International Currency Competition

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Abstract

We study how countries compete to become an international safe asset provider. Governments in our model issue debt to a common set of investors, resulting in competition as issuance by one country raises required yields for all countries. Governments are tempted ex post to engage in expropriation or capital controls, and can build reputation as a safe asset provider by resisting temptation to do so. We show how increased competition deters countries from building reputation, leaving more countries stuck at low reputation levels and unable to supply safe assets. We derive a model-implied measure of country reputation. We estimate this reputation measure using micro-data on investor portfolio holdings, and use it to track the evolution of countries' reputation over time. We study how an incumbent safe asset provider, like the U.S., uses its issuance strategy to deter the emergence of competitors.

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1 Introduction

The internationalization of the Chinese Renminbi (RMB) in global bond markets has raised the possibility of a multipolar world for international currencies. Nevertheless, the existing dominance of the US dollar provides a long-established safe asset. In contrast, foreign holdings of Chinese bonds are both small and still come with questions about their safety, including risk of the Chinese government imposing outflow controls or otherwise making it difficult to repatriate investment. To understand how a multipolar world might or might not develop, it is important to understand how competition affects the incentives of a country to build credibility as a safe asset provider, and to measure in real time where different countries stand in terms of their reputation.

We provide a tractable model of competition between countries seeking to establish their reputations as international safe asset providers. We show how greater competition reduces the benefits of building reputation by diminishing the exorbitant privilege from doing so, leading existing lower-reputation countries to be less willing to bear the costs of foregoing use of capital controls or forms of expropriation in order to improve their reputation for investability. We show that our model generates a measure of reputation of all countries that can be estimated using micro data on investor portfolio holdings in real time. We estimate this measure to provide a time series index of the reputation of different countries. Finally, we show how an established high-reputation safe asset provider, like the U.S., can deter the emergence of a lower-reputation competitor by increasing competition through its issuance strategy. In doing so, the U.S. sacrifices some of its exorbitant privilege in order to prevent even greater losses that would emerge from the establishment of an alternative provider.

Our model builds upon the single-country reputation model of Clayton, Dos Santos, Maggiori and Schreger (2024a), henceforth CDMS, by allowing for a continuum of countries that compete with one another in international debt markets. Governments borrow from a common pool of investors. Competition arises because greater borrowing by one government worsens the residual demand curve for the bonds issued by all other countries. Within this competitive environment, each country can build up its reputation as a safe asset provider. In particular, at each date, governments borrow from international investors on behalf of their domestic economy. Governments cannot commit not to impose a tax on investors ex post, for example a capital control to manage a capital flight during a crisis. We assume governments are one of two types: a committed type that never imposes the tax by assumption, or an opportunistic government that strategically chooses whether to impose the tax. Investors do not know any particular government's true type, but update beliefs about the type from their observation of the government's behavior. By choosing not to impose the tax, opportunistic governments can build reputation for being safe bond issuers, which increases investor demand for their debt and lowers future borrowing costs. Opportunistic governments trade off the flow benefit of imposing the tax against the continuation value of building reputation when deciding whether to impose the tax.

Our model admits a tractable aggregation whereby competition is summarized by how sensitive investors' demand for a given country's debt is to the price of that debt. This sensitivity increases in how much debt other countries are issuing, which directly fuels competition. Moreover, it also increases when other countries have on average higher reputations: higher-reputation countries take advantage of favorable borrowing terms by issuing more, pushing up how much investors have to lend and raising borrowing costs for all countries. Competition is therefore shaped by the full distribution of reputations among all countries in the world.

After proving existence of a steady state of our competition model, we show how our model gives rise to an interesting two-way interaction between the extent of competition and the incentives for a country to build reputation. All else equal, as competition becomes more fierce, the incentives of an individual country to build reputation diminish. Intuitively, as its borrowing terms become less favorable in a more competitive environment, the benefits it gets from building reputation and increasing its future debt issuance fall. This leads opportunistic governments to be less willing to resist the temptation to impose the tax, and more governments impose the tax even at low levels of reputation. This leads to a pile up of countries at low reputation levels in the steady state stationary distribution. It is a low reputation trap in which countries are stuck issuing debt that the market perceives as unsafe, and their ex-post policy confirm the debt is indeed not safe. In more extreme cases of competition, we show that the possibility that some governments are committed is alone enough to lead all opportunistic governments to become permanently stuck at the lowest reputation level.

Measuring reputation in the data is a notoriously difficult problem. Based on the model, we derive a sufficient statistic to track countries' reputation over time and estimate it in nearly real-time using micro data on foreign investors' portfolios. Intuitively, for each country, say China, we track whether foreign investment funds that own Chinese bonds are specialists in investing in developed or emerging market bonds. Formally, we estimate at each point in time the correlation among investment funds between the share of the foreign portfolio invested in RMB bonds and the remaining share invested in a reference set of safe developed countries government bonds. We construct this measure of reputation for all countries. Based on the model, a higher correlation indicates that a country's reputation is closer to that of the reference set of countries with the highest reputation. We find high positive and stable correlations (reputations) for countries such as the U.S. and Eurozone, and negative stable correlations (reputations) for countries such as Brazil and South Africa. Interestingly, we find that China's reputation is in between emerging markets and developed countries. Consistent with the model's prediction that reputation builds after a country resists temptation to impose a tax, we show that China's measured reputation increased following its restraint in imposing restrictions on foreign investors during the 2015-16 capital flight (see also CDMS).

The model is tractable and can help make sense of players at both early (e.g., China) and late (e.g., the US) stages of reputation building. We study the strategic incentives of an established high-reputation dominant country to manipulate the incentives for lower-reputation competitors to establish themselves as competitors. Formally, we model the issuance strategy of a large country (e.g., the US) whose government is known to be the committed type forever. This country internalizes how its issuance decisions affect the reputation building of competitors that go through the reputation building problem of the baseline model. We show how the large incumbent can use increases in its debt issuance to crowd out opportunistic competitors from building reputation. In the limiting case, the incumbent can increase supply so much that all opportunistic competitors remain stuck at the lowest reputation forever. More generally, we show that increasing competition by using issuance to raise borrowing costs for all other countries can be valuable because it decreases the likelihood that opportunistic competitors make it to any point beyond the lowest reputation level in the cycle, and also decreases the reputation of those lowest-reputation competitors. This can complement the traditional Stackelberg motive, whereby the large incumbent wants to raise its issuance to directly crowd out issuance of its competitors. One interpretation is that the U.S. may want to set itself up as a world banker via a sovereign wealth fund, issuing safe debt to the rest of world to finance investment in domestic or foreign productive assets.

Related Literature. There is a recent theoretical literature on the international monetary system, mostly focusing on established international currencies like the U.S. Dollar and Euro (Farhi and Maggiori (2018), He et al. (2019), Chahrour and Valchev (2021), Gopinath and Stein (2021), Drenik, Kirpalani and Perez (2021), Choi, Kirpalani and Perez (2022), Coppola, Krishnamurthy and Xu (2024)). An important exception is Bahaj and Reis (2020) who focus on the early process of jump-starting the Renminbi as an international currency. They focus on the unit of account and payments role of a currency and examine the role of the introduction of People's Bank of China (PBoC) swap lines in leading the Chinese Renminbi to be adopted in the global payments system. Clayton et al. (2024a) documents empirically how China is liberalizing its bond market sequentially to different types of investors and shows theoretically how gradual liberalization provides a strategy for reputation building. Our paper builds upon their reputation building model to study how countries compete to become international safe asset provider, and we provide an empirical measure of country reputation.

Our model of dynamic reputation is related to foundational work by Kreps and Wilson (1982), Milgrom and Roberts (1982), and Barro and Gordon (1983). Diamond (1989, 1991) mixes dynamic reputation and adverse selection to study the dynamics of reputation acquisition in financial markets and the choice between bond and loan financing. Our modeling of reputation builds on the strand of literature that considers changes in type over time (Mailath and Samuelson (2001), Cripps et al. (2004), Phelan (2006), and Mailath et al. (2006)).[1] Our paper is related to the literature examining how reputational incentives can help sustain debt repayment by governments as in Amador and Phelan (2021) and Fourakis (2021).

Finally, our focus on the temptation that governments face in imposing ex-post capital controls

[1] See also Tadelis (1999) and Lu (2013).

is related to the literature studying fire sales and liquidity (Caballero and Simsek (2020), Clayton and Schaab (2022), Coppola (2021)).

2 A Model of International Currency Competition

An important feature of becoming an international currency is that a country at the beginning of the cycle faces competition from both other "aspirants" – those at the same low level of reputation – and from countries that are already established – those at high levels of reputation. For example, China is entering now, but faces competition from the U.S. as an established reserve currency issuer. Theoretically, the interaction between reputation building and competition is an interesting area due to complementarities. For example, the value to a country of future higher reputation increases if current competitors lose reputation but decreases if entrenched players issue more. Both occur because the actions of others affect the residual demand curve that the country faces for its debt at future levels of reputation. Our theoretical framework allows us to study competition among potential reserve currency issuers in a simple and tractable manner.

Time is discrete and infinite horzion, $t = 0, 1, \ldots$. Each date is a stage game that is divided into a Beginning and an End. There is a unit continuum of countries, $j \in [0, 1]$, each with a risk neutral government. There is a total measure 1 of investors equally distributed across a set of (observable) types $i \in \{1, \ldots, \mathcal{I}\}$. At the beginning of date t, the government of country j is either committed or opportunistic. Government type is not observable to foreign investors or to other governments. We denote π_{jt} to be the common beginning-of-t investor beliefs that the government of country j is the committed type.

2.1 Stage Game

At the beginning of date t, governments choose how much to borrow from each type of investor. In the end, governments can impose a tax $\tau_t \in \{0, \overline{\tau}\}$ on repayment to investors. The tax can be interpreted, for example, as imposition of a capital control, a partial default, or a sanction. A committed government always sets $\tau_t = 0$ by assumption, whereas an opportunistic government optimally selects a choice of τ_t. The structure of our stage game builds upon the reduced form version of the one-country stage game of CDMS by extending it to a multi-country environment. For tractability, we also introduce an asset S that is in fixed supply $\overline{S}$ and that is sold competitively. Its endogenously determined return is R_t^S. This asset serves as a common factor across investors, who allocate their wealth among government debt and the outside asset.

Government Payoff. At the beginning of date t, the government of country j has an endowment of productive assets that pay off $A > 0$ in the end. The government can borrow an amount $D_{jt}^i \geq 0$ from investors $i \in \{1, \ldots, \mathcal{I}\}$ at an endogenous interest rate R_{jt}^i. Borrowed money is invested in productive assets with expected return Q per unit of investment.[2] Define government j's total debt to be $D_{jt} = \frac{1}{\mathcal{I}} \sum_{i \in \mathcal{I}} D^i_{jt}$ and define the average interest rate that government j pays on its debt to be $R_{jt} = \frac{1}{\mathcal{I}} \sum_{i \in \mathcal{I}} R^i_{jt} \frac{D^i_{jt}}{D_{jt}}$. At the end of date t, the government chooses whether or not to impose the tax on investors $\tau_t \in \{0, \overline{\tau}\}$, yielding its stage game payoff,

$$c_{jt} = g(\tau_t)\Big(A + (Q - R_{jt})D_{jt}\Big), \tag{1}$$

where $g(\overline{\tau}) > g(0) = 1$. CDMS microfounds equation 1 in a model in which the government faces the possibility of a crisis in which investors flee the country, resulting in a costly liquidation of productive assets to repay the fleeing investors. In this interpretation, the tax τ_t is a capital control employed to mitigate investor flight and prevent costly liquidations, while $g(\overline{\tau})$ is a net worth multiplier reflecting the efficacy of capital controls in preventing costly liquidations.

Investor Payoff and Demand Schedule. An investor of type i in the beginning of t makes a portfolio choice out of her wealth level w_i. Each investor has three investment options. First, she can invest in government debt, where D^i_{jt} denotes her investment in government j's debt at promised interest rate R^i_{jt}. Second, she can invest in her own project, with exogenous return $\overline{R}$. Third, she can invest in the outside asset S^i_t with return R^S_t.

At the beginning of date t, investors have a common belief $M_{jt} \in [0, 1]$ that the government of country j will not impose the tax in the end. We refer to M_{jt} as country j's *reputation* at date t. Investor i chooses her portfolio to solve

$$\max_{\{D^i_{jt}\}, S^i_t} \ \overline{R}w_i + (R^S_t - \bar{R})S^i_t + \int_j [R^i_{jt} E[1 - \tau_{jt}] - \bar{R}] D^i_{jt} dj - \frac{1}{8} b\left(S^i_t + \int_j \omega_i(M_{jt})^{-1} D^{i2}_{jt} dj\right)^2. \tag{2}$$

The first three terms are the expected monetary payoff from investor i's portfolio holdings. The final term is a utility holding cost of investment. Our key assumption is that this holding cost is interdependent across assets: buying more of any type of debt or of the asset S increases marginal holding costs for all assets. This interdependency gives rise to interconnected demand curves and a role for issuer competition. Following CDMS, we allow the holding cost from country j's debt to be inversely proportional to an exogenous cost/taste function $\omega_i(M_{jt})$ that is continuous and weakly increasing in M_{jt}. Although for most of the paper we think of ω_i as being on average constant and equal to 1, heterogeneity in ω_i across investors allows us to capture heterogeneity in investor specialization of preferences across countries of different risk profits M_{jt}. This heterogeneity in investor portfolios enables us to derive our empirical measure of reputation in Section 4.

