Platform Money*

Emre Ozdenoren — London Business School

Yuan Tian — LSE

Kathy Yuan — LSE

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Abstract

This paper studies how a platform's ability to create its own money affects its pricing decisions, the search and matching dynamics, and social welfare. By issuing its currency, the platform extracts seigniorage. A legacy market using outside money cannot recoup seigniorage and thus operates at a competitive disadvantage, even when inflation costs are less salient than fees. Equilibrium is socially inefficient both in terms of resulting seller/buyer ratios and the number of traders on the platform. When the platform has superior matching technology, introducing platform money can move the equilibrium outcome closer to the social optimum relative to a fee-only platform. When some buyers exhibit biased preference towards outside money, the platform might still choose not to accept any outside money payment options due to strategic considerations.

Keywords: Platform Money, Platform Tokens, Private Money, Platform Competition, Search and Matching, BigTech, Platform Payments, Digital Platforms.

JEL classification: G10, G01

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*Ozdenoren (eozdenoren@london.edu), Tian (y.tian11@lse.ac.uk), and Yuan (k.yuan@lse.ac.uk).

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1 Introduction

The rise of the platform economy is evidenced by the growing popularity of super-apps such as Amazon, Facebook (Meta), X, Uber, WeChat, Alibaba, Gala Games, and many others. Empowered by advanced data processing technologies and machine learning algorithms, these applications deliver superior matching capabilities between buyers and sellers, offering a user experience that traditional brick-and-mortar marketplaces cannot match. Many of these apps have evolved into lifestyle platforms with integrated ecosystems, enabling users to conduct all their economic activities - from work to consumption - without ever leaving the platform.

At the same time, platforms are increasingly developing their own digital payment systems to align with the inherently digital nature of economic activity in their marketplaces. Examples include Diem (formerly Libra), WeChat Money, AliPay, and Gala Token. In many cases, these tokens and digital wallets have become fully embedded in a platform's commercial and financial transactions; indeed, for some platforms (e.g. WeChat Pay and Alipay), it is nearly impossible to conduct transactions without using their proprietary payment systems.

Consider, for instance, WeChat's digital wallet. Originally launched with a red packet service that allowed users to send money as gifts, the system became especially popular during the Chinese New Year in 2014. Sponsored cash drops during the annual gift-giving season spurred rapid growth; according to the Wall Street Journal, just one month after its launch, WeChat Pay's user base expanded from 30 million to 100 million, and 20 million red packets were distributed during the holiday. Today, WeChat has transformed from a social media platform into a comprehensive lifestyle app with a payment system that facilitates shopping, dining, transportation, education, donations, payments, financing, investment, and more. As of the first quarter of 2023, WeChat boasted 1.33 billion active users, and by 2021, the platform supported 3.7 million mini programs with 2.7 trillion RMB in transactions.

Traditionally, payment services in the brick-and-mortar economy have been managed by trusted third parties, such as financial service providers. However, digital technology now enables platforms to secure payment transfers and capitalize on transaction data to boost economic activity within their ecosystems. Digital payment systems and digital private money are functionally equivalent, as payment providers control the interest rates on their digital wallets. The income generated from the interest rates in these wallets denominated in the native currency, in effect, seigniorage is typically captured by financial institutions in the traditional economy.[1] With the rise of the platform economy, however, a new scope economy is emerging at the intersection of payments and core platform activities.[2]

In this paper, we examine one source of that scope economy: how the ability to issue platform money as a medium of exchange affects competitive dynamics between platforms and legacy markets. In our model, the platform competes with a legacy market where money growth (inflation) is determined by a central bank to meet broader macroeconomic objectives. We explore how the platform's control over its own money supply influences its pricing decisions, the search and matching process, and overall economic welfare.

