \left[\sum_j \omega_j (p_{jt})^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = 1 \tag{15}$$

$$y_{jt} = \omega_j p_{jt}^{-\gamma} G_t \tag{16}$$

B.1.2 Producers (15(=3×2[$v, k/l$]+9×1[x/l]) equations)

$$v_{jt} = \left(\frac{1}{z_{jt}\varrho_j}(r_t^K)^{\alpha_k^j}(w_t)^{\alpha_l^j}\prod_{m\neq j} p_{mt}^{\alpha_m^j}\right)^{\frac{1}{1-\alpha_j^j}} \tag{17}$$

$$\frac{k_{jt}}{l_{jt}} = \frac{\alpha_k^j}{\alpha_l^j}\frac{w_t}{r_t^K} \tag{18}$$

$$\frac{x_{jmt}}{l_{jt}} = \begin{cases} \frac{\alpha_j^j}{\alpha_l^j}\frac{w_t}{v_{jt}} & (m=j) \\ \frac{\alpha_m^j}{a_l^j}\frac{w_t}{p_{mt}} & (m \neq j) \end{cases} \tag{19}$$

B.1.3 Customer Market: Producers (12(=3×4) equations)

$$\frac{h_t W_{jt}}{\sum_j m_{jt}} = \begin{cases} \frac{A_{jt}v_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1}v_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j=1) \\ \frac{A_{jt}p_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}}-\delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1}p_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j \neq 1) \end{cases} \tag{20}$$

$$d_{jt} = (1-\delta_H)d_{jt-1} + \frac{m_{jt}}{\sum_j m_{jt}}h_t \tag{21}$$

$$m_{jt} = (1-\delta_m^j)m_{jt-1} + a_{jt} - \frac{\psi_j}{2}m_{jt-1}\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)^2 \tag{22}$$

$$W_{jt} = q_{jt} - v_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}W_{jt+1}] \tag{23}$$

B.1.4 Customer Market: Retailers (7(=3×2[J, q]+1) equations)

$$J_{jt} = p_{jt} - q_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}J_{jt+1}] \tag{24}$$

$$q_{jt} = \theta p_{jt} + (1-\theta)v_{jt} \tag{25}$$

$$\sum_j \frac{m_{jt}}{\sum_j m_{jt}} J_{jt} = \chi p_{1t} \tag{26}$$

B.1.5 Market Cleaning (8(=3×2[F, d]+2) equations)

$$z_{jt}F_{jt} = \begin{cases} y_{jt} + \sum_m x_{mjt} + \sum_m a_{mt} + \chi h_t & (j=1) \\ y_{jt} + \sum_m x_{mjt} & (j \neq 1) \end{cases} \tag{27}$$

$$d_{jt} = \begin{cases} y_{jt} + \sum_{m\neq j} x_{mjt} + \sum_{m\neq j} a_{mt} + \chi h_t & (j=1) \\ y_{jt} + \sum_{m\neq j} x_{mjt} & (j \neq 1) \end{cases} \tag{28}$$

$$\sum_j l_{jt} = l_t \tag{29}$$

$$\sum_j k_{jt} = k_{t-1} \tag{30}$$

B.1.6 Exogenous process (6(=3×2) equations)

$$\ln z_{jt} = (1-\rho_z^j)\ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt} \tag{31}$$

$$\ln A_{jt} = \rho_A \ln A_{jt-1} + \varepsilon_{jt}^A \tag{32}$$

B.1.7 Auxiliary variables (4(=3×1[F]+1) equations)

$$F_{jt} = k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j} \tag{33}$$

$$G_t = \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_{jt})^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{34}$$

B.2 Variables

There are 62 (=15+18+15+4+6+4) variables

B.2.1 Household

$$\left\{c, l, w, i, k, R, r^K, P, \Omega\right\} + 3 \times \{y_j, p_j\} = 15$$

B.2.2 Producers

$$3 \times \{v_j, k_j, l_j\} + 9 \times \{x_{ij}\} = 18$$

B.2.3 Customer capital: Producers

$$3 \times \{d_j, a_j, m_j, q_j, W_j\} = 15$$

B.2.4 Customer Market: Retailers

$$\{h\} + 3 \times \{J_j\} = 4$$

B.2.5 Exogenous

$$3 \times \{z_j, A_j\} = 6$$

B.2.6 Auxiliary variables

$$\{G\} + 3 \times \{F_j\} = 4$$

C Deterministic Steady State

C.1 Summary of Steady State

C.1.1 Household

$$\begin{aligned}
c &= G - i \\
l &= 1 \\
R &= \frac{1}{\beta} \\
r^K &= \frac{1}{\beta} - 1 + \delta \\
i &= \delta k \\
k &= \text{equation}(72) \\
w &= \text{equation}(71) \\
P &= 1 \\
\Omega &= \beta \\
y_j &= \omega_j p_j^{-\gamma} G
\end{aligned}$$

C.1.2 Producers

$$\begin{aligned}
v_j &= \frac{\theta}{\mathcal{U}_j - (1-\theta)} p_j \\
k_j &= \frac{\alpha_k^j \tau_j}{\varrho_j r^K} F_j \\
l_j &= \frac{\alpha_l^j \tau_j}{\varrho_j w} F_j \\
x_{jm} &= \begin{cases} \frac{\alpha_j^j \tau_j}{\varrho_j v_j} F_j & (m = j) \\ \frac{\alpha_m^j \tau_j}{\varrho_j p_m} F_j & (m \neq j) \end{cases}
\end{aligned}$$

C.1.3 Customer capital: Producers

$$
\begin{aligned}
d_j &= \text{equation}(72) \\
a_j &= \delta_m m_j \\
m_j &= \frac{\delta_H(\mathcal{U}_1 - 1)}{(1-\beta(1-\delta_m^j)(1-\beta(1-\delta_H)}d_j \\
q_j &= \mathcal{U}_j v_j \\
W_j &= \frac{\mathcal{U}_j - 1}{1-\beta(1-\delta_H)}v_j
\end{aligned}
$$

C.1.4 Customer Market: Retailers

$$
\begin{aligned}
h &= \delta_H \sum_j d_j \\
J_j &= \frac{1-\theta}{\theta}W_j \\
p_1 &= \left(\omega_1 + \omega_2\left(\frac{\mathcal{U}_2 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1}\right)^{1-\gamma} + \omega_3\left(\frac{\mathcal{U}_3 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1}\right)^{1-\gamma}\right)^{\frac{1}{\gamma-1}} \\
p_2 &= \frac{\mathcal{U}_2 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1}p_1 \\
p_3 &= \frac{\mathcal{U}_3 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1}p_1
\end{aligned}
$$

C.1.5 Exogenous

$$
\begin{aligned}
z_j &= z_j^* \\
A_j &= 1
\end{aligned}
$$

C.1.6 Auxiliary variables

$$
\begin{aligned}
\mathcal{U}_2 &= \text{equation}(71) \\
\mathcal{U}_3 &= \text{equation}(71) \\
F_j &= \text{equation}(72) \\
G &= \text{equation}(72)
\end{aligned}
$$

C.1.7 Endogenous Parameters

$$\xi l^{\eta} = wc^{-\sigma}$$
$$\chi = \frac{1}{p_1} \sum_j \frac{m_j}{\sum_j m_j} J_j$$

D Log-linearization

Define $\hat{\cdot}_t = (\cdot_t - \cdot)/\cdot$

D.1 Household

$$\eta \hat{l}_t + \sigma \hat{c}_t = \hat{w}_t \tag{35}$$
$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - \frac{1}{\sigma}\hat{R}_t \tag{36}$$
$$-\sigma \hat{c}_t + \phi\delta(\hat{i}_t - \hat{k}_{t-1}) = \mathbb{E}_t\Big[-\sigma\hat{c}_{t+1} + \beta\phi\delta(\hat{i}_{t+1} - \hat{k}_t) + (1-\beta(1-\delta))\hat{r}^K_{t+1}\Big] \tag{37}$$
$$\hat{\Omega}_{t,t+1} = -\sigma(\hat{c}_{t+1} - \hat{c}_t) \tag{38}$$
$$\frac{c}{G}\hat{c}_t + \frac{i}{G}\hat{i}_t = \hat{G}_t \tag{39}$$
$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{i}_t \tag{40}$$
$$\hat{P}_t = \sum_j \omega_j p_j^{1-\gamma}\hat{p}_{jt} = 0 \tag{41}$$
$$\hat{y}_{jt} = -\gamma\hat{p}_{jt} + \hat{G}_t \tag{42}$$

D.2 Producers

$$(1-\alpha_j^j)\hat{v}_{jt} = -\hat{z}_{jt} + \alpha_k^j\hat{r}_t^K + \alpha_l^j\hat{w}_t + \sum_{m\neq j}\alpha_m^j\hat{p}_{mt} \tag{43}$$
$$\hat{k}_{jt} - \hat{l}_{jt} = \hat{w}_t - \hat{r}_t^K \tag{44}$$
$$\hat{x}_{jmt} - \hat{l}_{jt} = \begin{cases} \hat{w}_t - \hat{v}_{jt} & (m = j) \\ \hat{w}_t - \hat{p}_{mt} & (m \neq j) \end{cases} \tag{45}$$

D.3 Customer Market: Producers

$$
(1-\beta(1-\delta_m^j))\left(\hat{h}_t+\hat{W}_{jt}-\sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt}\right)=
$$

$$
\begin{cases}
\hat{A}_{jt}+\hat{v}_{1t}+\psi_j\delta_m^j(\hat{a}_{jt}-\hat{m}_{t-1}) \\
-\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{v}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big] & (j=1) \\
\hat{A}_{jt}+\hat{p}_{1t}+\psi_j\delta_m^j(\hat{a}_{jt}-\hat{m}_{t-1}) \\
-\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{p}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big] & (j\neq 1)
\end{cases} \tag{46}
$$

$$
\hat{d}_{jt}=(1-\delta_H)\hat{d}_{jt-1}-\delta_H\sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt}+\delta_H(\hat{m}_{jt}+\hat{h}_t) \tag{47}
$$

$$
\hat{m}_{jt}=(1-\delta_m^j)\hat{m}_{jt-1}+\delta_m^j\hat{a}_{jt} \tag{48}
$$

$$
\hat{W}_{jt}=\frac{1}{W_j}(q_j\hat{q}_{jt}-v_j\hat{v}_{jt})+(1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1}+\hat{W}_{jt+1}\Big] \tag{49}
$$