We now characterize the demand schedules for the outside asset S and government debt from investors' first order conditions. The investor i first order condition for the outside asset S^i_t yields

[2] We assume that the government does not invest in asset S or the debt of other countries. The former can be ensured by assuming $Q > R^S_t$. The latter is ensured because, as we show below, the return Q on productive assets is higher than the return R_{jt} on other governments' debt.

a demand schedule

$$R_t^S - \bar{R} = \frac{1}{4} b \left(S_t^i + \int_j \omega_i (M_{jt})^{-1} D_{jt}^{i2} dj \right). \tag{3}$$

Equation 3 equates the marginal benefit of purchasing asset S, the excess return $R_t^S - \overline{R}$, with the marginal increase in overall portfolio holding cost, the right hand side. This average portfolio holding cost of investor i across assets aggregates each part of the holding cost to the investor across her various assets. The first order condition implies a common factor that equalizes average portfolio holding costs across all investors to the endogenous return $R_t^S - \overline{R}$ and introduces much tractability. We define this average holding cost, which we denote b_t^*, to be

$$b_t^* = 4(R_t^S - \bar{R}). \tag{4}$$

Finally, the first order condition with respect to debt D_{jt}^i yields

$$R_{jt}^i = \frac{\bar{R} + \frac{1}{2} b_t^* \omega_i (M_{jt})^{-1} D_{jt}^i}{1 - (1 - M_{jt})\overline{\tau}}. \tag{5}$$

In our model, an increase in debt issuance by one country will have to be bought by investors, which pushes up their average holding costs. This increases b_t^*, or equivalently pushes up the return on the outside asset. This in turn worsens (steepens) the residual demand curve faced by any specific country for its debt. The effect occurs through a common component, b_t^*, to which countries of varying reputation M_{jt} are heterogeneously exposed via their reputation (both through the direct loss in expected payoff due to the tax and also through the taste $\omega_i(M_{jt})$). Countries at levels of reputation that investors on average find less attractive, a low $\omega_i(M_{jt})$, are more exposed to increases in b_t^*.

Equilibrium of the Debt and Asset Markets at the Beginning of t. The markets for debt and the outside asset must clear. Market clearing for the outside asset S is given by

$$\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} S_t^i = \overline{S}. \tag{6}$$

Each government j is small globally but is a monopolist in the market for its own debt. That is, government j takes as given the issuance choices and interest rates of other governments as well as the return R_t^S on asset S, but internalizes the effect of its issuance choice on the price of its own debt. The equilibrium of the government debt market is determined by these optimal issuance choices, and an equilibrium of the entire market is a return R_t^S such that equation 6 is satisfied (given government's optimal issuance choices). The equilibrium can therefore be analyzed by first determining debt issuance of governments for given returns R_t^S. In particular, analysis follows by first determining the optimal issuance of the committed type of government of each country. To avoid immediately revealing themselves, opportunistic governments then mimic committed governments of their own country (see below). As a result, the equilibrium of the debt market is identical whether the government of a country is committed or opportunistic.

Optimal Debt Policy of the Committed Type. We follow much of the reputation literature by assuming that the committed type does not internalize the impact of its issuance choices on the strategy of opportunistic governments. In particular, the committed government myopically maximizes its static payoff, taking its reputation M_t as given. The committed government therefore chooses debt issuance $\{D^i_{jt}\}$ to maximize its stage game payoff (equation 1 with $\tau_t = 0$), subject to investors' interest rate schedules (equation 5) and taking b^*_t as given. The following Lemma characterizes the committed government's optimal debt policy.

Lemma 1 *The optimal debt policy of a committed government is*

$$D^i_{jt} = D^i(M_{jt}, b^*_t) = \frac{1}{b^*_t \omega_i(M_{jt})^{-1}} \left[Q\left(1 - (1 - M_{jt})\overline{\tau}\right) - \bar{R} \right] \tag{7}$$

$$R^i_{jt} = R(M_{jt}) = \frac{1}{2} \frac{\overline{R}}{1 - (1 - M_{jt})\overline{\tau}} + \frac{1}{2} Q \tag{8}$$

All proofs are in Appendix A.I. Lemma 1 derives the policy of the committed government as functions of its reputation M_{jt} and the average holding cost b^*_t. In particular, Lemma 1 shows that the committed government offers the same interest rate, $R(M_{jt})$, to all investors, which is based on its reputation but not on the average holding cost b^*_t. Investors' debt purchases are then pinned down by their demand schedule, which is affected by both by the country's reputation and the average holding cost b^*_t. In particular, debt issuance rises in reputation M_{jt} and falls in the holding cost b^*_t. Investors with lower holding costs at the government's current reputation level also purchase more debt, an observation we leverage in Section 4 when we derive a measure of reputation based on investor portfolio holdings.

Finally, we can write the stage game payoff to the committed government as

$$V(M_{jt}, b^*_t) = A + \frac{1}{b^*_t \omega(M_{jt})^{-1}} \left(\frac{1}{2} Q - \frac{1}{2} \frac{\overline{R}}{1 - (1 - M_{jt})\overline{\tau}} \right) \left(Q\left(1 - (1 - M_{jt})\overline{\tau}\right) - \bar{R} \right) \tag{9}$$

where we have defined $\omega(M_{jt}) = \frac{1}{\mathcal{I}} \sum_{i \in \mathcal{I}} \omega_i(M_{jt})$ to be the average taste across investors. The committed government's payoff is therefore analogous to the case in which there is a representative investor with a taste $\omega(M_{jt})$ and an average holding cost b^*_t.

Opportunistic Government Payoff and Strategies. A country j opportunistic government chooses the debt policy of the committed government j so as to avoid immediately revealing its type.[3] The opportunistic government also chooses the tax $\tau_t \in \{0, \overline{\tau}\}$. Given preferences, the op-

[3]This is sustained by the assumption that investors' off-path beliefs are $\pi_{jt} = M_{jt} = 0$ if government j does not choose the issuance that would be chosen by the committed government.

portunistic government's stage game payoff as a function of its reputation M_{jt}, the average holding cost b_t^*, and its tax policy τ_t is

$$V^{Opp}(M_{jt}, b_t^*, \tau_t) = g(\tau_t)V(M_{jt}, b_t^*). \tag{10}$$

An opportunistic government takes the path of prices $\{R_t^S\}$ as given, or equivalently takes the path of slopes $\{b_t^*\}$ as given. For a given a path $\{b_t^*\}$ of slopes, we study strategies of opportunistic governments that are Markov in the common beginning-of-period belief π_{jt} of foreign investors that the country j government is the committed type. Investors have beliefs $m_{jt}(\pi_{jt})$ that an opportunistic government will impose the tax, meaning that $M_{jt} = M_{jt}(\pi_t) = \pi_{jt} + (1-\pi_{jt})m_{jt}(\pi_{jt})$. For notational simplicity, we suppress the dependency on π_{jt} whenever it is otherwise clear, and we leave the dependency on the path of b_t^* implicit. A strategy of an opportunistic government is the probability $m_{jt}^o(\pi_{jt}) \in [0,1]$ of not imposing the tax.

Asset Demand, Aggregation, and Competition. The market clearing conditions allow us to re-represent b_t^* (equation 4) in terms of the amount of debt issued across countries. Summing equation 3 over i and combining with equation 4, we obtain

$$b_t^* = b\bigg(\overline{S} + \int_j \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \omega_i(M_{jt})^{-1} D_{jt}^{i2} dj\bigg).$$

Finally, we can make use of the as-if representative investor aggregation of our model. In particular, summing equation 7 over investors we obtain $\omega(M_{jt})^{-1}D_{jt} = \omega_i(M_{jt})^{-1}D_{jt}^i$. Thus substituting in,

$$b_t^* = b\bigg(\overline{S} + \int_j \omega(M_{jt})^{-1} D_{jt}^2 dj\bigg). \tag{11}$$

Our model with competition is, from the perspective of an individual country j, akin to a model in which the country faces a representative investor with a demand curve slope b_t^* instead of b and a holding cost ω. Importantly, the slope b_t^* is endogenous to the equilibrium debt issuance and reputation of all other countries. All else equal, when countries issue more debt (D_{jt} increases) or when the supply of the outside asset is higher ($\overline{S}$), investor holding costs are higher and the demand curve slope increases. This raises the cost to any individual country of issuing debt and reduces their issuance (Lemma 1). Moreover, the reputational composition of countries also affects competition: a higher reputation countries' undertakes a greater issuance of debt than a low-reputation counterpart, and raises average holding costs by more. This means that a world economy in which a larger share of countries have high reputation levels generates a more competitive market for debt, reflected in a higher b_t^*. Formally, this is reflected in that $\omega(M_{jt})^{-1}D_{jt}(M_{jt}, b_t^*)^2$ is increasing in M_{jt} (even if ω is constant). On net, any individual country therefore faces lower borrowing costs not only if other countries issuing less debt, but also if other countries are at lower reputation levels.

For expositional purposes, we will refer to b_t^* as the level of competition that countries face, with higher values of b_t^* reflecting greater competition in the sense that the slope of residual demand curve faced by country j steepens. We will also heuristically refer to the case of $b_t^* = b$ as a "model without competition," as it corresponds to the case in which the slope of the investor demand schedule is fixed at the primitive slope parameter b irrespective of countries' debt issuance and reputation.

2.2 Repeated Game and Reputation Building

We now characterize the dynamic reputation building game. At the end of date t, after the stage game payoff is realized, the country j government may be dissolved (Phelan, 2006; Amador and Phelan, 2021; Clayton et al., 2024a). Dissolutions are not observable to investors, but investors know the probability of dissolution. Government dissolution is i.i.d. across countries, with probability ϵ^C (ϵ^O) that a committed (opportunistic) government is dissolved, and with $\epsilon^C + \epsilon^O < 1$. All governments have a discount factor $\beta^* < 1$, but place no value on future governments after a dissolution. Define $\beta = \beta^*(1 - \epsilon^O)$. Investor beliefs update according to Bayes' rule. If country j does not impose the tax, then end-of-t posterior beliefs (i.e., beginning-of-$t+1$ prior beliefs) are

$$\pi_{j,t+1} = \epsilon^O + (1 - \epsilon^C - \epsilon^O)\frac{\pi_{jt}}{M_{jt}(\pi_{jt})}. \tag{12}$$

If country j imposes the tax, then $\pi_{j,t+1} = \epsilon^O$, which accounts for the possibility the government was dissolved and switched types.

The decision problem of an opportunistic government can be defined according to the Bellman equation,

$$W_{jt}(\pi_{jt}) = \max_{m_{jt}^o \in [0,1]} m_{jt}^o \Big(V^{Opp}(M_{jt}(\pi_{jt}), b_t^*, 0) + \beta W_{j,t+1}(\pi_{j,t+1}) \Big) + (1 - m_{jt}^o) \Big(V^{Opp}(M_{jt}(\pi_{jt}), b_t^*, \bar{\tau}) + \beta W_{j,t+1}(\epsilon^0) \Big), \tag{13}$$

where the value function W_{jt} depends implicitly on the current and future values of b_t^*. The government of country j is willing to play a mixed strategy at date t if and only if it receives the same payoff whether or not it imposes the tax, that is,

$$V^{Opp}(M_{jt}(\pi_{jt}), b_t^*, 0) + \beta W_{j,t+1}(\pi_{j,t+1}) = V^{Opp}(M_{jt}(\pi_{jt}), b_t^*, \bar{\tau}) + \beta W_{j,t+1}(\epsilon^0).$$

The government is willing to play a pure strategy of imposing (not imposing) the tax if it has a weak preference for imposing it (not imposing it). Substituting in the payoff function of equation 10, we obtain that under a mixed strategy the government's continuation value after not imposing the tax must satisfy

$$W_{j,t+1}(\pi_{j,t+1}) = \frac{1}{\beta}\Big(g(\bar{\tau}) - 1\Big) V(M_{jt}(\pi_{jt}), b_t^*) + W_{j,t+1}(\epsilon^0). \tag{14}$$

Intuitively, for an opportunistic government to be willing to forego the flow payoff benefit of imposing the tax, it must be awarded a higher continuation value from the buildup in investor beliefs that it is the committed type.

Equilibrium Definition. We restrict our analysis to Markov strategies of committed and opportunistic government that are symmetric in investor beliefs π_{jt}: governments j and k of the same type play the same strategy if $\pi_{jt} = \pi_{kt}$. We now define an equilibrium of the model.

Definition 1 *An equilibrium of the model is a path of debt issuances of committed governments $\{D^i_{jt}\}$, a path of debt and outside asset purchases $\{S^i_t, D^i_{jt}\}$ of investors such that markets for government debt and the outside asset clear at interest rates $\{R_{jt}, R^S_t\}$, a path of strategies $m^o_t(\pi_{jt})$ of opportunistic governments, a path of investor beliefs about government types $\{\pi_{jt}\}$, opportunistic government strategies $\{m_t(\pi_{jt})\}$, and government reputation $\{M_t(\pi_{jt})\}$, and a path of slopes $\{b^*_t\}$ such that: (1) Debt issuances are optimal for committed governments; (2) Debt and asset purchases are optimal for investors; (3) $m^o_t(\pi_{jt})$ is an optimal strategy of opportunistic government j at date t; (4) π_{jt} is consistent with Bayes' rule in equation (12); (5) Investor beliefs are consistent with the opportunistic government optimal strategy; (6) $m_t(\pi_{jt}) = m^o_t(\pi_{jt})$; (7) Slope b^*_t is consistent with equation (4).*

3 Competition for Safe Asset Provision

Our main results characterize the effects of competition on countries' incentives to build reputation for being a safe asset provider. We show how increases in competition lower the gains from reputation building, leading fewer opportunistic governments to proceed through the reputation cycle and resulting in a pileup of countries at low reputation levels.