Our model also permits the possibility that consumers perceive inflation costs as less salient than direct fees when choosing between the platform and legacy marketplaces. This assumption, combined with the ability to control its own money supply, grants the platform two key advantages over the legacy market. First, by controlling its own money supply, the platform can moderate the inflation cost experienced by its users, whereas the legacy market is limited to adjusting fees - since the supply of outside money is managed by central banks. This is particularly disadvantageous in high-inflation environments, as fees cannot be set below zero. Second, if consumers are less sensitive to inflation, the platform can collect seigniorage income through its private payment system. It is important to note, however, that inflation salience is double-edged: When the legacy marketplace experiences high inflation with outside money, consumers in the legacy marketplace tend to be less sensitive to the resulting costs. As a result, even if the platform offers a lower inflation rate, these consumers are less inclined to switch to the platform.

We cast these tradeoffs in a new monetarist model, following the approach of Lagos and Wright (2005), in which money acts as a medium of exchange in a two-sided platform

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[1] Platforms often have hidden fees for withdrawals from digital wallets which effectively results in an exchange rate between platform money and fiat. Platforms have other ways to expand money supply, e.g., sending coupons and rewards in platform money, that can only be used/redeemed on the platform, e.g. uber cash.

[2] There are other sources of this scope economy. There might be tax advantages to issuing private money as opposed to charging fees. Platforms can take advantage of alternative investment opportunities of cash balances and bundle payment and financial services.

competing with a legacy market that uses fiat currency. In these two-sided marketplaces, the entry of an additional buyer reduces the matching probability for other buyers while increasing it for sellers - or vice versa in the competing market.

We show that by leveraging its private money and advanced matching technology, the platform can lower the cost of attracting buyers while generating cross-group network externalities. Importantly, the effectiveness of using private money to attract buyers depends on several factors including the inflation regime in the legacy marketplace, the inflation salience among platform participants, the relative bargaining power of buyers and sellers regarding consumption goods, and other market-specific parameters affecting the choice of trading venue. In particular, we find that inflation on the platform increases less than one-for-one with the inflation in outside money. As a result, in a low inflation environment platform experiences higher inflation than the legacy marketplace, and the opposite holds in a high inflation environment.

Moreover, the platform's superior matching capabilities further intensify the reinforcing network effects between buyers and sellers. In equilibrium, our analysis reveals that the platform attracts more buyers, imposes higher fees on sellers (yet still attracts more sellers overall), and higher profit compared to the legacy market. Market tightness (seller to buyer ratio) on the platform is lower than the legacy market when the two have similar matching technologies but can be higher if the platform has much superior matching technology. We derive closed-form results characterising these equilibrium properties and offer additional insights through numerical examples.

We also ask whether a planner who wants to maximize trade surplus should allow the use of private money or require the platform to use public money. We find that if both the platform and the legacy marketplace have identical matching technologies, private money is always inferior to public money from the planner's perspective. The reason is that with private money the platform attracts too many buyers relative to sellers (market tightness on the platform is too low) which lowers the trade probabilities both on the platform and on the legacy market. However, when the platform holds a technological advantage in matching buyers and sellers, allowing it to use private money can increase the probability of trade despite the countervailing effect on market tightness, and yield a superior equilibrium outcome for the planner.

Literature This paper most directly relates to works that model platform network effects. One line of the platform literature focuses on platforms with exogenous network effects. For example, Rochet and Tirole (2002, 2003, 2006) and Caillaud and Jullien (2001, 2003) examine pricing structures where platforms charge below marginal cost on one side of the market and above marginal cost on the other. Other works such as Spulber (1999, 2017), Weyl (2010), and Evans and Schmalensee (2016) also feature this pricing asymmetry. Armstrong (2006), in particular, studies competing platforms and the determinants of equilibrium pricing.

Another line of the platform literature considers endogenous network effects. In this context, Chen and Huang (2012) and Goos et al. (2014) model a single two-sided platform where search and matching frictions determine outcomes. These models typically do not feature prices below the marginal cost for sellers. Gautier et al. (2023) explores directed search where the platform acts as a middleman. Our paper contributes to this literature by examining platform competition through endogenous network effects, emphasizing the role of money in generating such effects via search and matching.

In addition, our paper contributes to the growing literature on platform tokens. Rogoff and You (2023) model token issuance as a mechanism to promote customer loyalty but show that non-tradable tokens - lacking monetary function - lead to higher platform profits. Brunnermeier and Payne (2023) propose a ledger-keeper framework in which enforcement of repayment relies on exclusion from future trades.