D.4 Customer Market: Retailers

$$
\hat{J}_{jt}=\frac{1}{J_j}(p_j\hat{p}_{jt}-q_j\hat{q}_{jt})+(1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1}+\hat{J}_{jt+1}\Big] \tag{50}
$$

$$
q_j\hat{q}_{jt}=\theta p_j\hat{p}_{jt}+(1-\theta)v_j\hat{v}_{jt} \tag{51}
$$

$$
\sum_j m_jJ_j(\hat{m}_{jt}+\hat{J}_{jt})=\chi\sum_j p_1m_j(\hat{p_{1t}}+\hat{m}_{jt}) \tag{52}
$$

D.5 Market Cleaning

$$
\hat{z}_{jt}+\hat{F}_{jt}=\begin{cases}
\frac{y_j}{F_j}\hat{y}_{jt}+\sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt}+\sum_m \frac{a_m}{F_j}\hat{a}_{mt}+\frac{\chi h}{F_j}\hat{h}_t & (j=1) \\
\frac{y_j}{F_j}\hat{y}_{jt}+\sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt} & (j\neq 1)
\end{cases} \tag{53}
$$

$$
\hat{d}_{jt}=\begin{cases}
\frac{y_j}{d_j}\hat{y}_{jt}+\sum_{m\neq j} \frac{x_{mj}}{d_j}\hat{x}_{mjt}+\sum_{m\neq j} \frac{a_m}{d_j}\hat{a}_{mt}+\frac{\chi h}{d_j}\hat{h}_t & (j=1) \\
\frac{y_j}{d_j}\hat{y}_{jt}+\sum_{m\neq j} \frac{x_{mj}}{d_j}\hat{x}_{mjt} & (j\neq 1)
\end{cases} \tag{54}
$$

$$
\sum_j \frac{l_j}{l}\hat{l}_{jt}=\hat{l}_t \tag{55}
$$

$$
\sum_j \frac{k_j}{k}\hat{k}_{jt}=\hat{k}_{t-1} \tag{56}
$$

D.6 Exogenous process

$$\hat{z}_{jt} = \rho_z^j \hat{z}_{jt-1} + \varepsilon_{jt} \tag{57}$$

$$\hat{A}_{jt} = \rho_A \hat{A}_{jt-1} + \varepsilon_{jt}^A \tag{58}$$

D.7 Auxiliary variables

$$\hat{F}_{jt} = \alpha_k^j \hat{k}_{jt} + \alpha_l^j \hat{l}_{jt} + \sum_m \alpha_m^j \hat{x}_{jmt} \tag{59}$$

$$\hat{G}_t = \sum_j \omega_j p_j^{1-\gamma} \hat{y}_{jt} \tag{60}$$

E Marginal Cost of Cobb Douglas Function

Consider

$$\min \sum_j p_{jt} x_{jt} \quad \text{s.t.} \quad Y_t = z_t \prod_j x_{jt}^{\alpha_j} = z_t \varrho \prod_j \left(\frac{x_{jt}}{\alpha_j}\right)^{\alpha_j}, \quad \sum_j \alpha_j = 1, \quad \varrho = \prod_j \alpha_j^{\alpha_j}$$

FOCs yields

$$p_{jt} = \lambda_t \left[z_t \varrho \prod_j \left(\frac{x_{jt}}{\alpha_j}\right)^{\alpha_j} \right] \left(\frac{x_{jt}}{\alpha_j}\right)^{-1} \iff \frac{\lambda_t Y_t}{p_{jt}} = \frac{x_{jt}}{\alpha_j}$$

Insert in production function

$$Y_t = z_t \varrho \prod_j \left(\frac{\lambda_t Y_t}{p_{jt}}\right)^{\alpha_j} = z_t \varrho \frac{\lambda_t Y_t}{\prod_j p_{jt}^{\alpha_j}} \iff \lambda_t = \frac{\prod_j p_{jt}^{\alpha_j}}{z_t \varrho} = \frac{p_t}{z_t \varrho}$$

where $p_t \equiv \prod_j p_{jt}^{\alpha_j}$
Insert in FOCs

$$p_{jt} = \frac{p_t}{z_t \varrho} Y_t \left(\frac{x_{jt}}{\alpha_j}\right)^{-1} \iff p_{jt} x_{jt} = \alpha_j \frac{p_t}{z_t \varrho} Y_t$$

Taking sum over j

$$\sum_j p_{jt} x_{jt} = \frac{p_t}{z_t \varrho} Y_t$$

Therefore, the cost function is given by

$$C(\{p\}, Y) = \frac{p_t}{z_t \varrho} Y_t$$

Eventually the marginal cost is given by

$$MC(Y) = \frac{p_t}{z_t \varrho} = \frac{1}{z_t} \frac{\prod_j p_{jt}^{\alpha_j}}{\prod_j \alpha_j^{\alpha_j}}$$

Note that the implied factor demand is given by

$$x_{jt} = \frac{\alpha_j}{\prod_j \alpha_j^{\alpha_j}} \frac{\prod_j p_{jt}^{\alpha_j}}{p_{jt}} \frac{Y_t}{z_t}$$

F Derivation: Deterministic Steady State

Normalize

$$P = 1$$

From equation (12),

$$\Omega = \beta$$

From Euler Equations (10) and (11)

$$R = \frac{1}{\beta}, \quad r^K = \frac{1}{\beta} - 1 + \delta$$

From equations (31) and (32)

$$z_j = z_j^*, \quad A_j = 1$$

From law of motion for capital, equation (14),

$$i = \delta k \tag{61}$$

Assume gross wholesale markup as

$$\frac{q_j}{v_j} \equiv \mathcal{U}_j$$

Note that we treat $\mathcal{U}_1$ as calibration targets and determine χ as the endogenous parameter. From Nash Bargaining, equation (25), it yields retail prices

$$p_j = \frac{\mathcal{U}_j - 1 + \theta}{\theta} v_j \tag{62}$$

From the value for retailer, equation (24) and Nash Bargaining, equation (25)

$$J_j = \frac{p_j - q_j}{1 - \beta(1 - \delta_H)} = \frac{1-\theta}{\theta} \frac{q_j - v_j}{1 - \beta(1 - \delta_H)} = \frac{1-\theta}{\theta} W_j$$

From the law of motion for marketing capital equation (22) and make use of the fact that the adjustment cost is 0 in the steady state

$$a_j = \delta_m^j m_j \tag{63}$$

From equation (21), we obtain

$$\delta_H d_j = \frac{m_j}{\sum_j m_j} h \Longrightarrow \delta_H \sum_j d_j = h \tag{64}$$

Also, we obtain

$$\frac{d_i}{d_j} = \frac{m_i}{m_j}$$

Using equation (20)

$$\frac{hW_j}{\sum_j m_j} = \begin{cases} (1 - \beta(1 - \delta_m^j))v_1 & (j = 1) \\ (1 - \beta(1 - \delta_m^j))p_1 & (j \neq 1) \end{cases} \tag{65}$$

From equation (23)

$$W_j = \frac{\mathcal{U}_j - 1}{1 - \beta(1 - \delta_H)} v_j \tag{66}$$

Combining equations (65) and (66) for $j = 1$

$$h = \frac{(1 - \beta(1 - \delta_m^1))(1 - \beta(1 - \delta_H))}{\mathcal{U}_1 - 1} \sum_j m_j \tag{67}$$

and using the above

$$\frac{hW_j}{\sum_j m_j} = \frac{\mathcal{U}_j - 1}{\mathcal{U}_1 - 1}(1 - \beta(1 - \delta_m^1))v_j$$

Comparison with equation (65) gives

$$v_j = \frac{\mathcal{U}_1 - 1}{\mathcal{U}_j - 1}\frac{1 - \beta(1 - \delta_m^j)}{1 - \beta(1 - \delta_m^1)}p_1, \quad (j \neq 1) \tag{68}$$

Using equation (62) and (68)

$$p_j = \frac{\mathcal{U}_j - 1 + \theta}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_j - 1}\frac{1 - \beta(1 - \delta_m^j)}{1 - \beta(1 - \delta_m^1)}p_1, \quad (j \neq 1) \tag{69}$$

Substitute into the aggregate price function, equation (15), we obtain the form of p_1 by $\mathcal{U}_j$

$$\begin{aligned}
1 &= \omega_1(p_1)^{1-\gamma} + \omega_2(p_2)^{1-\gamma} + \omega_3(p_3)^{1-\gamma} \\
&= \Bigg(\omega_1 + \omega_2\left(\frac{\mathcal{U}_2 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1}\frac{1 - \beta(1 - \delta_m^2)}{1 - \beta(1 - \delta_m^1)}\right)^{1-\gamma} \\
&\qquad + \omega_3\left(\frac{\mathcal{U}_3 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1}\frac{1 - \beta(1 - \delta_m^3)}{1 - \beta(1 - \delta_m^1)}\right)^{1-\gamma}\Bigg)p_1^{1-\gamma} \\
\iff p_1 &= \Bigg(\omega_1 + \omega_2\left(\frac{\mathcal{U}_2 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_2 - 1}\frac{1 - \beta(1 - \delta_m^2)}{1 - \beta(1 - \delta_m^1)}\right)^{1-\gamma} \\
&\qquad + \omega_3\left(\frac{\mathcal{U}_3 - (1-\theta)}{\theta}\frac{\mathcal{U}_1 - 1}{\mathcal{U}_3 - 1}\frac{1 - \beta(1 - \delta_m^3)}{1 - \beta(1 - \delta_m^1)}\right)^{1-\gamma}\Bigg)^{\frac{1}{\gamma-1}}
\end{aligned} \tag{70}$$