We focus on characterizing a steady state of the model in which the path of slopes (i.e., the asset return R^S_t) is constant over time: $b^*_t = b^*$. We construct an equilibrium proceeding as follows.

Constructing Government Strategies. First, we construct a strategy of an individual country government j that faces a constant path b^* of slopes over time. We consider an equilibrium strategy that takes the form of a graduation step Markov equilibrium (Phelan, 2006; Clayton et al., 2024a). This strategy is characterized by a reputation cycle, $n = 0, \dots, N$ for $N \geq 0$, where at any date t the government is at some step in this cycle. We thus move from calendar time t to cycle steps n from here on when describing opportunistic government strategies. At step $n < N$ of the cycle, opportunistic governments play a mixed strategy $m(\pi_n) \in (0, 1)$. As a government progresses upward in the cycle by continuing to not impose the tax, investors become more confident it is the committed type (π_n rises) and its reputation builds (M_n rises). On the other hand, opportunistic governments that impose the tax fall back to the first step $n = 0$ of the cycle, losing reputation. At the "graduation step" N, any remaining opportunistic governments play a pure strategy $m(\pi_N) = 0$

of imposing the tax for sure. Committed types that continue to steps $n > N$ either continue or switch types and play the pure strategy of imposing the tax. Therefore, we have $\pi_0 = \epsilon^O$, $\pi_N = M_N$, and $\pi_n = M_n = 1 - \epsilon^C$ for $n > N$. Under this conjectured equilibrium, for $n < N$ equation 14 reduces to

$$V(M_{n+1}, b^*) = \rho V(M_n, b^*) + V(M_0, b^*), \tag{15}$$

where $\rho = \frac{1}{\beta}\frac{g(\overline{\tau})-1}{g(\overline{\tau})}$ reflects the speed at which reputation must build in order for opportunistic governments to be willing to play the mixed strategy. The condition for graduation to occur at step N is

$$V(1 - \epsilon^C, b^*) \leq \rho V(M_N, b^*) + V(M_0, b^*), \tag{16}$$

which states that the required growth in reputation to sustain mixing exceeds the highest possible probability of the government being the committed type $V(1 - \epsilon^C)$. To show that, taking as given a constant path of slopes b^*, there is an equilibrium in which opportunistic governments follow this strategy, we can apply Proposition 2 of CDMS, which is proven in that paper in an environment with a single government (i.e., without competition) and is restated in the lemma below in the environment of this paper.

Lemma 2 (CDMS Proposition 2) *Let $b_t^* = b^*$ be constant over time. For each country j, there exists a unique graduation step Markov equilibrium.*

Lemma 2 completes the construction of strategies of individual governments faced with a constant path of slopes $b_t^* = b^*$. This construction satisfies parts (1)-(6) of Definition 1.

Completing the Equilibrium. To complete the conjectured equilibrium of the model, we must relate the equilibrium value of b^* to the strategies of governments and investors under our conjectured equilibrium (i.e., we must verify part (7) of Definition 1). We construct an equilibrium slope b^* as follows. Imagine a unit mass of countries each separately in a graduation step Markov equilibrium, in which the demand curve slope is b^*. Denote $\mathbf{M}(b^*) = \{M_0(b^*), ..., M_N(b^*), 1 - \epsilon^C\}$ to be the reputation cycle associated with the unique graduation step Markov equilibrium without competition when the slope is b^*. What remains to verify is that given conjectured equilibrium issuance D_{jt} and reputation cycle $\mathbf{M}(b^*)$ the right hand side of equation (11) indeed equals the conjectured value of b^*.

Given the reputation cycle $\mathbf{M}(b^*)$, the steady state (stationary) distribution μ_{b^*} over reputation levels $[0, 1]$ is atomic with atoms at each point in $\mathbf{M}(b^*)$ and with no mass at any subset of $[0, 1]$ that is disjoint with $\mathbf{M}(b^*)$. Appendix A.I.B shows that the mass points of this stationary distribution are given by $\mu_{b^*,n} = \frac{\delta_n^*}{\sum_{x=0}^{N+1} \delta_x^*}$ for $0 \leq n \leq N+1$, where $\delta_0^* = 1$, $\delta_n^* = \prod_{k=0}^{n-1} M_k$ for $0 < n < N+1$, and $\delta_{N+1}^* = \frac{1}{\epsilon^C}\prod_{k=0}^{N} M_k^*$. δ_n^* is the cumulative unconditional probability that a government that starts at step 0 goes through its next n crises without imposing the tax (with an adjustment at $N+1$ for the absorbing state for committed types). Given this stationary distribution, we can write equation 11 as

$$b^* = b\left(\overline{S} + \int_M \omega(M)^{-1} D(M, b^*)^2 d\mu_{b^*}\right), \tag{17}$$

where we employed a Lebesgue integral over μ_{b^*}, and where $D(M, b^*) = \frac{1}{b^*\omega(M)^{-1}}[\gamma Q(1-(1-M)\overline{\tau})-\bar{R}]$. An equilibrium of the model (of our conjectured form) exists if there is a b^* such that the above condition holds. The proposition below formalizes existence of our conjectured equilibrium, which we refer to as a steady state symmetric graduation step Markov equilibrium.

Proposition 1 *There exists a steady state symmetric graduation step Markov equilibrium of the competition model.*

3.1 Competition and Incentives to Build Reputation

Competition affects the model both by affecting the optimal debt policy for countries at a given reputation level, and by affecting the dynamics of how a country builds reputation. Intuitively, competition lowers the value of building reputation because the residual demand curve for debt is not as attractive (steeper) for the issuer. Most potential candidate countries stay at low levels of reputation, that is they do not become reserve currencies, and even those that emerge as reserve currencies find being one less valuable than in the absence of competition. To unpack these effects it is useful to consider some special cases before turning to the full effect of competition on the stationary distribution.

We consider first the special case of no domestic endowment of productive assets, so that all projects are fully debt financed.

Proposition 2 *Assume that the domestic endowment is zero, $A = 0$. Then, there exists a unique steady state symmetric graduation step Markov equilibrium of the model with competition. The reputation vector $\boldsymbol{M}$ and distribution μ do not depend on b^*. Competition lowers the optimal debt issuance but does not affect the evolution of reputation.*

In this limiting case, competition lowers equilibrium debt issuance but has no direct impact on the reputational dynamics. The reason is that absent a domestic endowment, the entire value of the government comes from debt issuance. Because b^* has the same proportional impact on the demand curves of investors at all reputation levels, it drops out of the dynamics of reputation building when $A = 0$, leading to the limiting result.

In the general case with $A \geq 0$, the transition dynamics are

$$V(M_n, b) = \rho Q A \frac{b^* - b}{b} + \rho V(M_{n-1}, b) + V(M_0, b), \tag{18}$$

where $V(M_n, b)$ is the indirect utility function of the committed government in the model "without competition" in which the slope is b.[4] In the limiting case of $A = 0$, these transition dynamics

[4]See the proof of Proposition 2 in the Appendix for the derivation of equation 18.

collapse to those of the model without competition, as highlighted by Proposition 2. When $A > 0$, the above equation shows that reputation builds more quickly when competition is higher, that is b^* increases relative to b. Intuitively, the government's payoff can be thought of as a combination of value from the domestic endowment and value from external debt. As competition becomes more fierce, the value of external debt declines relative to the value of the domestic endowment, making it less costly for a government to forego its current reputation level (all else equal). This means that a larger reputational gain is required to induce the opportunistic government to be willing to forego imposing the tax today, leading to a faster buildup of reputation.

The above observation gives rise to a second interesting limiting case: committed governments can provide sufficiently fierce competition to force immediate graduation by opportunistic governments.

Proposition 3 *There exists a threshold $\overline{b}^*$ such that if and only if $b^* > \overline{b}^*$, there is a crowd out equilibrium of the competition model in which $\boldsymbol{M} = \{\epsilon^O, 1 - \epsilon^C\}$ and all opportunistic governments immediately graduate.*

Intuitively, competition in this case is sufficiently fierce that opportunistic governments cannot build sufficient value from reputation. As a result, they immediately impose the tax and graduate, and so remain stuck at the lowest reputation level forever. Proposition 3 expresses the result in terms of a threshold on the sufficient statistic b^*.

Numerical Illustration. We now turn to a numerical illustration of the general case. For simplicity, we assume $\omega(M)$ is constant in M. Figure 1 plots the equilibrium cycle and distribution of reputation for a country in the model under two configurations. The first configuration (red) illustrates a one-country model with $b^* = b$, that is a model without competition. The second configuration (blue) illustrates the equilibrium of the model with competition (i.e., b^* is endogenous). All parameters are otherwise identical between the two configurations.

Panel (a) plots the reputation cycle under the two configurations. It shows that when faced with greater competition, countries start the reputation cycle at $n = 0$ at a lower reputation level, but then reputation builds more quickly and countries graduate sooner (N is lower). The faster growth is consistent with the lower mimicking probability at $n = 0$ under competition (Panel (b)). Intuitively, when faced with greater competition, countries get less value from building reputation and so require larger buildups in reputation to do so. This leads a larger fraction of opportunistic types to deviate early in the reputation cycle, leading to low initial reputation but a faster buildup. Panel (c) plots debt issuance, and confirms that debt issuance per country falls at each point in the reputation cycle due to competition. The presence of competition b^* leads to a decline in debt issuance even at steps in the reputation cycle in which a country's reputation is higher, which reflects the decline in how much countries value building up reputation when faced with greater competition. Panel (d) plots the stationary frequency that a country, drawn at random ex-ante, spends at each level of reputation. Given the law of large numbers, this frequency also coincides with the stationary cross-sectional distribution μ. A country spends most of the time at low levels of reputation, highlighting how difficult it is to emerge as a reserve currency in the model. In the presence of competition, a greater fraction of opportunistic types reveal themselves at $n = 0$ leading to a higher stationary mass point there.[5]

The proof of Proposition 1 shows that several of the key qualitative features of Figure 1 are generic properties of the model with competition. Generically, higher competition leads countries to start at a lower reputation level at $n = 0$, eventually build to a higher reputation level, and graduate faster. Higher competition always leads opportunistic governments to mimic less early in the reputation cycle (an implication of Proposition 6 below). Equilibrium debt issuance is lower for any given reputation level.

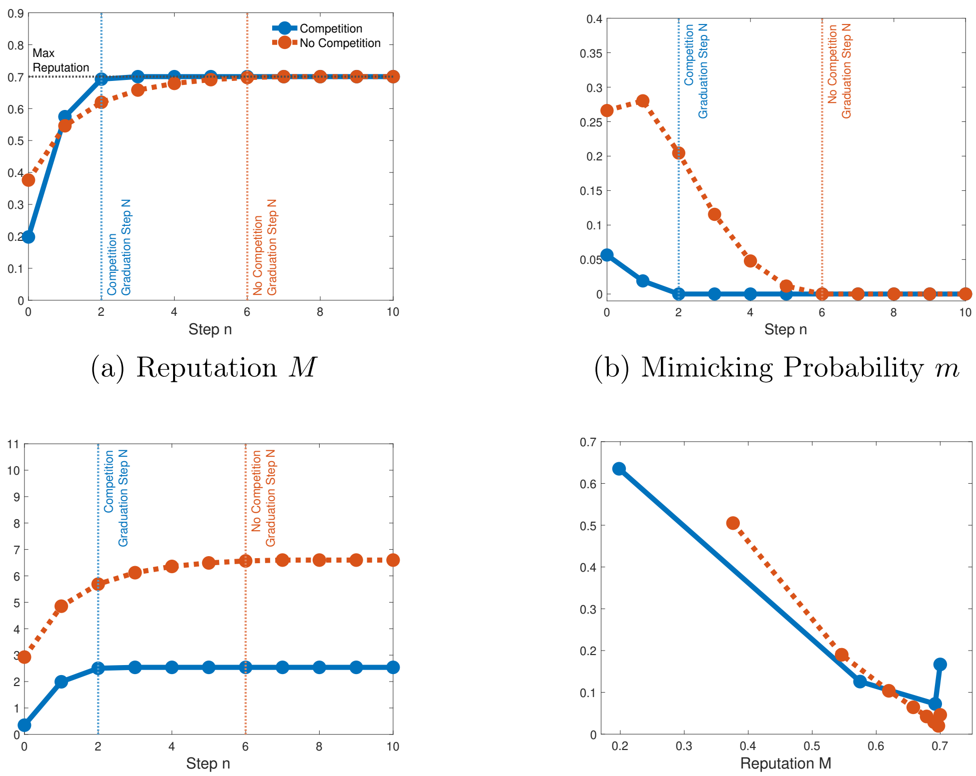
Overall, our results highlight how competition can deter countries that are currently at lower levels of reputation, like China, from building reputation up into being a reserve currency. The prospect of greater competition reduces the attractiveness of borrowing at any given reputation level. Moreover, increases in reputation are less valuable for a country when faced with higher competition. This intuition can be seen most clearly from the fact that the marginal reduction in borrow cost from an increase in reputation (holding issuance fixed) is *decreasing* in competition, that is $\frac{\partial^2 R^i_{jt}}{\partial b^*_t \partial M_{jt}} < 0$. This means that an increase in competition lowers the returns to building reputation.

Discussion of modeling choices. Our tax τ, which imposes a loss on investors, represents an alteration to borrowing terms ex post. There are a number of ways in practice that governments alter borrowing terms after the fact. The tax can represent an ex post capital control on outflows that investors have to pay when taking their money out of the country. It can represent a partial default, reflecting the cost to investors of a haircut, maturity lengthening, or other consequences of that partial default and any associated renegotiation. It can represent a sanction, such as a freezing of reserves, that prevents investors from accessing their funds. It could also reflect inflation or exchange rate depreciation.