Sockin and Xiong (2023) view tokenization as a commitment device to prevent platforms from exploiting users. While tokens in their framework serve as a financing tool - especially for platforms with weak fundamentals - they do not function as a means of payment. In contrast, our paper integrates monetary economics with platform economics, including fee-setting and market tightness. Platforms compete with legacy systems by indirectly altering market tightness, generating seigniorage and charging entry fees to both buyers and sellers. Goldstein et al. (2024) also find that tokenizing a platform commits a firm to give up monopolistic rents associated with the control of the platform, leading to long-run competitive prices. This is because in a competitive secondary market for tokens, future buyers and sellers buy and sell tokens and hence put competitive pressure on the platform. In an extension, they study when two platforms compete a la Bertrand, in the presence of positive network effects, welfare might be lower than that of a tokenized platform. We differ from their approach by modelling the effect of money on two-sides market competition and resulting searching and matching dynamics. Furthermore, we incorporate behavioural insights in modelling the role of native token in platform competition by allowing for the possibility that inflation costs are less salient to consumers compared to direct fees. This bias is empirically documented by Liston-Heyes (2002), who finds that consumers systematically overestimate the purchasing power of air miles. A broader literature on salience, such as the review by Bordalo et al. (2022), finds that consumers often underweight non-salient features in decision-making. Supporting this, Blake et al. (2021) demonstrate in a field experiment that disclosing fees upfront can reduce the quantity of purchases. Relatedly, Shafir et al. (1997) discuss the concept of money illusion, which is further explored by Brunnermeier and Julliard (2008) in the context of housing prices.

Finally, our work contributes to the new monetarist literature Lagos and Wright (2005); Lagos et al. (2017) by embedding money in an environment with search frictions. We differ by modelling platform competition and endogenizing demand for private money, enriching the connection between payment friction and network effects.

The rest of the paper is organized as follows. Section 2 sets up the model and provides the equilibrium definition. Section 3 analyses equilibrium properties. Section 4 presents comparative statics and numerical exercises. Section 5 solves the planner's problem. Section 7 concludes.

2 The Model

2.1 The Environment

Time is discrete, lasts forever, and is indexed by $t \in \{0, 1, ...\}$. There are three types of agent: a measure $\bar{N}_b$ of consumers, a measure $\bar{N}_s$ of sellers, and two owners of the two trading marketplaces: a private platform P and a legacy market L, where the decentralized search and matching between buyers and sellers occurs.

We consider an economy where the discount factor between periods is $\beta \in (0,1)$ and each period is divided into two stages. In the first stage, the decentralized marketplaces (DM) are open to trade a perishable consumption good y that only sellers can produce at zero marginal cost. Buyers obtain u from consuming one unit and do not value more units. In each period, buyers and sellers are able to participate in one and only one of the trading marketplaces. When a buyer and a seller match, the transaction is executed using money: on platform P, trades are conducted using platform money, whereas on legacy market L, outside money is used. We also refer to this DM stage as the consumption goods market.

In the second stage, a centralized, frictionless settlement market (CM) is established. In this market, the owners of the trading platforms set buyer and seller fees, and agents decide which future DM consumption good marketplace to join, pay the corresponding fee, and rebalance their portfolios of platform and outside money. Consequently, the platform money – as a medium of exchange – is priced. We assume that all types of agents consume the perishable CM good, x, and can supply labor, h, to produce the good x via a linear production technology with a 1:1 ratio. All agents obtain utility $U(x,h)$ by consuming x of the CM good but incur dis-utility from labor. To simplify the exposition, we assume that $U(x,h) = x - h$. We also refer to this CM stage as money market.

Both market owners impose fees denoted by k_t^j to sellers and f_t^j to buyers, where $j \in \{P, L\}$ (with the fees measured in units of x) while the owner of the platform (P) also chooses the amount of additional platform money to issue. Sellers and buyers observe these fees, pay the fee associated with the market they choose to enter and adjust their money portfolios accordingly.