From the marginal cost of producers and the relationship between the retail price and the marginal cost

$$\begin{aligned}
v_j^{1-\alpha_j^j} &= \frac{1}{\varrho_j}(r^K)^{\alpha_k^j}(w)^{\alpha_l^j}\prod_{m\neq j} p_m^{\alpha_m^j} \\
\iff \left(\frac{\theta}{\mathcal{U}_j - 1 + \theta}p_j\right)^{1-\alpha_j^j} &= \frac{1}{z_j^*\varrho_j}(r^K)^{\alpha_k^j}(w)^{\alpha_l^j}\prod_{m\neq j} p_m^{\alpha_m^j}
\end{aligned}$$

Thus we obtain

$$(1 - \alpha_1^1)(\ln\theta - \ln(\mathcal{U}_1 - 1 + \theta) + \ln p_1) = -\ln z_1^* - \ln\varrho_1 + \alpha_k^1 \ln r^k + \alpha_l^1 \ln w + \alpha_2^1 \ln p_2 + \alpha_3^1 \ln p_3$$
$$(1 - \alpha_2^2)(\ln\theta - \ln(\mathcal{U}_2 - 1 + \theta) + \ln p_2) = -\ln z_2^* - \ln\varrho_2 + \alpha_k^2 \ln r^k + \alpha_l^2 \ln w + \alpha_1^2 \ln p_1 + \alpha_3^2 \ln p_3$$

$$(1-\alpha_3^3)(\ln\theta - \ln(\mathcal{U}_3 - 1 + \theta) + \ln p_3) = -\ln z_3^* - \ln \varrho_3 + \alpha_k^3 \ln r^k + \alpha_l^3 \ln w + \alpha_1^3 \ln p_1 + \alpha_2^3 \ln p_2 \tag{71}$$

Given $\{\mathcal{U}_j\}, \{\delta_m^j\}, \{\omega_j\}, \theta, \beta$, we can compute $\{p_j\}$ using equations (69) and (70). The equilibrium values of $\{\mathcal{U}_2, \mathcal{U}_3, w\}$ solve the above system. $\{v_j, q_j\}$ are given by

$$\begin{aligned} v_j &= \frac{\theta}{\mathcal{U}_j - (1-\theta)} p_j \\ q_j &= \mathcal{U}_j v_j \end{aligned}$$

From equations (27), (63), (64), and (67)

$$\begin{aligned} F_j &= \begin{cases} y_j + \sum_m x_{mj} + \sum_j \delta_m^j m_j + \chi h & (j=1) \\ y_j + \sum_m x_{mj} & (j \neq 1) \end{cases} \\ &= \begin{cases} y_j + \sum_m x_{mj} + \Delta \sum_j \delta_m^j d_j + \chi \delta_H \sum_j d_j & (j=1) \\ y_j + \sum_m x_{mj} & (j \neq 1) \end{cases} \\ &= \begin{cases} y_j + \sum_m x_{mj} + \sum_j \left(\Delta \delta_m^j + \frac{\delta_H}{p_1} J_j\right) d_j & (j=1) \\ y_j + \sum_m x_{mj} & (j \neq 1) \end{cases} \end{aligned}$$

where we use

$$\delta_H d_j = \frac{m_j}{\sum_j m_j} h \Longrightarrow \delta_H d_j = \frac{m_j}{\sum_j m_j} \frac{(1-\beta(1-\delta_m^1))(1-\beta(1-\delta_H))}{\mathcal{U}_1 - 1} \sum_j m_j \Longrightarrow \Delta d_j = m_j$$

$$\frac{m_j}{\sum_j m_j} = \frac{d_j}{\sum_j d_j} \Longrightarrow \sum_j \frac{d_j}{\sum_j d_j} J_j = \chi p_1 \Longrightarrow \frac{\delta_H}{p_1} \sum_j J_j d_j = \chi \delta_H \sum_j d_j$$

$$\Delta = \frac{\delta_H(\mathcal{U}_1 - 1)}{(1-\beta(1-\delta_m^1))(1-\beta(1-\delta_H))}$$

Also from equations (28), (63), (64), and (67)

$$\begin{aligned} d_j &= \begin{cases} y_j + \sum_{m \neq j} x_{mj} + (\delta_m^2 m_2 + \delta_m^3 m_3) + \chi h & (j=1) \\ y_j + \sum_{m \neq j} x_{mj} & (j \neq 1) \end{cases} \\ &= \begin{cases} y_j + \sum_{m \neq j} x_{mj} + \Delta(\delta_m^2 d_2 + \delta_m^3 d_3) + \sum_j \frac{\delta_H}{p_1} J_j d_j & (j=1) \\ y_j + \sum_{m \neq j} x_{mj} & (j \neq 1) \end{cases} \end{aligned}$$

Combining the factor markets cleaning and the factor demands

$$\sum_j l_j = l, \quad \sum_j k_j = k$$

Therefore

$$\begin{aligned}
x_{11} + (\Delta\delta_m^1 + 1)d_1 &= z_1^* F_1 \\
x_{22} + d_2 &= z_2^* F_2 \\
x_{33} + d_3 &= z_3^* F_3 \\
y_1 + x_{21} + x_{31} + \Delta\delta_m^2 d_2 + \Delta\delta_m^3 d_3 + \frac{\delta_H}{p_1} J_1 d_1 + \frac{\delta_H}{p_1} J_2 d_2 + \frac{\delta_H}{p_1} J_3 d_3 &= d_1 \\
y_2 + x_{12} + x_{32} &= d_2 \\
y_3 + x_{13} + x_{23} &= d_3 \\
l_1 + l_2 + l_3 &= l \\
k_1 + k_2 + k_3 &= k
\end{aligned}$$

Note that

$$\begin{aligned}
y_j &= \omega_j p_j^{-\gamma} \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} = \omega_j p_j^{-\gamma} G \\
k_j &= \frac{\alpha_k^j \tau_j}{\varrho_j r^K} F_j \\
l_j &= \frac{\alpha_l^j \tau_j}{\varrho_j w} F_j \\
x_{jm} &= \begin{cases} \frac{\alpha_j^j \tau_j}{\varrho_j v_j} F_j & (m = j) \\ \frac{\alpha_m^j \tau_j}{\varrho_j p_m} F_j & (m \neq j) \end{cases}
\end{aligned}$$

Combining the above equations and normalizing $l = 1$ gives

$$\begin{pmatrix} 0 & \frac{\alpha_1^1\tau_1}{\varrho_1 v_1} - z_1^* & 0 & 0 & \Delta\delta_m^1 + 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_2^2\tau_2}{\varrho_2 v_2} - z_2^* & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\alpha_3^3\tau_3}{\varrho_3 v_3} - z_3^* & 0 & 0 & 1 & 0 \\ \omega_1 p_1^{-\gamma} & 0 & \frac{\alpha_1^2\tau_2}{\varrho_2 p_1} & \frac{\alpha_1^3\tau_3}{\varrho_3 p_1} & \frac{\delta_H}{p_1}J_1 - 1 & \Delta\delta_m^2 + \frac{\delta_H}{p_1}J_2 & \Delta\delta_m^3 + \frac{\delta_H}{p_1}J_3 & 0 \\ \omega_2 p_2^{-\gamma} & \frac{\alpha_2^1\tau_1}{\varrho_1 p_2} & 0 & \frac{\alpha_2^3\tau_3}{\varrho_3 p_2} & 0 & -1 & 0 & 0 \\ \omega_3 p_3^{-\gamma} & \frac{\alpha_3^1\tau_1}{\varrho_1 p_3} & \frac{\alpha_3^2\tau_2}{\varrho_2 p_3} & 0 & 0 & 0 & -1 & 0 \\ 0 & \frac{\alpha_l^1\tau_1}{\varrho_1 w} & \frac{\alpha_l^2\tau_2}{\varrho_2 w} & \frac{\alpha_l^3\tau_3}{\varrho_3 w} & 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha_k^1\tau_1}{\varrho_1 r^K} & \frac{\alpha_k^2\tau_2}{\varrho_2 r^K} & \frac{\alpha_k^3\tau_3}{\varrho_3 r^K} & 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} G \\ F_1 \\ F_2 \\ F_3 \\ d_1 \\ d_2 \\ d_3 \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{72}$$

A solution for the linear system above yields (if exists)

$$\{G, F_1, F_2, F_3, d_1, d_2, d_3, k\}$$

and then we obtain

$$c = G - \delta k$$

which determine the endogenous parameter ξ through the equation (9)

$$\xi l^\eta = wc^{-\sigma}$$

Using equations (6), (7), (8) and (16), we can obtain

$$k_j = \frac{\alpha_k^j\tau_j}{\varrho_j r^K}F_j$$

$$l_j = \frac{\alpha_l^j\tau_j}{\varrho_j w}F_j$$

$$x_{jm} = \begin{cases} \frac{\alpha_j^j\tau_j}{\varrho_j v_j}F_j & (m = j) \\ \frac{\alpha_m^j\tau_j}{\varrho_j p_m}F_j & (m \neq j) \end{cases}$$

$$y_j = \omega_j(p_j)^{-\gamma}G$$

Using equations (64) and (67), we can derive

$$m_j = \frac{\delta_H(\mathcal{U}_1 - 1)}{(1 - \beta(1 - \delta_m^1))(1 - \beta(1 - \delta_H))}d_j$$

$$
\begin{aligned}
a_j &= \delta_m^j m_j \\
h &= \delta_H \sum_j d_j
\end{aligned}
$$