We assume the committed type has market power over its own debt in order to capture monopoly rents a country can potentially achieve, particularly those that become safe asset providers (Farhi and Maggiori, 2018; Choi et al., 2022). Our model captures that monopoly rents are larger for countries of higher reputation, reflected in that the slope of investors' required yield becomes less sensitive to quantity issued as reputation builds (i.e., $\frac{\partial R^i_{jt}}{\partial D^i_{jt}}$ falls in M_{jt}).

[5]Both distributions feature an increase in mass at the highest reputation that is achieved after graduation. This level of reputation is identical in the two configurations and given by $1 - \epsilon^C$. The graduation step is an absorbing state for committed types, so that a mass of probability builds up in the model at that level of reputation. Although visually this mass is larger under competition, as in Panel (a) competition means there is a smaller set of high reputation levels a country can reach prior to graduation. This means the overall mass of countries at high reputation levels is actually similar or smaller in the model without competition.

Figure 1: Competition and the Stationary Distribution



(a) Reputation M

(b) Mimicking Probability m

(c) Debt Issuance D

(d) Stationary Distribution μ

Notes: Numerical illustration of the model with or without competition. Panel (a) plots the reputation cycle M. Panel (b) plots the mimicking probability m. Panel (c) plots debt issuance. In panels (a), (b), and (c), the dashed-blue and dashed-red lines are the graduation steps of the model with competition and no competition, respectively. Panel (d) plots the stationary distribution μ of the two models.

For simplicity, in our model governments are fully revealed to be opportunistic (up to dissolution probability) when they impose the tax. In practice, even "committed" governments might at times undertake such measures. We could capture partial loss of reputation by assuming even the committed government probabilistically imposes the tax. Similar to how in the current model reputation builds faster in the presence of greater competition, it is also possible that on the flip side reputation would be lost more quickly precisely because a competitive environment makes the costs of reputational losses less severe. This could give rise to smooth but rapid transitions between different countries being dominant safe asset providers. Competition could also endogenize how much investors diversify away from a high-reputation country that suffers a partial loss of reputation. In particular, the magnitude of the response would likely depend on the presence or absence of an alternative safe asset provider, with less of a diversification in absence of a viable alternative.

It is also interesting to reflect on how the model speaks to earlier episodes of countries building reputation toward becoming a global reserve currency. In this respect, we think of Alexander Hamilton's policy, when he was the first U.S. Secretary of the Treasury, of having the newly created federal government assume the debt of the states. The policy aimed at building a solid reputation as a borrower for the newly created United States (Sargent (2012)).[6] Similarly, we think of the later efforts by New York Federal Reserve Governor Benjamin Strong to build an investor base for the trade-bills (bankers acceptances) market in dollar in New York to rival the liquid and safe markets for these bills in sterling in London. Such efforts were instrumental into making the dollar a reserve currency (Eichengreen (2011); Broz (2018)). The need to maintain reputation was also a motivation behind England's misguided return to the gold standard at the pre-war exchange rate level in the 1920s.[7]

Countries have, at various times, suffered losses of reputation as providers of reserve currencies. England suffered a blow to its reputation with the sudden devaluation of the pound in 1931 and never recovered its role as a reserve currency provider. The U.S. went off gold in 1933 and then again in 1972. In particular, the Nixon administration in 1971 reneged on a promise of free convertibility of the dollar into gold, restricting this ability only to official ("stable") investors and excluding the private ("flighty") investors. Immediately after 1973 there was an attempt by foreign investors to diversify away from the dollar, but, perhaps due to the lack of viable alternatives, the dollar quickly regained and maintained its status.

[6] Hamilton (1790) extols the virtues of governments that maintain their promises to creditors: "States, like individuals, who observe their engagements, are respected and trusted: while the reverse is the fate of those, who pursue an opposite conduct. [...] The credit of the United States will quickly be established on the firm foundation of an effectual provision for the existing debt." Chernow (2004)[pg 298] remarks: "With this huge gamble, Hamilton laid the foundations for America's future financial preeminence".

[7] The Cunliffe Committee, charged in 1918 with studying the possible international monetary arrangements after WWI, stated in in its interim report: "The uncertainty of the monetary situation will handicap our industry, our position as an international financial centre will suffer and our general commercial status in the eyes of the world will be lowered." A strong dissenting voice was John Maynard Keynes (Keynes (1923)) who argued that these concerns were overblown compared to the economic cost of return to gold at a deflationary peg.

4 Measuring International Currency Reputation

Measuring reputation empirically is a notoriously difficult task. In this section we derive a model-implied sufficient statistic for reputation. We then empirically implement this new measure of reputation with detailed micro data on foreign investors' bond holdings. We begin by deriving the measure theoretically and then estimate it in the data.

4.1 Investor Specialization and a Theoretical Measure of Reputation

We show that our model can be used to derive a measure of country reputation based on observable portfolio holdings of investors. To derive our measure, we characterize what types of investors hold a country's debt at a given point in its reputation cycle. Consider a country j with reputation M_{jt}. As derived in Section 2, we have $D_i(M_{jt}) = \omega_i(M_{jt})\omega(M_{jt})^{-1}D(M_{jt})$. This means that the (infinitesimal) portfolio share of investor i in the debt of country j is given by

$$\alpha_i(M_{jt}) = \frac{\omega_i(M_{jt})\omega(M_{jt})^{-1}D(M_{jt})}{w_i}.$$

We consider a reference set of countries that have the highest reputation level $\bar{M}$. We show below that the cross-sectional correlation at date t of portfolio shares across investors between the portfolio share in the debt of country j and the debt of this reference set $\bar{M}$ reveals the correlation between investor taste $\omega_i(M_j)$ and $\omega_i(\bar{M})$. As long as investors are heterogeneous in this taste, i.e. they specialize in debt of varying reputation, the rank of these correlations reveals the issuers' reputation rank.[8] The proposition below formalizes this measure.[9]

Proposition 4 *The correlation of investors' portfolio shares in the debt issued by country j of reputation M_{jt} with a reference set of debt issue by countries with reputation $\bar{M}$ measured at a point in time across investors is*

$$corr_i(\alpha_i(M_{jt}), \alpha_i(\bar{M})) = corr_i\left(\omega_i(M_{jt}), \omega_i(\bar{M})\right).$$

Let $\omega_i(M_{jt}) \approx \phi_0^i + \phi_1^i(M_{jt} - M^r)$ be a first order Taylor approximation around point M^r and define $\sigma_0^2 = Var_i(\phi_0^i)$, $\sigma_1^2 = Var_i(\phi_1^i)$, and $\rho_{0,1} = corr_i(\phi_0^i, \phi_1^i)$. Provided a sufficiently small

[8] We assume that portfolio shares are invariant to the size of the fund w_i. In the model this can be done by assuming w_i to be constant across funds or by defining the taste functions ω_i up to a multiplicative constant w_i so that their ratio is independent of i. We also assume w_i is sufficiently high that there is positive investment in the outside asset.

[9] Proposition 4 does not rely on the economy having reached a steady state, that is it also holds along a transition path in which b_t^* is not constant.

approximation error, if $\sigma_0^2 > 0$*,* $\sigma_1^2 > 0$*, and* $|\rho_{0,1}| < 1$*, then we have for any two countries* j, k*:*

$$M_{jt} > M_{kt} \quad \iff \quad corr_i(\alpha_i(M_{jt}), \alpha_i(\bar{M})) > corr_i(\alpha_i(M_{kt}), \alpha_i(\bar{M}))$$

Intuitively, Proposition 4 says that if investors specialize in the debt of issuers with different reputation and there is a set of issuers for which the reputation level is known to the the highest, then all other issuers' reputation can be ranked by checking how similar are the portfolio shares in those issuers compared to the reference set. Heuristically, if investors that are observed to concentrate their holdings in high-reputation countries also tend to hold country j's debt, we can infer that country j is also a high reputation country. We show below that this measure can be taken directly to the data. We note that the measure does not require knowing the parameters of the function $\omega_i(M)$, is valid even if the aggregate $\omega(M)$ is constant, and does not require observing the universe of investors or rely on market clearing.

This measure is particularly useful in the context of a new asset, like Chinese bonds, for which time-series evidence on returns is of limited use or in situations when reforms or crises (like a default or imposition of controls) are likely to have changed the countries' reputation. By using portfolio quantities among many heterogeneous funds, it provides a cross-sectional estimate of what the investors believe about the asset (see also Koijen and Yogo (2019), Koijen et al. (2024), and Jiang et al. (2024)). Moreover, it can be assessed at different points in time to track how a country's reputation is evolving relative to others.

4.2 Empirical Implementation

The idea behind Proposition 4 is that heterogeneity in investor portfolios is driven by different relative preferences for investing in countries of various reputation levels. While we cannot observe this characteristic directly, if we know a set of countries to have a high reputation, then we can infer the relative ranking of other countries by seeing which other assets are held by the funds that also hold debt of these high reputation governments. In order to take this measure to the data, we need: (i) a sufficiently large and heterogeneous (in terms of reputational focus) set of portfolio investors for which we observe their complete portfolio; and, (ii) a choice of a reference set of high reputation countries. We take the reference set $\bar{M}$ in Proposition 4 to be a set of developed countries (DM) government bonds denominated in their local currency. We think of this reference set as having a high reputation $\bar{M}$.

4.2.1 Portfolio Holdings

We use micro-data on portfolio investment from foreign investors via mutual funds and ETFs from around the world. Investment funds are a useful set of investors for our purposes because: (i) they tend to specialize in specific markets, (ii) high quality data is available at the security level for many countries, and (iii) they are substantial private holders of foreign debt securities.[10] Our data include global mutual fund and exchange traded fund (ETF) holdings provided by Morningstar for each fund at the security level. We supplement it with information on the asset class, currency, market of issuance, nationality and residency of the issuer and its ultimate parent company, and other security characteristics.[11]

For each fund and currency, we calculate the share of the fund's total foreign currency bond investment in DM local currency bonds and the remaining share in a selected currency (with that currency omitted from the DM calculation if relevant).[12] In our baseline sample, we omit holdings of domestic currency bonds and any equities from the calculations because equities do not have a clear nominal currency component and domestic currency bonds play a special role for each country (see Maggiori et al. (2020)). We measure the correlation between the share of a foreign-currency bond portfolio invested in that currency with the share of the remaining foreign-currency bond portfolio invested in DM currencies across the universe of mutual funds and ETFs. More formally, for each fund i and currency c, we compute the share of the foreign-currency bond portfolio in that currency:

$$\alpha_{c,i} = \frac{\sum_{b \in B_c} \text{MV}_{b,i}}{\sum_{c \in FC_i} \sum_{b \in B_c} \text{MV}_{b,i}},$$

where $\text{MV}_{b,i}$ is the market value of holdings (measured in USD) that fund i has in bond b, B_c denotes the set of bonds denominated in currency c, and FC_i the super-set of bonds in foreign currency from the perspective of fund i. The denominator, therefore, is the value of holdings of foreign currency bonds by fund i. In addition, for each fund i and currency c we compute the share of the remaining foreign-currency bond portfolio in DM currencies as

$$\alpha_{DM,c,i} = \frac{\sum_{d \in \{DM_i/c\}} \alpha_{d,i}}{(1 - \alpha_{c,i})}.$$

We exclude currency c if it is a developed currency, so that $\{DM_i/c\}$ is the set of developed currencies excluding c. We re-scale shares by $(1 - \alpha_{c,i})^{-1}$ so that they reflect the composition of the remaining portfolio excluding currency c.[13] Finally, we compute the summary statistic of interest: the correlation across funds of the share invested in currency c and the remaining share invested in

[10]For many large developed countries mutual funds and ETFs are the largest foreign bond investors, usually followed by insurance companies, pension funds, and non-financial corporations. Our focus is on keeping the type of investor constant across many domiciles, so we use mutual funds and ETFs for which high quality data is available for many countries.

[11]See Maggiori et al. (2020), Coppola et al. (2021), Florez-Orrego et al. (2023) for details on the data and the many sources combined in assembling it.

[12]We define domestic currency to be the currency of the country in which the fund is domiciled.

[13]This re-scaling maps the estimates closer to the theory since there the composition of the residual portfolio is unaffected by the size of the share in bonds issued by country j given the assumption of a continuum of issuers each of measure zero.

(other) developed currencies

$$\rho_{c,DM} = corr_i\left(\alpha_{c,i}\ ,\alpha_{DM,c,i}\right), \tag{19}$$

where the notation $corr_i$ emphasizes that the correlation is cross-sectional over funds i at a point in time.

In bringing the model to the data, we make two further refinements. First, in our baseline analysis we restrict the focus to the government bonds[14] of the country issuing each particular currency. For example, for the dollar we restrict the attention to U.S. government bonds and exclude bonds denominated in dollar but issued by other sovereigns. The focus on local-currency sovereign bonds in our baseline empirical analysis follows the rationale of our model since, as discussed above, these assets are the most directly sensitive to the reputation of a government (as opposed to corporate bonds and equity, for example).

Second, we exclude from our analysis funds that specialize in any particular currency, which we define as funds that have more than 50% of their foreign-currency bond portfolio in a single currency. We do so because these funds are most likely to have too specific a mandate to reliably contribute to the correlation estimation. We also leave out funds with a small foreign currency portfolio, i.e. less than $20 million of foreign currency investment, since these small investments are more likely to be noisy and reflect residual positions. Based on our focus on foreign-currency bonds and sample cleaning, the resulting dataset includes approximately 600 investment funds, adding up to just over a trillion dollar of assets under management.