Under the assumption that consumption goods x and y are perishable, the only forms of money in this economy are platform money and outside money. We denote the money supply and the corresponding money price in each market by M_t^j and ϕ_t^j, respectively, where $j \in \{P, L\}$. In the legacy market, a central bank sets the money growth rate (that is, inflation) to achieve macroeconomic objectives. In contrast, the platform determines the growth rate of its own money to maximize profit.[3] In practice, the platform sets interest

[3]Since fewer people hold cash and use bank accounts for digital payments, this means that central bank influences aggregate money supply indirectly via affecting banks' deposit rates.

rates on its digital wallet - effectively expanding its money supply - and occasionally issues coupons or vouchers, which are equivalent to helicopter money.

In what follows, we specify the law of motion for the money supply in each marketplace:

$$M_{t+1}^{j} = \mu^{j} M_{t}^{j}. \tag{1}$$

We assume that $\mu^j > \beta$ is set in such a way that the money depreciation rate exceeds the discount factor; otherwise, agents' demand for money would be infinite. Hence, a seller doesn't carry any money across periods. A buyer doesn't carry the money of the market where she doesn't trade and carries the optimal amount of money necessary to trade in DM. Following the new monetarist literature, we focus on a stationary equilibrium where in steady state $M_t^j \phi_t^j$ is constant. Figure 2.1 summarizes the aforementioned events in this economic environment.

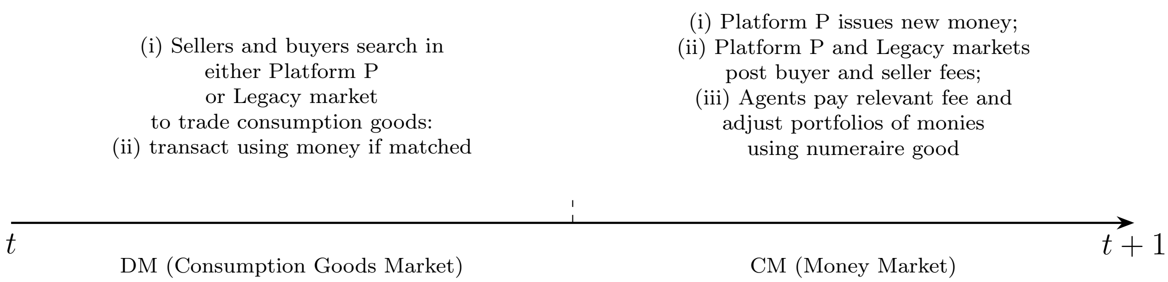


Figure 1: Timeline

2.2 Centralized Market

We denote an agent's value function in the CM by W_t and in the DM by V_t. The CM value function of a buyer denoted by subscript b is given by:

$$W_{b,t}(m_{b,t}^{P}, m_{b,t}^{L}) = \hat{\Pi}_{b,t}^{P} W_{b,t}^{P}(m_{b,t}^{P}, m_{b,t}^{L}) + \hat{\Pi}_{b,t}^{L} W_{b,t}^{L}(m_{b,t}^{P}, m_{b,t}^{L}), \tag{2}$$

where $\hat{\Pi}_{b,t}^{j}$ is buyer's probability of choosing market j in the CM. This formulation allows buyers to bring both types of money from CM to DM but in fact buyers will bring at most one type of money (or none) since holding money is costly if not for transaction purposes.

That is, buyers pay only the entry fee (f) and purchase the relevant money for the chosen marketplace. The continuation value of going to market j is:

$$W^j_{b,t}(m^P_{b,t}, m^L_{b,t}) = \max_{x_t, h_t, m^j_{b,t+1} \geq 0} x_t - h_t + \beta V^j_{b,t+1}(m^j_{b,t+1}, 0) \tag{3}$$

$$\text{s.t.} \qquad x_t + f_j + \phi^j_t m^j_{b,t+1} \leq h_t + \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t}, \tag{4}$$

where buyers choose consumption, labour, and money holding amount optimally. By substituting for $x_t - h_t$, we obtain

$$W^j_{b,t}(m^P_{b,t}, m^L_{b,t}) = \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t} + W^j_{b,t}(0,0),$$

where

$$W^j_{b,t}(0,0) = \max_{m^j_{b,t+1} \geq 0} -\phi^j_t m^j_{b,t+1} - f_j + \beta V^j_{b,t+1}(m^j_{b,t+1}, 0) \tag{5}$$