The free entry and exit condition, equation (26), gives an endogenous parameter of the search cost χ

$$
\chi = \frac{1}{p_1} \sum_j \frac{d_j}{\sum_j d_j} J_j
$$

The marketing expenditure to sale ratio is given by

$$
\begin{aligned}
\mathcal{M}_j &= \begin{cases} \frac{v_1 a_1}{q_1 d_1} & (j = 1) \\ \frac{p_1 a_j}{q_j d_j} & (j \neq 1) \end{cases} \\
&= \begin{cases} \frac{v_1}{q_1} \frac{\delta_m^1 m_1}{d_1} & (j = 1) \\ \frac{p_1}{q_j} \frac{\delta_m^j m_j}{d_j} & (j \neq 1) \end{cases} \\
&= \begin{cases} \frac{\delta_m^1 \delta_H (\mathcal{U}_1 - 1)}{\mathcal{U}_1 (1 - \beta(1 - \delta_m^1))(1 - \beta(1 - \delta_H))} & (j = 1) \\ \frac{\delta_m^j \delta_H (\mathcal{U}_j - 1)}{\mathcal{U}_j (1 - \beta(1 - \delta_m^j))(1 - \beta(1 - \delta_H))} & (j \neq 1) \end{cases}
\end{aligned}
$$

G Derivation: log-linearization

G.1 Household

G.1.1 Consumption Labor choice

$$
\begin{aligned}
\xi \frac{l_t^\eta}{c_t^{-\sigma}} &= w_t \\
\ln \xi + \eta \ln l_t &= \ln w_t - \sigma \ln c_t \\
\eta \hat{l}_t + \sigma \hat{c}_t &= \hat{w}_t
\end{aligned}
$$

G.1.2 EE for Bond

$$
\begin{aligned}
c_t^{-\sigma} &= \beta R_t \mathbb{E}_t [c_{t+1}^{-\sigma}] \\
-\sigma c^{-\sigma} \hat{c}_t &= \beta R c^{-\sigma} \Big(\hat{R}_t - \sigma \mathbb{E}_t [\hat{c}_{t+1}] \Big) \\
\hat{c}_t &= \mathbb{E}_t [\hat{c}_{t+1}] - \frac{1}{\sigma} \hat{R}_t
\end{aligned}
$$

G.1.3 EE for capital

$$\frac{c_t^{-\sigma}}{1-\phi_1(i_t,k_{t-1})} = \beta\mathbb{E}_t\Bigg[\frac{c_{t+1}^{-\sigma}}{1-\phi_1(i_{t+1},k_t)}\Big\{1-\delta+r_{t+1}^K(1-\phi_1(i_{t+1},k_t))-\phi_2(i_{t+1},k_t)\Big\}\Bigg]$$

Recall

$$\phi_1(i_t,k_{t-1}) = \phi\bigg(\frac{i_t}{k_{t-1}}-\delta\bigg), \quad \phi_2(i_t,k_{t-1}) = -\phi\bigg(\frac{i_t}{k_{t-1}}-\delta\bigg)\frac{i_t}{k_{t-1}}$$

Then

$$\begin{aligned}
\frac{c_t^{-\sigma}}{1-\phi\Big(\frac{i_t}{k_{t-1}}-\delta\Big)} &= \beta\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{1-\phi\Big(\frac{i_{t+1}}{k_t}-\delta\Big)}\left\{1-\delta+\phi\bigg(\frac{i_{t+1}}{k_t}-\delta\bigg)\frac{i_{t+1}}{k_t}\right\}+r_{t+1}^K c_{t+1}^{-\sigma}\right]\\
c^{-\sigma}\Big(-\sigma\hat{c}_t+\phi\delta(\hat{i}_t-\hat{k}_{t-1})\Big) &= \beta\mathbb{E}_t\Big[c^{-\sigma}\Big(-\sigma(1-\delta)\hat{c}_{t+1}+\phi\delta(\hat{i}_{t+1}-\hat{k}_t)+r^K\hat{r}_{t+1}^K-\sigma r^K\hat{c}_{t+1}\Big)\Big]\\
c^{-\sigma}\Big(-\sigma\hat{c}_t+\phi\delta(\hat{i}_t-\hat{k}_{t-1})\Big) &= \beta\mathbb{E}_t\Big[-\sigma(1-\delta+r^K)\hat{c}_{t+1}+\phi\delta(\hat{i}_{t+1}-\hat{k}_t)+r^K\hat{r}_{t+1}^K)\Big]\\
-\sigma\hat{c}_t+\phi\delta(\hat{i}_t-\hat{k}_{t-1}) &= \mathbb{E}_t\Big[-\sigma\hat{c}_{t+1}+\beta\phi\delta(\hat{i}_{t+1}-\hat{k}_t)+(1-\beta(1-\delta))\hat{r}_{t+1}^K\Big]
\end{aligned}$$

G.1.4 SDF

$$\begin{aligned}
\Omega_{t,t+1} &= \beta\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\\
\Omega\hat{\Omega}_{t,t+1} &= \beta\sigma(-\hat{c}_{t+1}+\hat{c}_t)\\
\hat{\Omega}_{t,t+1} &= -\sigma(\hat{c}_{t+1}-\hat{c}_t)
\end{aligned}$$

G.1.5 Goods expenditure

$$c_t+i_t = \left(\sum_j \omega_j^{\frac{1}{\gamma}}(y_{jt})^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} = G_t$$

$$\begin{aligned}
c\hat{c}_t+i\hat{i}_t &= G\hat{G}_t\\
\frac{c}{G}\hat{c}_t+\frac{i}{G}\hat{i}_t &= \hat{G}_t
\end{aligned}$$

G.1.6 LOM for capital

$$k_t = (1-\delta)k_{t-1}+i_t-\frac{\phi}{2}k_{t-1}\bigg(\frac{i_t}{k_{t-1}}-\delta\bigg)^2$$

$$
\begin{aligned}
k\hat{k}_t &= (1-\delta)k\hat{k}_{t-1} + i\hat{i}_t \\
\hat{k}_t &= (1-\delta)\hat{k}_{t-1} + \delta\hat{i}_t
\end{aligned}
$$

G.1.7 Aggregate Price

$$
\begin{aligned}
P_t &= \left[\sum_j \omega_j p_{jt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = 1 \\
P\hat{P}_t &= \frac{1}{1-\gamma}\left[\sum_j \omega_j p_j^{1-\gamma}\right]^{\frac{1}{1-\gamma}-1} (1-\gamma)\omega_j p_j^{1-\gamma}\hat{p}_{jt} \\
\hat{P}_t &= \sum_j \frac{\omega_j p_j^{1-\gamma}}{\sum_j \omega_j p_j^{1-\gamma}}\hat{p}_{jt} \\
\hat{P}_t &= \sum_j \omega_j p_j^{1-\gamma}\hat{p}_{jt} = 0
\end{aligned}
$$

G.1.8 Demand Schedule

$$
\begin{aligned}
y_{jt} &= \omega_j p_{jt}^{-\gamma}\left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_{jt})^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} = \omega_j p_{jt}^{-\gamma} G_t \\
\ln y_{jt} &= \ln \omega_j - \gamma \ln p_{jt} + \ln G_t \\
\hat{y}_{jt} &= -\gamma\hat{p}_{jt} + \hat{G}_t
\end{aligned}
$$

G.2 Producers

G.2.1 Marginal cost

$$
\begin{aligned}
v_{jt}^{1-\alpha_j^j} &= \frac{1}{z_{jt}\varrho_j}(r_t^K)^{\alpha_k^j}(w_t)^{\alpha_l^j}\prod_{m\neq j} p_{mt}^{\alpha_m^j} \\
(1-\alpha_j^j)\ln v_{jt} &= -\ln z_{jt} - \ln \varrho_j + \alpha_k^j \ln r_t^K + \alpha_l^j \ln w_t \sum_{m\neq j}\alpha_m^j \ln p_{mt} \\
(1-\alpha_j^j)\hat{v}_{jt} &= -\hat{z}_{jt} + \alpha_k^j \hat{r}_t^K + \alpha_l^j \hat{w}_t + \sum_{m\neq j}\alpha_m^j \hat{p}_{mt}
\end{aligned}
$$

G.2.2 Capital Labor choice

$$
\frac{k_{jt}}{l_{jt}} = \frac{\alpha_k^j}{\alpha_l^j}\frac{w_t}{r_t^K}
$$

$$
\begin{aligned}
\ln k_{jt} - \ln l_{jt} &= \ln \alpha_k^j - \ln \alpha_l^j + \ln w_t - \ln r_t^K \\
\hat{k}_{jt} - \hat{l}_{jt} &= \hat{w}_t - \hat{r}_t^K
\end{aligned}
$$

G.2.3 Intermediate Labor choice

$$
\begin{aligned}
\frac{x_{jmt}}{l_{jt}} &= \begin{cases} \frac{\alpha_j^j}{\alpha_l^j}\frac{w_t}{v_{jt}} & (m=j) \\ \frac{\alpha_m^j}{\alpha_l^j}\frac{w_t}{p_{mt}} & (m \neq j) \end{cases} \\
\ln x_{jmt} - \ln l_{jt} &= \begin{cases} \ln \alpha_j^j - \ln \alpha_l^j + \ln w_t - \ln v_{jt} & (m=j) \\ \ln \alpha_m^j - \ln \alpha_l^j + \ln w_t - \ln p_{mt} & (m \neq j) \end{cases} \\
\hat{x}_{jmt} - \hat{l}_{jt} &= \begin{cases} \hat{w}_t - \hat{v}_{jt} & (m=j) \\ \hat{w}_t - \hat{p}_{mt} & (m \neq j) \end{cases}
\end{aligned}
$$

G.3 Customer Market: Producers

G.3.1 Optimal Marketing Capital

$$
\frac{h_t W_{jt}}{\sum_j m_{jt}} = \begin{cases} \frac{A_{jt} v_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1} v_{1t+1} \Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}} - \delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j=1) \\ \frac{A_{jt} p_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)} - \mathbb{E}_t\left[\frac{A_{jt+1} p_{1t+1} \Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}} - \delta_m^j\right)}\left\{(1-\delta_m^j) - \frac{\psi_j}{2}\left((\delta_m^j)^2 - \left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right] & (j \neq 1) \end{cases}
$$