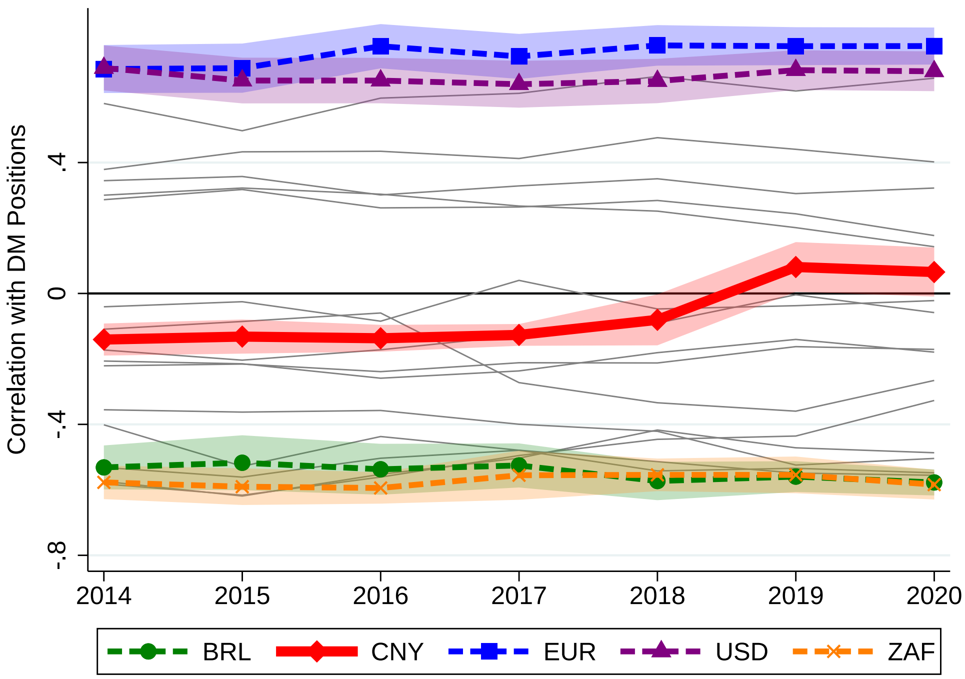
4.2.2 Heterogeneous Investment Portfolios and Country Reputation

Through the lens of our model, tracking the correlation measure over time allows us to infer the evolution of a country's reputation rank. Figure 2 plots this correlation (i.e., our reputation measure) annually from 2014 to 2020 for a set of countries. Consistent with their inclusion in the basket of high-reputation countries, our measure shows that the US and Eurozone rank as two of the highest reputation currencies across all years. In contrast, Brazil and South Africa both have low and stable reputation ranks. A country of particular interest is China, which has been gradually liberalization its bond market over the past decade. Figure 2 shows that China's portfolio correlation with developed markets has increased and so has its reputation rank in our model-implied measure. Consistent with our model, this reputational increase follows China's decision not to impose capital control restrictions on foreign investors following stock market turmoil and capital flight in 2015-2016 (see CDMS and Clayton et al. (2023a) for more on this episode).

Our measure of reputation can be used in future empirical and quantitative research in which a country's reputation plays a role. We are currently in the processing of updating our measure to extend it into more recent years. We intend to make our measure publicly available.

[14] In the case of China we classify Policy Banks' bonds as government debt, as these are assumed to be implicitly guaranteed by the central government.

Figure 2: A Rank Measure of Reputation: Sovereign Issuers in Local Currency



Notes: Figure reports, for a selected set of countries, the time series of the correlation between the foreign-bond portfolio shares invested in government bonds in each currency and in a reference set of Developed Markets (DM) local-currency government bonds. Each line, including the ones in gray, corresponds to a specific currency. We label select currencies for ease of comparison. 95% confidence intervals are computed via bootstrapping.

5 How Can the U.S. Deter China From Becoming a Reserve Currency?

In our baseline model, each government takes as given the reputation cycle and stationary distribution of both their own country and others, in the spirit of monopolistic competition models. It is interesting to extend this set-up to consider the incentives of a country to manipulate the reputation cycle and issuance strategies of its competitors, and the impact of such manipulation on its competitors' outcomes. For example, the US may want to change its issuance strategy in order to deter the build up of alternatives. We study this question in this section.

We model an incumbent large country that is known to be committed forever, so that its reputation is $M = 1$ and constant.[15] In the environment of our baseline model, this corresponds to there being zero probability that a committed government of the incumbent is dissolved ($\epsilon^O_{Incumbent} = 0$). It faces a measure one of entrants (potential new reserve currencies) going through the reputation game. For example, we might think of the U.S. as the incumbent and China as a possibly opportunistic new competitor. We consider the problem of the incument (a Stackelberg leader) choosing its debt issuance, accounting for the impact of its debt issuance on the entire equilibrium outcome (debt issuance, reputation cycle, and stationary distribution) of the entrant countries. It is analytically convenient to make this country ("the U.S.") the issuer of the outside safe asset S which we previously took as being supplied exogenously at $\bar{S}$. To simplify the problem, we abstract away from any transition dynamics and focus solely on the stationary point ("steady state"). As a result, the problem is static from the perspective of the incumbent, that is we assume entrants converge instantly to the stationary distribution.

The stage game payoff V^* of the incumbent takes the same basic form as that of entrants,

$$V^* = A^* + (Q^* - R^S)\overline{S} \tag{20}$$

where Q^* is the incumbent's return on productive assets. We assume that the incumbent does not purchase other countries' debt.[16] As derived in Sections 2 and 3, the incumbent faces a demand schedule for its asset defined by the two equations,

$$\overline{S}(b^*) = \frac{1}{b}\left(b^* - b\int \omega(M)^{-1}D(M,b^*)^2 d\mu_{b^*}\right) \tag{21}$$

$$R^S(b^*) = \overline{R} + \frac{1}{4}b^*. \tag{22}$$

[15] Taking the incumbent (e.g., the U.S.) as being known to be committed, while we think of entrants (e.g., China) as potentially opportunistic, is purely for convenience and sharpens the focus on the key forces we want to highlight.

[16] Assuming $Q^* \geq Q$ is sufficient to ensure that the hegemon would not invest in other countries' debt for reasons of monetary payoff. In principle, one could imagine that the hegemon might want to purchase debt to manipulate the reputation cycle, for example buying the debt of low-reputation countries to reduce the attractiveness of reputation building.

The first equation is a rearrangement of equation 11. Intuitively, it describes the asset supply $\overline{S}$ the incumbent must provide in order to achieve a given slope b^* of the demand curve faced by entrants. Formally, it describes the slack in market clearing for $\overline{S}$ that results from a given choice of b^* that must be filled by the incumbent, and hence determines $\overline{S}(b^*)$. The second equation rearranges equation 4, and gives the promised return $R^S(b^*)$ that is required to incentivize investors to buy $\overline{S}(b^*)$ along with their equilibrium debt purchases from entrants. That is to say, equations 21 and 22 describe the price and quantity of the incumbent's asset necessary to sustain a slope b^* in equilibrium.

Formally, the decision problem of the incumbent is to choose a promised return $R^S(b^*)$ in order to maximize its utility (equation 20), internalizing the resulting demand $\overline{S}(b^*)$ for incumbent debt and a slope b^* for entrants. Equations (21) and (22) therefore summarize the manner in which the incumbent implements a choice of b^*. The following proposition characterizes the optimal issuance of the incumbent.[17]

Proposition 5 *The optimal issuance of the incumbent satisfies*

$$\begin{aligned}\overline{S}(b^*) = \overbrace{\frac{4}{b}(Q^* - R^S(b^*))}^{\text{Standard Monopolist}} &\overbrace{- 4(Q^* - R^S(b^*)) \int \omega(M)^{-1} \frac{\partial D(M,b^*)^2}{\partial b^*} d\mu_{b^*}}^{\text{Standard Stackelberg}} \\ &\underbrace{- 4(Q^* - R^S(b^*)) \frac{\partial \int \omega(M)^{-1} D(M,b^*)^2 d\mu_{b'}}{\partial b'}\bigg|_{b'=b^*}}_{\text{Effect on Entrants' Reputations}}\end{aligned} \tag{23}$$

When considering whether to increase the supply of its own asset, the U.S. considers three distinct effects. The first line comprises two standard effects. The first is a standard monopolist effect: as the U.S. increases $\overline{S}$, it raises the amount that investors must purchase, pushing up their holding costs and so raising b^* (equation 21). This means that the required return $R^S(b^*)$ on the incumbent's asset rises, encouraging the incumbent to under-supply its asset to keep the required return low (i.e., to raise its asset's price). The second term is a standard Stackelberg effect of the incumbent's issuance on the issuance of entrants: as the incumbent's issuance pushes up b^*, the cost to entrants of issuing debt rises. This directly crowds out their debt issuance at any fixed reputation level (Lemma 1), which reduces the quantity of debt investors have to hold and so improves the incumbent's borrowing terms. This motivates the incumbent to increase its issuance $\overline{S}$ to directly crowd out its competitors. The multiplication of all terms on the right hand side by 4 reflects the amount that an increase in R^S raises b^* (equation 22).

The second line represents the effect of incumbent increasing its yield on the total issuance of its competitors, operating through changes in their reputation levels. As b^* raises, entrants'

[17] Equation 22 describes a bijection from $R^S(b^*)$ to b^*. Since the graduation step Markov equilibrium that describes government strategies is unique in b^*, this means that the incumbent is selecting a unique equilibrium of this form by choosing its promised yield. This means $\overline{S}(b^*)$ is also uniquely determined for a given choice of $R^S(b^*)$.

incentives to build reputation change (equation 18) and consequently the stationary distribution μ_{b^*} over reputation levels changes. Since debt issuance is increasing in the reputation level for entrants, the incumbent prefers to increase its issuance, all else equal, if raising b^* also pushes entrants to lower levels of reputation. The incumbent therefore seeks to crowd out entrants not only by directly competing with their debt issuance, but also by preventing them from building reputation for being credible safe asset providers. Our analysis in the next subsection expands upon this effect by studying how the incumbent can use increases in issuance to deter opportunistic entrants from building reputation.

5.1 How to Crowd Out Opportunistic Competitors

We next study how the incumbent can use its asset issuance to prevent opportunistic competitors from building reputation. As a first observation, an interesting corollary of Proposition 3 is that the incumbent (the U.S.) can choose sufficiently high issuance $\overline{S}$ such that all opportunistic competitors graduate immediately. Intuitively, the incumbent flooding the market with safe assets diminishes the value of building reputation for an opportunistic competitor (say China) sufficiently to completely discourage it from building any reputation. Formally, Proposition 3 provides a minimum value $\overline{b}^*$ that is required to fully crowd out opportunistic entrants, that is to ensure opportunistic competitors remain stuck at the first point in the reputation cycle. In particular, the incumbent can implement this outcome by offering a yield $R^S(\overline{b}^*)$ and corresponding issuance $\overline{S}(\overline{b}^*)$ (or any larger choice $R^S(b^*)$ with $b^* > \overline{b}^*$). This shows how the U.S. can completely deter opportunistic entrants from competing with the U.S. at higher reputation levels.

More generally, we show that the incumbent can always crowd out more opportunistic competitors by changing issuance to increase b^* (i.e., by raising its promised yield $R^S(b^*)$). Formally, we show that the probability that an opportunistic competitor, starting at the beginning of the reputation cycle (at step 0), goes through its first n crises without ever imposing the tax declines for any $n > 0$ as the incumbent raises b^* by issuing more safe debt. This means that the probability an opportunistic competitor builds to any reputation above the initial level declines. Moreover, as we show in the proof, the reputation level at the first step of the cycle decreases as b^* rises, enhancing the tendency for countries stuck at low steps to have even lower reputations. In this sense, increased issuance by the incumbent makes it harder for an emerging opportunistic competitor to establish itself as a competitor reserve currency. In practice, one important concern is that an incumbent like the U.S. issuing more debt to deter new entrants like China might risk a self-full-filling debt crisis in the U.S. itself (see Farhi and Maggiori (2018)). This risk is absent here since we imposed common knowledge that the US is a committed type.

Formally, we define $\delta_n = \prod_{k=0}^{n-1} m_k$ to be the probability that a government that is opportunistic at step 0 (and is not dissolved) does not impose the tax in any of the next n crises, and so reaches step n of its reputation cycle. We collect the result in the proposition below.

Proposition 6 *The probability that an opportunistic government (e.g. China) starting at step* 0 *reaches step* n *of its reputation cycle decreases in competition* b^* *for any* $n \geq 1$*, that is* $\frac{\partial \delta_n}{\partial b^*} < 0$.

The incumbent U.S. can use position to deter emergence of opportunistic competitors, like China, by offering higher yields to investors and reducing demand for alternative safe assets. Although doing so can reduce the monopoly rents the incumbent can extract, it nevertheless can be preferable to the alternative of another safe asset provider building up to meet that demand. Although this will generally require the U.S. to increase its debt issuance, it does not necessarily require the U.S. to increase its deficit. For example, the U.S. can use the proceeds of its debt issuance to set up and maintain domestic and foreign investments via a sovereign wealth fund. In doing so the U.S. could behave akin to a banker for the world, issuing safe debt liabilities to foreign investors while taking the proceeds and making productive investments.

In this set-up, the presence of an existing hegemon, like the U.S., makes it less likely that a multipolar international monetary system emerges. Much like in Stackelberg competition, the incumbent can use its dominant position to discourage entrance, in this case by oversupplying safe assets and shrinking the exorbitant privilege. To the extent that a multipolar system is desirable, this analysis opens up a role for multilateral policy agreements and points to the tools from the analysis of monopolies and competition as a way forward to analyze and reform the international monetary system.