For LHS

$$
\frac{h_t W_{jt}}{\sum_j m_{jt}} \Longrightarrow \frac{h W_j}{\sum_j m_j}\left(\hat{h}_t + \hat{W}_{jt} - \sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt}\right)
$$

For RHS

$$
\begin{aligned}
\frac{A_{jt} v_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)} &\Longrightarrow v_1\left(\hat{A}_{jt} + \hat{v}_{1t} + \psi_j \delta_m^j (\hat{a}_{jt} - \hat{m}_{t-1})\right) \\
\frac{A_{jt} p_{1t}}{1-\psi_j\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)} &\Longrightarrow p_1\left(\hat{A}_{jt} + \hat{p}_{1t} + \psi_j \delta_m^j (\hat{a}_{jt} - \hat{m}_{t-1})\right)
\end{aligned}
$$

and

$$
\mathbb{E}_t\left[\frac{A_{jt+1}v_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j)-\frac{\psi_j}{2}\left((\delta_m^j)^2-\left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right]
$$

$$
\Longrightarrow v_1\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{v}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big]
$$

$$
\mathbb{E}_t\left[\frac{A_{jt+1}p_{1t+1}\Omega_{t,t+1}}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\left\{(1-\delta_m^j)-\frac{\psi_j}{2}\left((\delta_m^j)^2-\left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)\right\}\right]
$$

$$
\Longrightarrow p_1\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{p}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big]
$$

Note that

$$
\frac{(1-\delta_m^j)-\frac{\psi_j}{2}\left((\delta_m^j)^2-\left(\frac{a_{jt+1}}{m_{jt}}\right)^2\right)}{1-\psi_j\left(\frac{a_{jt+1}}{m_{jt}}-\delta_m^j\right)}\Longrightarrow(\psi_j(\delta_m^j)^2+(1-\delta_m^j)\psi_j\delta_m^j)(\hat{a}_{jt+1}-\hat{m}_{jt})
$$

Thus

$$
(1-\beta(1-\delta_m^j))\left(\hat{h}_t+\hat{W}_{jt}-\sum_j\frac{m_j}{\sum_j m_j}\hat{m}_{jt}\right)=
$$

$$
\begin{cases}
\hat{A}_{jt}+\hat{v}_{1t}+\psi_j\delta_m^j(\hat{a}_{jt}-\hat{m}_{t-1}) & \\
-\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{v}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big] & (j=1)\\
\hat{A}_{jt}+\hat{p}_{1t}+\psi_j\delta_m^j(\hat{a}_{jt}-\hat{m}_{t-1}) & \\
-\beta\mathbb{E}_t\Big[(1-\delta_m^j)(\hat{A}_{jt+1}+\hat{p}_{1t+1}+\hat{\Omega}_{t,t+1})+\psi_j\delta_m^j(\hat{a}_{jt+1}-\hat{m}_{jt})\Big] & (j\neq 1)
\end{cases}
$$

G.3.2 LOM for Client List

$$
d_{jt}\sum_j m_{jt}=(1-\delta_H)d_{jt-1}\sum_j m_{jt}+m_{jt}h_t
$$

$$
\left(d_j\hat{d}_{jt}\sum_j m_j\right)+\left(d_j\sum_j m_j\hat{m}_{jt}\right)=\left((1-\delta_H)d_j\hat{d}_{jt-1}\sum_j m_j\right)+\left((1-\delta_H)d_j\sum_j m_j\hat{m}_{jt}\right)
$$

$$
+m_jh(\hat{m}_{jt}+\hat{h}_t)
$$

$$
\hat{d}_{jt}\sum_j m_j+\sum_j m_j\hat{m}_{jt}=(1-\delta_H)\hat{d}_{jt-1}\sum_j m_j+(1-\delta_H)\sum_j m_j\hat{m}_{jt}
$$

$$
+\delta_H\sum_j m_j(\hat{m}_{jt}+\hat{h}_t)
$$

$$\hat{d}_{jt} = (1-\delta_H)\hat{d}_{jt-1} - \delta_H \sum_j \frac{m_j}{\sum_j m_j}\hat{m}_{jt} + \delta_H(\hat{m}_{jt} + \hat{h}_t)$$

G.3.3 LOM for Marketing Capital

$$\begin{aligned} m_{jt} &= (1-\delta_m^j)m_{jt-1} + a_{jt} - \frac{\psi_j}{2}m_{jt-1}\left(\frac{a_{jt}}{m_{jt-1}} - \delta_m^j\right)^2 \\ m_j\hat{m}_{jt} &= (1-\delta_m^j)m_j\hat{m}_{jt-1} + a_j\hat{a}_{jt} \\ \hat{m}_{jt} &= (1-\delta_m^j)\hat{m}_{jt-1} + \delta_m^j\hat{a}_{jt} \end{aligned}$$

G.3.4 Wholesaler Value

$$\begin{aligned} W_{jt} &= q_{jt} - v_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}W_{jt+1}] \\ W_j\hat{W}_{jt} &= q_j\hat{q}_{jt} - v_j\hat{v}_{jt} + (1-\delta_H)\beta W_j\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{W}_{jt+1}\Big] \\ \hat{W}_{jt} &= \frac{1}{W_j}(q_j\hat{q}_{jt} - v_j\hat{v}_{jt}) + (1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{W}_{jt+1}\Big] \end{aligned}$$

G.4 Customer Market: Retailers

G.4.1 Retailer Value

$$\begin{aligned} J_{jt} &= p_{jt} - q_{jt} + (1-\delta_H)\mathbb{E}_t[\Omega_{t,t+1}J_{jt+1}] \\ J_j\hat{J}_{jt} &= p_j\hat{p}_{jt} - q_j\hat{q}_{jt} + (1-\delta_H)\Omega J_j\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{J}_{jt+1}\Big] \\ \hat{J}_{jt} &= \frac{1}{J_j}(p_j\hat{p}_{jt} - q_j\hat{q}_{jt}) + (1-\delta_H)\beta\mathbb{E}_t\Big[\hat{\Omega}_{t,t+1} + \hat{J}_{jt+1}\Big] \end{aligned}$$

G.4.2 Nash Bargaining

$$\begin{aligned} q_{jt} &= \theta p_{jt} + (1-\theta)v_{jt} \\ q_j\hat{q}_{jt} &= \theta p_j\hat{p}_{jt} + (1-\theta)v_j\hat{v}_{jt} \end{aligned}$$

G.4.3 Free entry condition

$$\begin{aligned} \sum_j \frac{m_{jt}}{\sum_j m_{jt}}J_{jt} &= \chi p_{1t} \\ \sum_j m_{jt}J_{jt} &= \chi p_{1t}\sum_j m_{jt} \end{aligned}$$

$$\sum_j m_j J_j(\hat{m}_{jt} + \hat{J}_{jt}) = \chi \sum_j p_1 m_j(\hat{p_{1t}} + \hat{m}_{jt})$$

G.5 Market Cleaning

G.5.1 Good Market

$$z_{jt}F_{jt} == \begin{cases} y_{jt} + \sum_m x_{mjt} + \sum_m a_{mt} + \chi h_t & (j = 1) \\ y_{jt} + \sum_m x_{mjt} & (j \neq 1) \end{cases}$$

$$F_j(\hat{z}_{jt} + \hat{F}_{jt}) = \begin{cases} y_j\hat{y}_{jt} + \sum_m x_{mj}\hat{x}_{mjt} + \sum_m a_m\hat{a}_{mt} + \chi h\hat{h}_t & (j = 1) \\ y_j\hat{y}_{jt} + \sum_m x_{mj}\hat{x}_{mjt} & (j \neq 1) \end{cases}$$

$$\hat{z}_{jt} + \hat{F}_{jt} = \begin{cases} \frac{y_j}{F_j}\hat{y}_{jt} + \sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt} + \sum_m \frac{a_m}{F_j}\hat{a}_{mt} + \frac{\chi h}{F_j}\hat{h}_t & (j = 1) \\ \frac{y_j}{F_j}\hat{y}_{jt} + \sum_m \frac{x_{mj}}{F_j}\hat{x}_{mjt} & (j \neq 1) \end{cases}$$

G.5.2 Retail Market

$$d_{jt} = \begin{cases} y_{jt} + \sum_{m\neq j} x_{mjt} + \sum_{m\neq j} a_{mt} + \chi h_t & (j = 1) \\ y_{jt} + \sum_{m\neq j} x_{mjt} & (j \neq 1) \end{cases}$$

$$d_j\hat{d}_{jt} = \begin{cases} y_j\hat{y}_{jt} + \sum_{m\neq j} x_{mj}\hat{x}_{mjt} + \sum_{m\neq j} a_m\hat{a}_{mt} + \chi h\hat{h}_t & (j = 1) \\ y_j\hat{y}_{jt} + \sum_{m\neq j} x_{mj}\hat{x}_{mjt} & (j \neq 1) \end{cases}$$

$$\hat{d}_{jt} = \begin{cases} \frac{y_j}{d_j}\hat{y}_{jt} + \sum_{m\neq j} \frac{x_{mj}}{d_j}\hat{x}_{mjt} + \sum_{m\neq j} \frac{a_m}{d_j}\hat{a}_{mt} + \frac{\chi h}{d_j}\hat{h}_t & (j = 1) \\ \frac{y_j}{d_j}\hat{y}_{jt} + \sum_{m\neq j} \frac{x_{mj}}{d_j}\hat{x}_{mjt} & (j \neq 1) \end{cases}$$

G.5.3 Labor

$$\sum_j l_{jt} = l_t$$

$$\sum_j l_j\hat{l}_{jt} = l\hat{l}_t$$

G.5.4 Capital

$$\sum_j k_{jt} = k_{t-1}$$

$$\sum_j k_j \hat{k}_{jt} = k \hat{k}_{t-1}$$

G.6 Exogenous process

G.6.1 Productivity

$$\begin{aligned}
\ln z_{jt} &= (1-\rho_z^j)\ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt} \\
\ln \frac{z_j + \Delta z_{jt}}{z_j} &= (1-\rho_z^j)\ln \frac{z_j^*}{z_j} + \rho_z^j \ln \frac{z_j + \Delta z_{jt-1}}{z_j} + \varepsilon_{jt} \\
\ln(1+\hat{z}_{jt}) &= \rho_z^j \ln(1+\hat{z}_{jt-1}) + \varepsilon_{jt} \\
\hat{z}_{jt} &= \rho_z^j \hat{z}_{jt-1} + \varepsilon_{jt}
\end{aligned}$$

G.6.2 Investment Productivity

$$\begin{aligned}
\ln A_{jt} &= \rho_A \ln A_{jt-1} + \varepsilon_{jt}^A \\
\ln \frac{A_j + \Delta A_{jt}}{A_j} &= \rho_A \ln \frac{A_j + \Delta A_{jt-1}}{A_j} + \varepsilon_{jt}^A \\
\ln\left(1+\hat{A}_{jt}\right) &= \rho_A \ln\left(1+\hat{A}_{jt-1}\right) + \varepsilon_{jt}^A \\
\hat{A}_{jt} &= \rho_A \hat{A}_{jt-1} + \varepsilon_{jt}^A
\end{aligned}$$

G.7 Auxiliary variables

G.7.1 F

$$\begin{aligned}
F_{jt} &= k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j} \\
\ln F_{jt} &= \alpha_k^j \ln k_{jt} + \alpha_l^j \ln l_{jt} + \sum_m \alpha_m^j \ln x_{jmt} \\
\hat{F}_{jt} &= \alpha_k^j \hat{k}_{jt} + \alpha_l^j \hat{l}_{jt} + \sum_m \alpha_m^j \hat{x}_{jmt}
\end{aligned}$$

G.7.2 G

$$G_t = \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_{jt})^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

$$G\hat{G}_t = G\sum_j \frac{\gamma}{\gamma-1}\left(\sum_j \omega_j^{\frac{1}{\gamma}}(y_{jt})^{\frac{\gamma-1}{\gamma}}\right)^{-1}\omega_j^{\frac{1}{\gamma}}\frac{\gamma-1}{\gamma}(y_j)^{\frac{\gamma-1}{\gamma}}\hat{y}_{jt}$$

$$G\hat{G}_t = \sum_j \frac{\omega_j^{\frac{1}{\gamma}}(y_j)^{\frac{\gamma-1}{\gamma}}}{\sum_j \omega_j^{\frac{1}{\gamma}}(y_j)^{\frac{\gamma-1}{\gamma}}}\hat{y}_{jt}$$

$$\hat{G}_t = \sum_j \frac{\omega_j^{\frac{1}{\gamma}}(y_j)^{\frac{\gamma-1}{\gamma}}}{\sum_j \omega_j^{\frac{1}{\gamma}}(y_j)^{\frac{\gamma-1}{\gamma}}}\hat{y}_{jt}$$

$$\hat{G}_t = \sum_j \frac{\omega_j^{\frac{1}{\gamma}}\left(\omega_j p_j^{-\gamma}\left(\sum_j \omega_j^{\frac{1}{\gamma}}(y_j)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}}}{\sum_j \omega_j^{\frac{1}{\gamma}}(y_j)^{\frac{\gamma-1}{\gamma}}}\hat{y}_{jt}$$

$$\hat{G}_t = \sum_j \omega_j p_j^{1-\gamma}\hat{y}_{jt}$$

H Different Detrending Methods of the TFP Process

Table 8: Parameters of productivity process

	ρ_i			σ_i		
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
KLEMS						
Hamilton filter ($h = 2, p = 4$)	0.498	0.552	0.506	0.016	0.010	0.011
HP filter ($\lambda = 100$)	0.358	0.507	0.569	0.011	0.008	0.006
Linear trend	0.871	0.728	0.761	0.014	0.009	0.006
Cubic trend	0.535	0.574	0.694	0.012	0.008	0.006

Figure 10: Hamilton filter 1987-2018

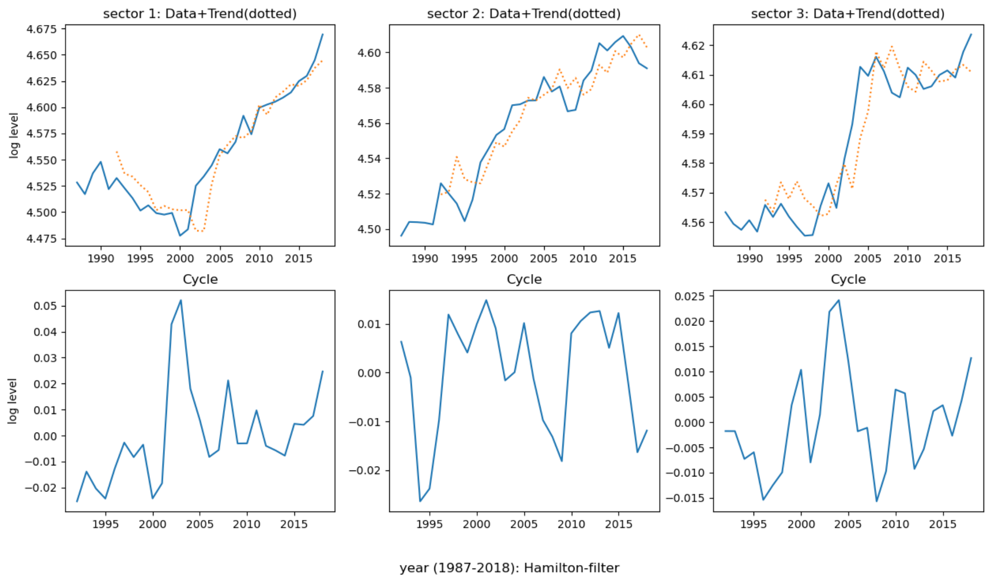

Figure 11: HP filter 1987-2018

Figure 12: Linear Trend 1987-2018

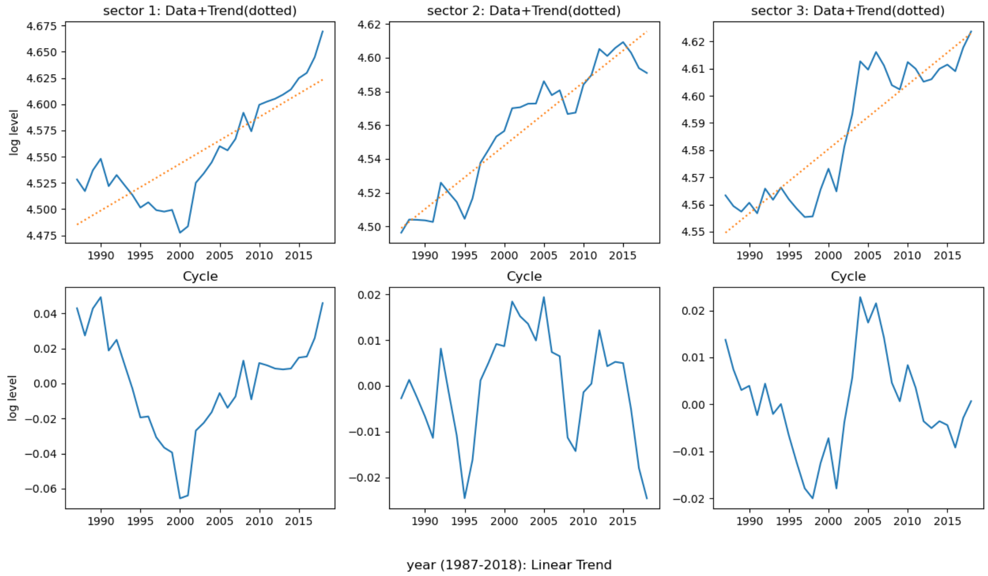

Figure 13: Cubic Trend 1987-2018

I Industry Classification

Table 9: Industry Classification based on AI-exposure

NAICS 3 digit	NAICS code (IO table)	Industry Description	Baseline	Conservative
111,112	111CA	Crop & animal production (Farms)	3	3
113-115	113FF	Forestry, fishing, and related activities	3	3
211	211	Oil and gas extraction	3	3
212	212	Mining, except oil and gas	3	3
213	213	Support activities for mining	3	3
22	22	Utilities	3	3
23	23	Construction	3	3
311,312	311FT	Food and beverage and tobacco products	3	3
313,314	313TT	Textile mills and textile product mills	3	3
315,316	315AL	Apparel and leather and applied products	3	3
322	322	Paper products	3	3
323	323	Printing and related support activities	3	3
324	324	Petroleum and coal products	3	3
325	325	Chemical products	3	3
326	326	Plastics and rubber products	3	3
321	321	Wood products	3	3
327	327	Nonmetallic mineral products	3	3
331	331	Primary metal products	3	3
332	332	Fabricated metal products	3	3
333	333	Machinery	3	3
334	334	Computer and electronic products	3	3
335	335	Electrical equipment, appliances, and components	3	3
3361-3363	3361MV	Motor vehicles, bodies and trailers, and parts	3	3
3364-3369	3364OT	Other transportation equipment	3	3
337	337	Furniture and related products	3	3
339	339	Miscellaneous manufacturing	3	3
42	42	Wholesale trade	2	2
44,45	441	Retail trade	2	2
481	481	Air transportation	3	3
482	482	Rail transportation	3	3
483	483	Water transportation	3	3
484	484	Truck transportation	3	3
485	485	Transit and ground passenger transportation	3	3
486	486	Pipeline transportation	3	3
487,488,492	487OS	Other transportation and support activities	3	3
493	493	Warehousing and storage	3	3
511	511	Publishing industries, except internet (includes software)	1	2
512	512	Motion picture and sound recording industries	1	2
515,517	513	Broadcasting and telecommunications	1	2
518,519	514	Data processing, internet publishing, and other information services	1	1
521,522	521CI	Federal reserve banks, credit intermediation, and related activities	2	2
523	523	Securities, commodity contracts, and other financial investments and related activities	2	2
524	524	Insurance carriers and related activities	2	2
525	525	Funds, trusts, and other financial vehicles	2	2
531	HS	Real estate	2	2
531	ORE	Real estate	2	2
532,533	532RL	Rental and leasing services and lessors of nonfinancial and intangible assets	2	2
5411	5411	Legal services	2	2
5415	5415	Computer systems design and related services	1	1
5412-5414,5416-5419	5412OP	Miscellaneous professional, scientific, and technical services	1	2
55	55	Management of companies and enterprises	1	1
561	561	Administrative and support services	1	1
562	562	Waste management and remediation services	3	3
61	61	Educational services	2	2
621	621	Ambulatory health care services	3	3
622,623	622	Hospitals and nursing and residential care facilities	3	3
622,623	623	Hospitals and nursing and residential care facilities	3	3
624	624	Social assistance	3	3
711,712	711AS	Performing arts, spectator sports, museums, and related activities	3	3
713	713	Amusements, gambling, and recreation industries	3	3
721	721	Accommodation	3	3
722	722	Food services and drinking places	3	3
81	81	Other services, except government	3	3