5.2 Extensions

We discuss extensions of our model that are part of our ongoing work.

Debt Issuance and Stability Risks. Our model assumes that both the incumbent and entrant governments can issue debt without stability risks. One extension would be to assume each period can have a good state in which no crisis occurs and there is no temptation to impose the tax, and a bad state in which a crisis occurs and there is temptation to impose the tax. To capture stability risk from debt issuance, one could model the probability of the crisis occurring as a function of debt issuance. This would raise the cost of debt issuance for all governments, and as a result might reduce the value to a government of building reputation. However, it would similarly raise the cost to an incumbent that sought to increase its own issuance to deter entrants from building reputation. The effects on the equilibrium both with and without an incumbent are interesting to analyze.

Non-Bayesian Updating. Our model assumes investors correctly update beliefs about the government's type and reputation according to Bayes' rule. In practice, it is possible that investors might over-extrapolate the safety of the debt of a government that has not imposed the tax for an extended period of time. We can capture this in reduced form in the current model using the curvature of the tastes ω_i, in particular assuming that the average weight ω is increasing (particularly at high reputation levels). More generally, we could embed an over-inference of safety in our model for example by assuming that investors update more aggressively than Bayes' rule would imply about the probability that the government is the committed type. This would be a form of over-extrapolation. Such beliefs would likely increase the returns to building reputation, and lead opportunistic governments to be more willing to build reputation. This could make it harder for the incumbent to deter opportunistic competitors. At the same time, it could also increase the returns to the incumbent from deterring rivals from building reputation, since higher-reputation entrants would present greater competition to the incumbent. It is interesting to study which force dominates and whether the incumbent tries to fight entrants more or less aggressively as a result.

Geopolitical Goals and Rivals. Our model focuses on economic benefits (monopoly rents) for a government that builds reputation as a safe asset provider. Another potential goal of a hegemonic country, such as the large incumbent or another high reputation government, is the ability to leverage its position for geopolitical gains. A hegemon could threaten to cut off targeted foreign entitites from accessing its safe assets in order to get those entities to comply with the hegemon's demands (see also Clayton, Maggiori and Schreger (2023b, 2024b)). We could extend our model to incorporate geopolitical goals in reduced form by assuming that a government (either incumbent or entrant) can demand a geopolitical action from foreigners in exchange for being able to buy its debt/asset. Demanding such an action would raise the cost of purchasing the country's debt, meaning that governments would face a trade off between geopolitical goals and economic rents. We conjecture that higher-reputation countries with better borrowing terms would be more willing to demand geopolitical actions, giving a link between reputation, safe asset provision, and geopolitical influence. A hegemonic incumbent would find it more valuable to deter rival entrants whose geopolitical demands were opposed to the hegemon's preferences. On the other hand, the hegemon might actually promote the ascent of a geopolitical ally, particularly if that ally's ascent helped to crowd out geopolitical rivals by saturating the market for safe assets. It is also interesting to study how a hegemon would trade off changes in geopolitical demands and increases in promised yields (asset supply) when trying to crowd out potential entrants. For example, a hegemon might selectively target foreigners that it expected entrants to try and make the most inroads with.

6 Conclusion

We provide a model of how countries compete to build reputation as international safe asset providers. Emerging lower-reputation countries' incentives to build reputation are muted when they face competition from established safe asset providers, leading more of these countries to remain stuck at low levels of reputation. Our model delivers an empirically estimable time series measure of countries' reputation. We estimate this measure using micro data, and it can be used by future researchers in empirical and quantitiative work in which a country's reputation plays a role. We show how established safe asset providers, like the U.S., can crowd out potential competitors by satiating demand for safe assets through its issuance strategy. Such a strategy can decrease the US's exorbitant privilege even while shielding it from the even greater losses that would result from competition.

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Online Appendix for "International Currency Competition"

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A.I Proofs

A.I.A Proof of Lemma 1

The committed government solves

$$\max_{D_{jt}^i} \sum_i (Q - R_{jt}^i) D_{jt}^i \quad s.t. \quad R_{jt}^i = \frac{\bar{R} + \frac{1}{2} b_t^* \omega_i (M_{jt})^{-1} D_{jt}^i}{1 - (1 - M_{jt})\overline{\tau}}$$

The decision problem is separable across investors, yielding first order condition

$$0 = (Q - R_{jt}^i) - \frac{\partial R_{jt}^i}{\partial D_{jt}} D_{jt}^i$$

$$D_{jt}^i = \frac{1}{b_t^* \omega_i (M_{jt})^{-1}} \left[Q \left(1 - (1 - M_{jt})\overline{\tau} \right) - \bar{R} \right]$$

Substituting into the interest rate schedule, we obtain

$$R_{jt}^i = \frac{1}{2} \frac{\overline{R}}{1 - (1 - M_{jt})\overline{\tau}} + \frac{1}{2} Q$$

A.I.B Derivation of Stationary Distribution

Consider the discrete set of reputations $\mathbf{M}$ that comes out of the unique graduation step Markov equilibrium of the model without competition with slope b^*. The stationary distribution μ over $\mathbf{M}$ from the reputation game is the atoms over $\mathbf{M}$, whose probabilities are $\{\mu_0, ..., \mu_N, \mu_{N+1}\}$, where μ_{N+1} is the measure of countries that have reached reputation $1 - \epsilon^C$ (i.e., that were committed types at cycle step N). We can characterize the stationary distribution as follows. First consider any step $0 < n < N + 1$. At step n, the mass μ_n of countries in the stationary distribution comes from countries at the prior step that do not exercise the capital control, $M_{n-1}\mu_{n-1}$. Observe that all countries at step n either move to step $n + 1$ or revert to step 0, meaning that

$$\mu_n = M_{n-1}\mu_{n-1}.$$

Step $n = N + 1$ is an absorbing state for committed governments that do not switch type. The flows of types are the same as at steps $0 < n < N + 1$, except that the mass $1 - \epsilon^C$ of committed types also remain at $N+1$. Therefore, we have μ_{N+1} given by $\mu_{N+1} = (1-\epsilon^C)\mu_{N+1} + M_N\mu_N$, which rearranges to

$$\mu_{N+1} = \frac{1}{\epsilon^C} M_N \mu_N.$$

Let us define $\delta_n^* = \prod_{k=0}^{n-1} M_k$ for $0 < n < N+1$, with $\delta_0^* = 1$ and $\delta_{N+1}^* = \frac{1}{\epsilon^C}\prod_{k=0}^{N} M_k^*$. δ_n^* is a cumulative unconditional probability that a government that starts at step 0 goes through its first n crises without exercising capital controls (with an adjustment at $N+1$ for the absorbing state).[1] From above, we have $\mu_n = \delta_n^* \mu_0$ for all n. Finally using that $\sum_{n=0}^{N+1} \mu_n = 1$, we obtain

$$\mu_0 = \frac{1}{\sum_{n=0}^{N+1} \delta_n^*}.$$

Thus, from here we can write for all $0 \leq n \leq N+1$

$$\mu_n = \frac{\delta_n^*}{\sum_{x=0}^{N+1} \delta_x^*}$$

This characterizes the stationary distribution that arises out of the dynamic reputation model.

A.I.C Proof of Proposition 1

For given b^*, Lemma 2 shows that there exists a unique graduation step Markov equilibrium, and Appendix A.I.B derives its stationary distribution μ_{b^*}. To prove existence, we need to verify that there is some b^* such that the consistency condition holds, that is

$$\Delta(b^*) \equiv b^* - b\left(\overline{S} + \int_M \omega(M)^{-1} D(M, b^*)^2 d\mu_{b^*}\right) = 0.$$

This equation depends on b only through b itself, since all other elements are functions of b^*.

The proof strategy has two steps. First, we show that there is are two threshold values $0 < \underline{b} < \overline{b} < \infty$ such that $\Delta(\underline{b}) < 0$ and $\Delta(\overline{b}) > 0$. We then prove that Δ is continuous on the interval $[\underline{b}, \overline{b}]$. This proves that Δ has a zero on the interval $[\underline{b}, \overline{b}]$ and hence an equilibrium exists.

Step 1: Interval Bounds

We first show there is a lower point $\underline{b} > 0$ such that $\Delta(\underline{b}) < 0$. From Lemma 1 and using $\omega_i(M_{jt})^{-1} D_{jt}^i = \omega(M_{jt})^{-1} D_{jt}$, we have

$$\omega(M)^{-1} D(M, b^*)^2 = \frac{1}{b^{*2}} \omega(M) \Big[Q(1 - (1-M)\overline{\tau}) - \bar{R} \Big]^2.$$

[1] Note the close relationship with δ_n defined in the main text, which defines the same object but condition on being opportunistic at step 0.

Since $\omega(M)$ is a nondecreasing function of M, then we have for all M

$$\omega(M)^{-1}D(M,b^*)^2 \geq \frac{1}{b^{*2}}\omega(0)\Big[Q(1-\overline{\tau})-\bar{R}\Big]^2 > 0.$$

Therefore, we can write

$$\Delta(b^*) \leq b^* - b\bigg(\overline{S} + \frac{1}{b^{*2}}\omega(0)\Big[Q(1-\overline{\tau})-\bar{R}\Big]^2\bigg).$$

Observe that $\Delta(0+) \to -\infty$, so that we have a $\underline{b} > 0$ such that $\Delta(\underline{b}) < 0$.

Next we construct an $\bar{b} > 0$ such that $\Delta(\bar{b}) > 0$. We have for all M

$$\omega(M)^{-1}D(M,b^*)^2 \leq \frac{1}{b^{*2}}\omega(1)\Big[Q-\bar{R}\Big]^2.$$

Therefore, we can write

$$\Delta(b^*) \geq b^* - b\bigg(\overline{S} + \frac{1}{b^{*2}}\omega(1)\Big[Q-\bar{R}\Big]^2\bigg) = b^* - \frac{1}{b^{*2}}b\omega(1)\Big[Q-\bar{R}\Big]^2 - b\overline{S}.$$

The RHS is an increasing function of b^*, with $RHS(0+) \to -\infty$ and $RHS(\infty) \to \infty$. Therefore we have a threshold $\bar{b}$ such that $RHS(\bar{b}) = 0$, and therefore $\Delta(\bar{b}) \geq 0$. Finally, observe that from the derivations above we can always construct a value $\underline{b}$ that lies below $\bar{b}$.

Therefore, there exist $0 < \underline{b} < \bar{b} < \infty$ with $\Delta(\underline{b}) < 0$ and $\Delta(\bar{b}) \geq 0$.

Step 2: Proof of Continuity

The proof that $\Delta(b^*)$ is continuous amounts to a proof that $\bar{D}(b^*) \equiv b\int_M \omega(M)^{-1}D(M,b^*)^2 d\mu_{b^*}$ is continuous in b^*.

Let $\{M_0(b^*), ..., M_N(b^*)\}$ be the equilibrium reputation cycle of the graduation step Markov equilibrium without competition and with slope b^*. It is helpful to define the extended path $\{M_n(b^*)\}$ for all $n \geq 0$, where $M_n(b^*) = 1-\epsilon^C$ for all $n > N$.[2]

We break the proof of continuity of $\overline{D}(b^*)$ into two steps. First, we prove that for every n in the extended path, M_n is continuous in b^*. We then use this result to prove that $\overline{D}(b^*)$ is continuous.

We begin with an initial Lemma establishing a notion of monotonicity.

Lemma 3 *Let $b_1^* > b_2^*$. If $M_n(b_1^*) \geq M_n(b_2^*)$, then $M_{n+s}(b_1^*) \geq M_{n+s}(b_2^*)$ for all $s \geq 0$.*

[2]The extended path is useful because it allow us to define the stationary distribution over all steps $n \geq 0$ of governments, including after the graduation step, rather than defining it over reputation at each step. This has the advantage that the set of steps $n \in \{0, \ldots\}$ of the extended path is invariant to b^*, which eases proof of continuity by avoiding discrete changes in graduation steps, which change discretely the number of atoms in the stationary distribution over reputation.

Proof of Lemma 3. Suppose that $M_n(b_1^*) \geq M_n(b_2^*)$. Recall we can write the transition equation as

$$V(M_n) = \rho V(M_{n-1}) + B^* + V(M_0)$$

where for notational compactness we have defined $B^* = \rho Q A \frac{b^*-b}{b}$. Note that $B_1^* > B_2^*$ when $b_1^* > b_2^*$. Suppose first, by way of contradiction, that $B_1^* + V(M_0(b_1^*)) \leq B_2^* + V(M_0(b_2^*))$. Since $B_1^* > B_2^*$, then $M_0(b_1^*) < M_0(b_2^*)$. Then, we have

$$V_1(M_1^*) = B_1^* + V(M_0(b_1^*)) + \rho V(M_0(b_1^*)) < B_2^* + V(M_0(b_1^*)) + \rho V(M_0(b_2^*)) = V_2(M_1^*).$$

By induction, if $B_1^* + V(M_0(b_1^*)) \leq B_2^* + V(M_0(b_2^*))$ and $V(M_n(b_1^*)) < V(M_n(b_2^*))$, then

$$V(M_n(b_1^*)) = \rho V(M_{n-1}(b_1^*)) + B_1^* + V(M_0(b_1^*)) < V(M_n(b_2^*))$$

for all n, a contradiction. Therefore if $M_n(b_1^*) \geq M_n(b_2^*)$ for some n, then $B_1^* + V(M_0(b_1^*)) > B_2^* + V(M_0(b_2^*))$. But then by induction, $M_{n+s}(b_1^*) \geq M_{n+s}(b_2^*)$ for all $s \geq 0$. This completes the proof.

Lemma 3 allows us to prove the next result.