J PCE Bridge Table

Table 10: An excerpt of PCE Bridge Table of 2020 (Millions of dollars)

NIPA Line	PCE Category	Commodity Code (NAICS 3 digit)	Commodity Description	Producers' Value	Transportation Costs	Trade Margins Wholesale	Trade Margins Retail	Purchasers' Value
5	New motor vehicles	3361MV	Motor vehicles, bodies and trailers, and parts	176,213	2,664	6,076	107,861	292,814
6	Net purchases of used motor vehicles	Used	Scrap, used and secondhand goods	82,508	1,759	2,396	78,906	165,570
7	Motor vehicles parts and accessories	325	Chemical products	213	4	31	125	373
7	Motor vehicles parts and accessories	326	Plastics and rubber products	14,153	321	5,100	17,907	37,481
7	Motor vehicles parts and accessories	327	Nonmetallic mineral products	190	30	32	242	493
7	Motor vehicles parts and accessories	331	Primary metals	12	0	1	15	28
7	Motor vehicles parts and accessories	332	Fabricated metal products	760	24	402	970	2,155
7	Motor vehicles parts and accessories	333	Machinery	0	0	0	0	0
7	Motor vehicles parts and accessories	334	Computer and electronic products	10	0	8	13	31
7	Motor vehicles parts and accessories	335	Electrical equipment, appliances, and components	1,394	17	247	1,778	3,436
7	Motor vehicles parts and accessories	3361MV	Motor vehicles, bodies and trailers, and parts	16,766	215	2,845	21,379	41,205
7	Motor vehicles parts and accessories	339	Miscellaneous manufacturing	133	19	36	169	357
7	Motor vehicles parts and accessories	Used	Scrap, used and secondhand goods	-1,701	25	1,492	3,249	3,064
9	Furniture and furnishings	313TT	Textile mills and textile product mills	11,164	615	4,185	14,684	30,647
9	Furniture and furnishings	315AL	Apparel and leather and allied products	64	3	29	67	163
9	Furniture and furnishings	321	Wood products	1,629	129	297	1,703	3,758
9	Furniture and furnishings	323	Printing and related support activities	264	13	43	275	595
9	Furniture and furnishings	326	Plastics and rubber products	954	20	341	1,104	2,417
9	Furniture and furnishings	327	Nonmetallic mineral products	1,753	269	505	442	2,969
9	Furniture and furnishings	331	Primary metals	91	3	9	442	545
9	Furniture and furnishings	332	Fabricated metal products	554	17	275	2,690	3,536
9	Furniture and furnishings	334	Computer and electronic products	907	25	272	946	2,149
9	Furniture and furnishings	335	Electrical equipment, appliances, and components	2,748	403	819	2,899	6,869
9	Furniture and furnishings	337	Furniture and related products	54,321	8,007	10,369	53,845	126,542
9	Furniture and furnishings	339	Miscellaneous manufacturing	5,221	633	1,955	4,496	12,305
9	Furniture and furnishings	532RL	Rental and leasing services and lessors of intangible assets	5,375	0	0	0	5,375
9	Furniture and furnishings	Used	Scrap, used and secondhand goods	8,205	3,781	2,996	2,130	17,112
10	Household appliances	326	Plastics and rubber products	27	1	8	41	77
10	Household appliances	331	Primary metals	30	1	3	18	52
10	Household appliances	333	Machinery	5,448	170	2,479	3,212	11,309
10	Household appliances	335	Electrical equipment, appliances, and components	30,418	1,425	9,571	16,766	58,180
10	Household appliances	Used	Scrap, used and secondhand goods	-84	38	26	35	14
11	Glassware, tableware and household utensils	321	Wood products	568	38	104	40	750
11	Glassware, tableware and household utensils	326	Plastics and rubber products	18,455	385	3,212	1,285	23,336
11	Glassware, tableware and household utensils	327	Nonmetallic mineral products	7,608	1,180	2,471	2,971	14,230
11	Glassware, tableware and household utensils	331	Primary metals	530	16	51	37	634
11	Glassware, tableware and household utensils	332	Fabricated metal products	4,118	126	1,223	317	5,784
11	Glassware, tableware and household utensils	339	Miscellaneous manufacturing	188	2	122	13	325
11	Glassware, tableware and household utensils	Used	Scrap, used and secondhand goods	-558	65	52	10	-432
12	Tools and equipment for house and garden	313TT	Textile mills and textile product mills	39	3	17	77	136
12	Tools and equipment for house and garden	321	Wood products	40	3	7	194	243
12	Tools and equipment for house and garden	325	Chemical products	4,477	130	925	3,613	9,146
12	Tools and equipment for house and garden	326	Plastics and rubber products	382	7	102	752	1,244
12	Tools and equipment for house and garden	327	Nonmetallic mineral products	325	71	63	191	651
12	Tools and equipment for house and garden	331	Primary metals	7	0	1	36	44
12	Tools and equipment for house and garden	332	Fabricated metal products	1,635	50	851	7,651	10,187
12	Tools and equipment for house and garden	333	Machinery	3,859	120	1,088	18,738	23,805
12	Tools and equipment for house and garden	334	Computer and electronic products	0	0	0	1	1
12	Tools and equipment for house and garden	335	Electrical equipment, appliances, and components	59	1	18	286	363
12	Tools and equipment for house and garden	337	Furniture and related products	118	19	43	233	413
12	Tools and equipment for house and garden	339	Miscellaneous manufacturing	212	32	73	905	1,223
12	Tools and equipment for house and garden	532RL	Rental and leasing services and lessors of intangible assets	1,535	0	0	0	1,535
12	Tools and equipment for house and garden	Used	Scrap, used and secondhand goods	61	28	39	133	260
14	Video, audio, photographic, and information processing equipment and media	315AL	Apparel and leather and allied products	0	0	0	0	0
14	Video, audio, photographic, and information processing equipment and media	333	Machinery	704	22	701	346	1,773
14	Video, audio, photographic, and information processing equipment and media	334	Computer and electronic products	69,155	778	18,502	34,028	122,463
14	Video, audio, photographic, and information processing equipment and media	335	Electrical equipment, appliances, and components	1,080	13	322	888	2,303
14	Video, audio, photographic, and information processing equipment and media	339	Miscellaneous manufacturing	6	1	1	0	8
14	Video, audio, photographic, and information processing equipment and media	511	Publishing industries, except internet (includes software)	81,107	865	19,282	24,339	125,593
14	Video, audio, photographic, and information processing equipment and media	512	Motion picture and sound recording industries	5,136	48	1,213	5,772	12,169
14	Video, audio, photographic, and information processing equipment and media	514	Data processing, internet publishing, and other information services	32,114	0	0	0	32,114
14	Video, audio, photographic, and information processing equipment and media	Used	Scrap, used and secondhand goods	15	39	64	43	161
15	Sporting equipment, supplies, guns, and ammunition	313TT	Textile mills and textile product mills	2,340	145	1,002	1,873	5,360
15	Sporting equipment, supplies, guns, and ammunition	315AL	Apparel and leather and allied products	350	17	155	280	802
15	Sporting equipment, supplies, guns, and ammunition	325	Chemical products	1,670	52	390	1,337	3,449
15	Sporting equipment, supplies, guns, and ammunition	331	Primary metals	97	3	9	78	188
15	Sporting equipment, supplies, guns, and ammunition	332	Fabricated metal products	12,115	366	4,820	9,530	26,832
15	Sporting equipment, supplies, guns, and ammunition	3364OT	Other transportation equipment	3,786	41	333	1,309	5,468
15	Sporting equipment, supplies, guns, and ammunition	339	Miscellaneous manufacturing	19,418	2,649	7,785	22,765	52,616
15	Sporting equipment, supplies, guns, and ammunition	Used	Scrap, used and secondhand goods	-125	0	0	0	-125
16	Sports and recreational vehicles	326	Plastics and rubber products	1,326	30	468	1,541	3,365
16	Sports and recreational vehicles	332	Fabricated metal products	128	4	75	36	243
16	Sports and recreational vehicles	333	Machinery	1,867	60	370	524	2,822
16	Sports and recreational vehicles	335	Electrical equipment, appliances, and components	44	1	8	12	64
16	Sports and recreational vehicles	3361MV	Motor vehicles, bodies and trailers, and parts	25,999	309	768	10,104	37,181
16	Sports and recreational vehicles	3364OT	Other transportation equipment	26,859	292	2,400	9,093	38,644
16	Sports and recreational vehicles	Used	Scrap, used and secondhand goods	970	1,926	635	1,173	4,703
⋮	⋮	⋮	⋮	⋮	⋮	⋮		

K Alternative Industry Classification

K.1 Calibration

Table 11: Input share α_m^j based on the conservative classification in Table 9.