Lemma 4 *The following comparative statics are true in the model without competition and slope b^*:*

- *(a) The graduation step N is weakly decreasing in b^*.*
- *(b) Take $b_1^* > b_2^*$ such that $N(b_2^*) > 0$. Then, there exists $n' > 0$ such that $M_n(b_1^*) < M_n(b_2^*)$ for $n < n'$ and $M_n(b_1^*) \geq M_n(b_2^*)$ for $n \geq n'$.*
- *(c) The path of beliefs $\pi_n(b^*)$ is nondecreasing in b^*.*

Proof of Lemma 4.

(a) Take slope b_2^*, and let $N_2 \equiv N(b_2^*)$ be the graduation step and $M_n(b_2^*)$ be the path of reputation associated with b_2^*. Now consider $b_1^* > b_2^*$. Suppose by way of contradiction that $N_1 \equiv N(b_1^*) > N(b_2^*)$. From Lemma 3, we know we must have $M_{N_2}(b_1^*) < M_{N_2}(b_2^*)$ to avoid graduation at N_2. From Lemma 3, we know that $M_n(b_1^*) < M_n(b_2^*)$ for all $n \leq N_2$. But then Bayes' rule implies that

$$M_{N_2}(b_1^*) < M_{N_2}(b_2^*) = \pi_{N_2}(b_2^*) < \pi_{N_2}(b_1^*)$$

meaning that beliefs exceed reputation at N_2, a contradiction. Therefore, $N_1 \leq N_2$, and the graduation step is non-increasing in b^*.

(b) First, note that Lemma 3 implies that if there is some n' such that $M_{n'}(b_1^*) \geq M_{n'}(b_2^*)$, then $M_n(b_1^*) \geq M_n(b_2^*)$ for all $n > n'$. We therefore necessarily have some such $n' \leq N_2 + 1$. From part (a), we know that $N_1 \leq N_2$. Now, suppose that we had $n' = 0$, and hence we had $M_n(b_2^*) < M_n(b_1^*)$ for all $n \leq N_1$. But then, from Bayes' rule we have

$$M_{N_1}(b_2^*) < M_{N_1}(b_1^*) = \pi_{N_1}(b_1^*) < \pi_{N_1}(b_2^*)$$

a contradiction. So, $M_0(b_1^*) < M_0(b_2^*)$, giving the result for some $n' > 0$.

(c) Let $b_1^* > b_2^*$. From part (a), we know that $N_1 \leq N_2$. From part (b), we know that there exists an $n' > 0$ such that $M_n(b_1^*) < M_n(b_2^*)$ iff $n < n'$. Therefore, from Bayes' rule the result holds trivially for $n \leq n'$ (recall that $\pi_{n'}$ is determined from $M_0, ..., M_{n'-1}$). If $n' \geq N_2$ then we have finished the proof. Suppose instead that $n' < N_2$ and consider $n > n'$. We know that at $n = N_2$,

$$\pi_{N_2}(b_2^*) = M_{N_2}(b_2^*) \leq M_{N_2}(b_1^*) = \pi_{N_2}(b_1^*),$$

so we also have beliefs that are higher at N_2.[3] Now recall Bayes' rule,

$$\pi_n = \epsilon^C + (1 - \epsilon^C - \epsilon^O)\frac{\pi_{n-1}}{M_{n-1}}.$$

From Bayes' rule, we know that if $\pi_n(b_1^*) \geq \pi_n(b_2^*)$ and $M_{n-1}(b_1^*) \geq M_{n-1}(b_2^*)$, then it must be the case that $\pi_{n-1}(b_1^*) \geq \pi_{n-1}(b_2^*)$. Therefore, by induction we have $\pi_n(b_1^*) \geq \pi_n(b_2^*)$ for all $n > n'$. This completes the proof.

We are now ready to prove that each point on the extended path is continuous.

Lemma 5 *For every $n \geq 0$ of the extended path, $M_n(b^*)$ is continuous in b^*.*

Proof of Lemma 5: *We begin by showing that if $M_0(b^*)$ is continuous, then $M_n(b^*)$ is continuous for all $n > 0$. Recall that the path M_n for $n > 0$ is generated in equilibrium by the transition equation*

$$V(M_n) = \rho QA\frac{b^* - b}{b} + \rho V(M_{n-1}) + V(M_0)$$

for $0 < n \leq N$, and by $M_n = 1 - \epsilon^C$ for $n > N$. Define $\underline{V} = V(\epsilon^O)$ and $\overline{V} = V(1 - \epsilon^C)$. Define the hypothetical path of indifference-sustaining utilities as

$$V_n^* = \rho QA\frac{b^* - b}{b} + \rho V_{n-1}^* + V_0^*$$

with $V_0^* = V(M_0)$. Define V^{-1} the inverse function of V, which is continuous and well defined on $[\underline{V}, \overline{V}]$. Thus we can define the path of reputation as

$$M_n = \begin{cases} V^{-1}(V_n^*), & V_n^* \leq \overline{V} \\ 1 - \epsilon^C, & V_n^* > \overline{V} \end{cases}$$

[3] Note that if $N_1 < N_2$, the above equation holds because $M_{N_2}(b_1^*) = \pi_{N_2}(b_1^*) = 1 - \epsilon^C$.

Since V_n^* is a continuous function of (b^*, V_0^*), since V_0^* is a continuous function of M_0, and since $V^{-1}(\overline{V}) = 1 - \epsilon^C$, then a sufficient condition for M_n to be a continuous function of b^* for any $n > 0$ is that M_0 is a continuous function of b^*. It thus remains to prove that M_0 is a continuous function of b^*.

The proof that M_0 is continuous proceeds by contradiction. Suppose, hypothetically, that M_0 were not a continuous function of b^*. We have already proven that M_0 is a decreasing function of b^* (Lemma 4), so the discontinuity must be downward. Hence, a discontinuity implies there is some $b_1^* \in [\underline{b}, \overline{b}]$ such that $M_0(b_1^*-) > M_0(b_1^*+) + \delta$ for some $\delta > 0$, where $M_0(b_1^*-)$ and $M_0(b_1^*+)$ denote the left and right limits respectively. We have also proven that N is a nonincreasing function of b^* (Lemma 4). Therefore, we must have either $N(b_1^*-) = N(b_1^*+)$ or else $N(b_1^*-) > N(b_1^*+)$.

Suppose first that $N(b_1^*-) = N(b_1^*+) \equiv N$, that is the graduation step is continuous at the discontinuity. But since the transition dynamics determining M_n are continuous in b^* for given M_0 (V_0^*), then we have $M_n(b_1^*-) > M_n(b_1^*+)$ for all $n \leq N$. But then from Bayes rule, we have

$$\pi_N(b_1^*-) < \pi_N(b_1^*+),$$

contradicting that both $M_n(b_1^*-) = \pi_N(b_1^*-)$ and $M_n(b_1^*+) = \pi_N(b_1^*+)$, and hence contradicting that N is the graduation step of both. Thus, we cannot have $N(b_1^*-) = N(b_1^*+)$ at a discontinuity.

Suppose, then, that the discontinuity is accompanied with a discontinuity in the graduation step, $N(b_1^*-) > N(b_1^*+)$. By the same argument as above, the transition dynamics imply $M_n(b_1^*-) > M_n(b_1^*+)$ for all $n \leq N(b_1^*-)$. But then since $N(b_1^*-) > N(b_1^*+)$,

$$M_{N(b_1^*+)+1}(b_1^*+) < M_{N(b_1^*+)+1}(b_1^*-) \leq 1 - \epsilon^C$$

contradicting that $N(b_1^*+) < N(b_1^*-)$. Therefore, a discontinuity in M_0 also cannot be accompanied by a discontinuity in the graduation step.

Therefore, $M_0(b^*)$ is continuous. This completes the proof of this lemma.

Having proven that $M_n(b^*)$ is continuous for all $n \geq 0$ on the extended path, we are now ready to prove that $\overline{D}(b^*)$ is continuous and hence $\Delta(b^*)$ is continuous. It is helpful to redefine the stationary distribution over steps, $\mu_n(b^*)$, rather than over reputation levels, so that we can make use of the extended path. Under this definition, we can rewrite for any b^*

$$\bar{D}(b^*) = \sum_{n=0}^{\infty} \omega(M_n(b^*))^{-1} D(M_n(b^*), b^*)^2 \mu_n(b^*).$$

Now, we can define the stationary distribution analogous to its definition in Appendix A.I.B, except that we no longer have an absorbing state. Therefore defining $\delta_0^*(b^*) = 1$ and defining

$$\delta_n^*(b^*) = \prod_{k=0}^{n-1} M_k(b^*)$$

for all $n > 0$. Note that mass at steps $n > N$ captures committed types that have not switched type. Given this definition of $\delta_n^*(b^*)$, we have the stationary distribution over steps

$$\mu_n(b^*) = \frac{\delta_n^*(b^*)}{\sum_{k=0}^{\infty} \delta_k^*(b^*)}.$$

Thus substituting in above, we can write

$$\bar{D}(b^*) = \frac{\sum_{n=0}^{\infty} \omega(M_n(b^*))^{-1} D(M_n(b^*), b^*)^2 \delta_n^*(b^*)}{\sum_{n=0}^{\infty} \delta_n^*(b^*)}$$

Observe that the numerator and denominator are continuous functions of the path $\{M_n(b^*)\}$ and the slope b^*, and that the denominator is strictly positive and bounded away from zero since $\delta_0^*(b^*) = 1$. Therefore, $1/\sum_{n=0}^{\infty} \delta_n^*(b^*)$ is also a continuous function of $\{M_n(b^*)\}$. Therefore, $\overline{D}(b^*)$ is the product of two functions that are continuous in $(\{M_n(b^*)\}, b^*)$, and so is itself continuous. Therefore since by Lemma 5 $M_n(b^*)$ is continuous in b^* for every n, then $\overline{D}$ is continuous. This completes the proof of continuity.

Summarizing Argument

We have shown that there is an interval $[\underline{b}, \bar{b}]$ such that $\Delta(\underline{b}) < 0$ and $\Delta(\bar{b}) > 0$ (Step 1). We have further shown that $\Delta(b^*)$ is continuous on $[\underline{b}, \bar{b}]$ (Step 2). Therefore, there is a value $\hat{b} \in [\underline{b}, \bar{b}]$ such that $\Delta(\hat{b}) = 0$. Therefore, an equilibrium of the competition model, as described, exists.

A.I.D Proof of Proposition 2

The transition dynamics of the equilibrium of this model are identical to those of the model without competition but slope $b' = b^*$. Thus we can write

$$V(M_n, b^*) = \rho V(M_{n-1}, b^*) + V(M_0, b^*).$$

Recall that we have

$$V(M_n, b^*) = QA + \Big(Q - R(M_n)\Big) D(M_n, b^*)$$

Using the issuance solutions, we have

$$D(M_n, b^*) = \frac{\omega(M_n)}{b^*}\Big[\gamma Q(1 - (1 - M_n)\overline{\tau}) - \bar{R}\Big] = \frac{b}{b^*} D(M_n, b).$$

Thus, substituting in we can write

$$V(M_n, b^*) = QA + \Big(Q - R(M_n)\Big) D(M_n) \frac{b}{b^*} = QA \frac{b^* - b}{b^*} + \frac{b}{b^*} V(M_n).$$

From here, we can substitute in to the transition equation and rearrange to obtain

$$V(M_n) = \rho QA \frac{b^* - b}{b} + \rho V(M_{n-1}) + V(M_0)$$

Finally, suppose that $A = 0$. Then, this transition equation is exactly the same as the transition equation in the model without competition and slope b. Thus, we obtain the same graduation step Markov equilibrium $\mathbf{M}$ and the same stationary distribution μ. However, issuance is affected since we have $D(M_n, b^*) = \frac{b}{b^*} D(M_n, b)$.

A.I.E Proof of Proposition 3

Recall that the transition equation is

$$V(M_n) = \rho QA \frac{b^* - b}{b} + \rho V(M_{n-1}) + V(M_0)$$

and that there is a unique graduation step Markov equilibrium associated of the model without competition and slope b^*. Conjecture an equilibrium with immediate graduation, $N = 0$. This means that $M_0 = \epsilon^O$ and that

$$V(1 - \epsilon^C) \leq \rho QA \frac{b^* - b}{b} + (1 + \rho) V(\epsilon^O)$$

Rearranging, we have

$$\left(1 + \frac{V(1 - \epsilon^C) - (1 + \rho) V(\epsilon^O)}{\rho QA}\right) b \leq b^*$$

which gives the result.

A.I.F Proof of Proposition 4

Given that we have $\alpha_i(M_{jt}) = \frac{\omega_i(M_{jt}) \frac{1}{\omega(M_{jt})} D(M_{jt})}{w_i}$ and taking $w_i = w$ to be constant, then we have

$$corr_i(\alpha_i(M_{jt}), \alpha_i(\overline{M})) = corr_i \left(\omega_i(M_{jt}) \frac{\omega(M_{jt})^{-1} D(M_{jt})}{w}, \omega_i(\overline{M}) \frac{\omega(\overline{M})^{-1} D(\overline{M})}{w} \right) = corr_i \left(\omega_i(M_{jt}), \omega_i(\overline{M}) \right)$$

given that $\frac{\omega(M_{jt})^{-1} D(M_{jt})}{w}$ and $\frac{\omega(\bar{M})^{-1} D(\bar{M})}{w}$ are constant across i. This gives the first part of the result.

Now to the second part of the result. Employing a Taylor series approximation around M^r, we have

$$\omega_i(M_{jt}) = \omega_i(M^r) + \omega_i'(M^r)(M_{jt} - M^r) + \mathcal{O}(M_{jt} - M^r)^2.$$

Denote $\phi_0^i = \omega_i(M^r)$, $\phi_1^i = \omega_i'(M^r)$, and $\mathcal{O}_{ijt}$ to be the approximation error. Denote $\mathcal{O}_t = \sup_{i,j} \mathcal{O}_{ijt}$ and $o_{ijt} = \mathcal{O}_{ijt}/\mathcal{O}_t \leq 1$ then

$$\omega_i(M_{jt}) = \phi_0^i + \phi_1^i(M_{jt} - M^r) + o_{ijt}\mathcal{O}_t.$$

Thus as $\mathcal{O}_t \to 0$,

$$corr_i(\omega_i(M_{jt}), \omega_i(\overline{M})) \to corr_i\Big(\phi_0^i + \phi_1^i(M_{jt} - M^r), \phi_0^i + \phi_1^i(\overline{M} - M^r)\Big).$$

We now look to show that the correlation is strictly increasing in M as the approximation error converges to zero. Denote $\hat{\omega}_i(M_{jt}) = \phi_0^i + \phi_1^i(M_{jt} - M^r)$ the first order approximations. Then, the correlation applied to $\hat{\omega}_i$ is

$$\rho(M_{jt}) = corr_i(\hat{\omega}_i(M_{jt}), \hat{\omega}_i(\overline{M})) = \frac{\text{cov}_i(\hat{\omega}_i(M_{jt}), \hat{\omega}_i(\overline{M}))}{\sqrt{var_i(\hat{\omega}_i(M_{jt}))var_i(\hat{\omega}_i(\overline{M}))}}.$$

Differentiating the correlation, we have

$$\begin{aligned}\frac{\partial\rho(M_{jt})}{\partial M_{jt}} &= \frac{\frac{\partial \text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M}))}{\partial M_{jt}}\sqrt{var_i(\hat{\omega}_i(M_{jt}))} - \frac{\text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M}))}{2\sqrt{var_i(\hat{\omega}_i(M_{jt}))}}\frac{\partial var_i(\hat{\omega}_i(M_{jt}))}{\partial M_{jt}}}{var_i(\hat{\omega}_i(M_{jt}))\sqrt{var_i(\hat{\omega}_i(\overline{M}))}} \\ &= \frac{\frac{\partial \text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M}))}{\partial M_{jt}} var_i(\hat{\omega}_i(M_{jt})) - \frac{1}{2}\text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M}))\frac{\partial var_i(\hat{\omega}_i(M_{jt}))}{\partial M_{jt}}}{var_i(\hat{\omega}_i(M_{jt}))^{3/2} var_i(\hat{\omega}_i(\overline{M}))^{1/2}}\end{aligned}$$

We have the covariance and variance

$$cov_i(\hat{\omega}_i(M_{jt}), \hat{\omega}_i(\overline{M})) = \sigma_0^2 + \sigma_1^2(M_{jt} - M^r)(\overline{M} - M^r) + \sigma_{01}(M_{jt} + \overline{M} - 2M^r)$$

$$var_i(\hat{\omega}_i(M_{jt})) = \sigma_0^2 + \sigma_1^2(M_{jt} - M^r)^2 + 2\sigma_{01}(M_{jt} - M^r)$$

where $\sigma_0^2 = var_i(\phi_0^i)$, $\sigma_1^2 = var_i(\phi_1^i)$, and $\sigma_{01} = cov_i(\phi_0^i, \phi_1^i)$. Therefore, we have

$$\frac{\partial cov_i(\hat{\omega}_i(M_{jt}), \hat{\omega}_i(\overline{M}))}{\partial M_{jt}} = \sigma_1^2(\overline{M} - M^r) + \sigma_{01}$$

$$\frac{\partial var_i(\hat{\omega}_i(M_{jt}))}{\partial M_{jt}} = 2\sigma_1^2(M_{jt} - M^r) + 2\sigma_{01}$$

Substituting these expressions back into the derivative of the correlation,

$$\frac{\partial \rho(M_{jt})}{\partial M_{jt}} = \frac{\sigma_1^2\Big[(\overline{M}-M^r)var_i(\hat{\omega}_i(M_{jt})) - \text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M}))(M_{jt}-M^r)\Big]}{var_i(\hat{\omega}_i(M_{jt}))^{3/2}var_i(\hat{\omega}_i(\overline{M}))^{1/2}} + \frac{\sigma_{01}\Big[var_i(\hat{\omega}_i(M_{jt})) - \text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M}))\Big]}{var_i(\hat{\omega}_i(M_{jt}))^{3/2}var_i(\hat{\omega}_i(\overline{M}))^{1/2}}$$

Now, using the formulas for variance and covariance, we have

$$var_i(\hat{\omega}_i(M_{jt})) - \text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M})) = -(\overline{M}-M_{jt})\Big[\sigma_1^2(M_{jt}-M^r)+\sigma_{01}\Big]$$

$$(\overline{M}-M^r)var_i(\hat{\omega}_i(M_{jt})) - \text{cov}_i(\hat{\omega}_i(M_{jt}),\hat{\omega}_i(\overline{M}))(M_{jt}-M^r) = (\overline{M}-M_{jt})\Big[\sigma_0^2+\sigma_{01}(M_{jt}-M^r)\Big]$$

Substituting these expressions into the derivative of the correlation gives

$$\begin{aligned}\frac{\partial \rho(M_{jt})}{\partial M_{jt}} &= \frac{\sigma_1^2(\overline{M}-M_{jt})\Big[\sigma_0^2+\sigma_{01}(M_{jt}-M^r)\Big] - \sigma_{01}(\overline{M}-M_{jt})\Big[\sigma_1^2(M_{jt}-M^r)+\sigma_{01}\Big]}{var_i(\hat{\omega}_i(M_{jt}))^{3/2}var_i(\hat{\omega}_i(\overline{M}))^{1/2}}\\ &= (\overline{M}-M_{jt})\frac{\sigma_1^2\sigma_0^2-\sigma_{01}^2}{var_i(\hat{\omega}_i(M_{jt}))^{3/2}var_i(\hat{\omega}_i(\overline{M}))^{1/2}}\\ &= (\overline{M}-M_{jt})\frac{\Big(1-\rho_{01}^2\Big)\sigma_1^2\sigma_0^2}{var_i(\hat{\omega}_i(M_{jt}))^{3/2}var_i(\hat{\omega}_i(\overline{M}))^{1/2}}\end{aligned}$$

where $\rho_{01} = \frac{\sigma_{01}}{\sigma_0\sigma_1}$ is the correlation between ϕ_0^i and ϕ_1^i. Thus, we have $\frac{\partial \rho(M_{jt})}{\partial M_{jt}} > 0$ provided that: (i) $\overline{M} > M_{jt}$; (ii) $|\rho_{01}| < 1$; (iii) $\sigma_0^2, \sigma_1^2 > 0$.

Finally, provided that $\frac{\partial \rho(M_{jt})}{\partial M_{jt}} > 0$, then as $\mathcal{O}_t \to 0$ we have that $corr_i(\alpha_i(M_{jt}), \alpha_i(\overline{M}))$ increases in M_{jt}. Therefore, $M_{jt} > M_{kt} \iff corr_i(\alpha_i(M_{jt}), \alpha_i(\overline{M})) > corr_i(\alpha_i(M_{kt}), \alpha_i(\overline{M}))$, and hence the portfolio correlation rank also ranks countries by reputation. This concludes the proof.

A.I.G Proof of Proposition 5

Internalizing equations 21 and 22 into the incumbent's objective, we have the optimization problem

$$\max_{b^*} A^* + \frac{1}{b}(Q^* - \overline{R} - \frac{1}{4}b^*)\bigg(b^* - b\int \omega(M)^{-1}D(M,b^*)^2 d\mu_{b^*}\bigg)$$

Differentiating, we obtain

$$0 = -\frac{1}{4}\overline{S}(b^*) + (Q^* - R^S(b^*))\frac{1}{b}\bigg(1 - b\frac{\partial \int \omega(M)^{-1}D(M,b^*)^2 d\mu_{b^*}}{\partial b^*}\bigg).$$

Splitting apart the derivative and rearranging, we obtain

$$\overline{S}(b^*) = \frac{4}{b}(Q^* - R^S(b^*)) - 4(Q^* - R^S(b^*))\left[\int \omega(M)^{-1}\frac{\partial D(M,b^*)^2}{\partial b^*}d\mu_{b^*} + \frac{\partial \int \omega(M)^{-1}D(M,b^*)^2 d\mu_{b'}}{\partial b'}\bigg|_{b'=b^*}\right]$$

A.I.H Proof of Proposition 6

Define the extended path as in the proof of Proposition 1. Define $\delta_n = \Pi_{k=0}^{n-1} m_k$. Note that since $m_n(b_1^*) = 0$ for all $n \geq N_1$, the result holds trivially for all $N_1 \leq n \leq N_2$ and we can restrict attention to $n < N_1$.

Starting with δ_1 ($n = 1$). We know from part (b) of Lemma 4 that M_0 declines in b^*, so m_0 also declines in b^* given that initial beliefs are $\pi_0 = \epsilon^O$. Therefore, $\delta_1 = m_0$ declines in b^*.

We now consider δ_n for $n > 1$. Reputation is given by $M_n = \pi_n + (1 - \pi_n)m_n$ and beliefs are given by $\pi_{n+1} = \epsilon^O + (1 - \epsilon^C - \epsilon^O)\frac{\pi_n}{M_n}$. Using the first equation to write $m_n = \frac{M_n - \pi_n}{1 - \pi_n}$ and the second to write $M_n = \frac{(1-\epsilon^C - \epsilon^O)\pi_n}{\pi_{n+1} - \epsilon^O}$, we can substitute the second equation into the first to obtain

$$m_n = \frac{\pi_n}{1 - \pi_n}\frac{1 - \epsilon^C - \pi_{n+1}}{\pi_{n+1} - \epsilon^O}.$$

From here, we can substitute into δ_n to obtain

$$\delta_n = \prod_{k=0}^{n-1}\frac{\pi_k}{1 - \pi_k}\frac{1 - \epsilon^C - \pi_{k+1}}{\pi_{k+1} - \epsilon^O}.$$

Taking logs and regrouping terms, we can write

$$\log \delta_n = \log\frac{\pi_0}{1 - \pi_0} + \sum_{k=1}^{n-1}\log\frac{\pi_k}{1 - \pi_k}\frac{1 - \epsilon^C - \pi_k}{\pi_k - \epsilon^O} + \log\frac{1 - \epsilon^C - \pi_n}{\pi_n - \epsilon^O}$$

$$\log \delta_n = \log\frac{\pi_0}{1 - \pi_0} + \sum_{k=1}^{n-1}\left[\log\frac{\pi_k}{\pi_k - \epsilon^O} + \log\frac{1 - \epsilon^C - \pi_k}{1 - \pi_k}\right] + \log\frac{1 - \epsilon^C - \pi_n}{\pi_n - \epsilon^O}$$

To complete the proof, we show that every term on the RHS is weakly decreasing in b^*. The first term, $\log\frac{\pi_0}{1-\pi_0}$, is constant since $\pi_0 = \epsilon^O$. Every component in the sum declines, since beliefs are weakly increasing in b^* and since we have

$$\frac{\partial}{\partial \pi_k}\left[\frac{\pi_k}{\pi_k - \epsilon^O}\right] = \frac{-\epsilon^O}{(\pi_k - \epsilon^O)^2} < 0$$

$$\frac{\partial}{\partial \pi_k}\frac{1 - \epsilon^C - \pi_k}{1 - \pi_k} = \frac{-\epsilon^C}{(1 - \pi_k)^2} < 0$$

The last term is also weakly decreasing in b^*, since

$$\frac{\partial}{\partial \pi_n}\left[\frac{1-\epsilon^C-\pi_n}{\pi_n-\epsilon^O}\right] = \frac{-(1-\epsilon^C-\epsilon^O)}{(\pi_n-\epsilon^O)^2} < 0$$

where the inequality follows since $1-\epsilon^C-\epsilon^O > 0$ by assumption. Therefore, we obtain the result for all n, concluding the proof.

A.I.I Competition Solutions

The following proposition associates solutions of the model with competition with the no-competition models that generate them.

Proposition 7 *For every b^*, there exists a unique b (holding all other parameters fixed) such that there is an equilibrium of the model with competition that generates slope b^*.*

Proposition 7 provides a simple way of mapping a model with competition back into the parameters of the model without competition that generates it, in particular the original slope b. To understand Proposition 7, begin with a choice of b^*. From Lemma 2, we obtain the unique graduation step Markov equilibrium and cycle $\mathbf{M}$. From there, we obtain the stationary distribution μ over $\mathbf{M}$. Finally, we can rearrange the consistency condition to $b = \frac{b^*}{\int \omega(M)^{-1} D(M)^2 d\mu(M)}$, which gives us the value of b. Given the graduation step Markov equilibrium and its stationary distribution are both unique, b is also unique. From here, reversing the steps starting from b yields an equilibrium of the model with competition that generates slope b^*.[4]

A.I.I1 Proof of Proposition 7

Take a given b^*, then from Lemma 2 we know there exists a unique graduation step Markov equilibrium of the model without competition. We can then find the stationary distribution of this equilibrium (see Appendix A.I.B). The consistency condition $b^* = b\left(\overline{S} + \int \omega(M)^{-1} D(M)^2 d\mu(M)\right)$ is a linear equation in b, and therefore has a single solution (given knowledge of b^* and the unique graduation date Markov equilibrium). Therefore, there is a unique b such that the proposition holds.

[4]Note that Proposition 7 shows that each b^* is uniquely associated with a b, but not that b uniquely maps into b^*.