	Sector 1	Sector 2	Sector 3
α_1^j	0.09	0.06	0.04
α_2^j	0.17	0.23	0.10
α_3^j	0.10	0.10	0.39
Labor α_L^j	0.49	0.25	0.29
Capital α_k^j	0.15	0.36	0.18

Table 12: Spending share ω_j based on the conservative classification in Table 9.

	Sector 1	Sector 2	Sector 3
ω_j	0.007	0.325	0.668

Table 13: Price volatility based on the conservative classification in Table 9

	Sector 1	Sector 2	Sector 3
σ_{PPI}	0.004	0.014	0.032
σ_{PCE}	0.022	0.011	0.012
$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	0.163	1.300	2.575

Table 14: Parameters of productivity process based on the conservative classification in Table 9.

	ρ_i			σ_i		
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
KLEMS	0.609	0.277	0.694	0.017	0.006	0.006

Jointly calibrated parameters

Parameter	Symbol	Value	Target		Model	
Physical Capital Adjustment cost	ϕ	0.9965	$\sigma_i/\sigma_{\text{GDP}}$	2.830	$\sigma_i/\sigma_{\text{rGDP}}$	2.5788
Marketing Capital Adjustment cost	ψ_1	34.9928	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	0.163	$\sigma_{q_1}/\sigma_{p_1}$	0.7663
Marketing Capital Adjustment cost	ψ_2	29.9967	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	1.300	$\sigma_{q_2}/\sigma_{p_2}$	2.5842
Marketing Capital Adjustment cost	ψ_3	34.9928	$\sigma_{\text{PPI}}/\sigma_{\text{PCE}}$	2.575	$\sigma_{q_3}/\sigma_{p_3}$	3.5436
Persistence of productivity in Sector 1	ρ_1	0.5117	KLEMS	0.609	rGDP_1/F_1	0.4602
Persistence of productivity in Sector 2	ρ_2	0.6007	KLEMS	0.277	rGDP_2/F_2	0.6235
Persistence of productivity in Sector 3	ρ_3	0.3998	KLEMS	0.694	rGDP_3/F_3	0.4044
Standard deviation of productivity in Sector 1	σ_1	0.0070	KLEMS	0.017	rGDP_1/F_1	0.0210
Standard deviation of productivity in Sector 2	σ_2	0.0070	KLEMS	0.006	rGDP_2/F_2	0.0064
Standard deviation of productivity in Sector 3	σ_3	0.0070	KLEMS	0.006	rGDP_3/F_3	0.0066
Parameter of labor disutility	ξ	0.6894	Normalized steady state labor supply	1		
Search cost	χ	0.1423	Gross wholesale markup	10%		
Marketing Capital Depreciation	δ_m^1	0.1628	Marketing expenditure to Sales ratio	7%		
Marketing Capital Depreciation	δ_m^2	0.2591	Marketing expenditure to Sales ratio	7%		
Marketing Capital Depreciation	δ_m^3	0.2744	Marketing expenditure to Sales ratio	7%		

K.2 Full Set of IRFs

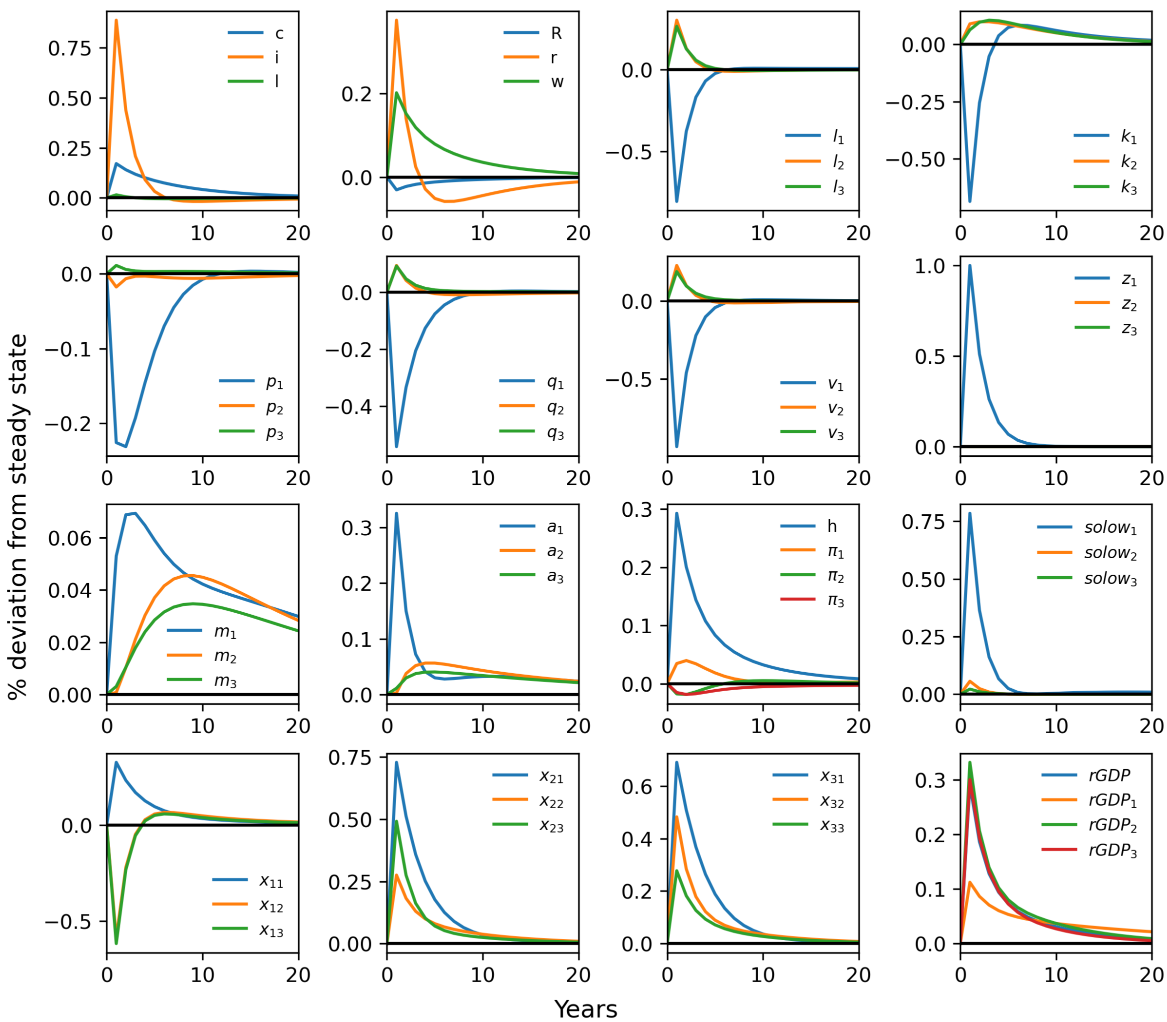

Figure 14: IRFs after a Gen-AI shock

L Model Summary: No Customer Search

Household

$$\xi \frac{l_t^{\eta}}{c_t^{-\sigma}} = w_t \tag{73}$$

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t[c_{t+1}^{-\sigma}] \tag{74}$$

$$\frac{c_t^{-\sigma}}{1-\phi\left(\frac{i_t}{k_{t-1}}-\delta\right)} = \beta \mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)}\left\{1-\delta+r_{t+1}^K\left(1-\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\right)+\phi\left(\frac{i_{t+1}}{k_t}-\delta\right)\frac{i_{t+1}}{k_t}\right\}\right] \tag{75}$$

$$\Omega_{t,t+1} = \beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \tag{76}$$

$$c_t + i_t = G_t \tag{77}$$

$$k_t = (1-\delta)k_{t-1} + i_t - \frac{\phi}{2} k_{t-1} \left(\frac{i_t}{k_{t-1}} - \delta \right)^2 \tag{78}$$

$$P_t = \left[\sum_j \omega_j (p_{jt})^{1-\gamma} \right]^{\frac{1}{1-\gamma}} = 1 \tag{79}$$

$$y_{jt} = \omega_j p_{jt}^{-\gamma} G_t \tag{80}$$

Producers

$$p_{jt} = v_{jt} \tag{81}$$

$$p_{jt} = \left(\frac{1}{z_{jt}\varrho_j} (r_t^K)^{\alpha_k^j} (w_t)^{\alpha_l^j} \prod_{m \neq j} p_{mt}^{\alpha_m^j} \right)^{\frac{1}{1-\alpha_j^j}} \tag{82}$$

$$\frac{k_{jt}}{l_{jt}} = \frac{\alpha_k^j}{\alpha_l^j} \frac{w_t}{r_t^K} \tag{83}$$

$$\frac{x_{jmt}}{l_{jt}} = \frac{\alpha_m^j}{a_l^j} \frac{w_t}{p_{mt}} \tag{84}$$

Market Cleaning

$$z_{jt} F_{jt} = y_{jt} + \sum_m x_{mjt} \tag{85}$$

$$\sum_j l_{jt} = l_t \tag{86}$$

$$\sum_j k_{jt} = k_{t-1} \tag{87}$$

Exogenous process

$$\ln z_{jt} = (1-\rho_z^j) \ln z_j^* + \rho_z^j \ln z_{jt-1} + \varepsilon_{jt} \tag{88}$$

Auxiliary variables

$$F_{jt} = k_{jt}^{\alpha_k^j} l_{jt}^{\alpha_l^j} \prod_m x_{jmt}^{\alpha_m^j} \tag{89}$$

$$G_t = \left(\sum_j \omega_j^{\frac{1}{\gamma}} (y_{jt})^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{90}$$