Thus, a buyer's optimization problem in CM is not history dependent and her value function in CM can be written as

$$W_{b,t}(m^P_{b,t}, m^L_{b,t}) = \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t} + \left[\hat{\Pi}^P_{b,t} W^P_{b,t}(0,0) + \hat{\Pi}^L_{b,t} W^L_{b,t}(0,0)\right]. \tag{6}$$

Similarly, a seller's value function denoted by subscript s can be written as:

$$W_{s,t}(m^P_{s,t}, m^L_{s,t}) = \phi^P_t m^P_{s,t} + \phi^L_t m^L_{s,t} + \left[\hat{\Pi}^P_{s,t} W^P_{s,t}(0,0) + \hat{\Pi}^L_{s,t} W^L_{s,t}(0,0)\right]. \tag{7}$$

where k is the seller entry fee and

$$W^j_{s,t}(0,0) = \max_{m^j_{s,t+1} \geq 0} -\phi^j_t m^j_{s,t+1} - k_j + \beta V^j_{s,t+1}(m^j_{s,t+1}, 0). \tag{8}$$

2.3 Decentralized Market

Trading in a decentralized marketplace is subject to search frictions that we capture with a matching function. For any market j with N_s sellers and N_b buyers, the matching function $Q^j(N_s, N_b)$ represents the total number of successful matches in j. We assume that the function $Q^j(\cdot)$ exhibits the constant-return-to-scale property. We also assume that Q^j is concave in both variables. It is also useful to define the market tightness in market j, denoted by n_j, as the ratio of sellers to buyers in this market, i.e., $n_j \equiv N_s/N_b$. Using this definition, the probability that a *buyer* successfully finds a match in market j is related to the market tightness in the following manner:

$$a_{jb}(n_j) \equiv \frac{Q^j(N_s, N_b)}{N_b} = Q^j(n_j, 1) \tag{9}$$

Similarly, the probability that a *seller* finds a match in market j is:

$$a_{js}(n_j) \equiv \frac{Q^j(N_s, N_b)}{N_s} = \frac{1}{n_j} Q^j(n_j, 1) \tag{10}$$

We assume that the platform has (weakly) better matching technology so that for any market tightness n buyers and sellers are (weakly) more likely to find a match on the platform: $a_{Pb}(n) \geq a_{Lb}(n)$ and $a_{Ps}(n) \geq a_{Ls}(n)$. Furthermore, the marginal increase (decrease) in matching probability is weakly larger for buyers (sellers) on the platform if market tightness increases: $a'_{Pb}(n) \geq a'_{Lb}(n)$ ($a'_{Ps}(n) \leq a'_{Ls}(n)$).

Conditional on a successful match, we assume that buyer and seller bargain over the terms of trade. The literature uses two solution concepts to obtain a bargaining outcome. These concepts are the generalized Nash bargaining (Nash (1950, 1953)) and Kalai's proportional bargaining (Kalai (1977)). When the buyers are not liquidity constrained, the two approaches yield the same solution but become distinct when buyers are liquidity constrained which is critical in applications to money (Hu and Rocheteau (2020)). We adopt the latter approach for three reasons. First, Kalai's proportional bargaining is shown to be more empirically relevant over the generalized Nash solution when the buyers are liquidity constrained (See the experimental evidence in Duffy et al. (2021)). Second, under proportional bargaining buyers use money even when the sellers do not have cost of production which we find reasonable. And third, proportional bargaining provides expositional clarity.[4]

Suppose that the seller's cost of producing x units of the good is xc where $0 \leq c < u$. The buyer obtains utility xu if the seller produces $0 \leq x \leq 1$ and does not value more than

[4]We provide the generalized Nash bargaining solution in the appendix and show our main results are robust to the bargaining approaches.

one unit.[5] We can also interpret x as probability of trade.

First, suppose the buyer is not liquidity constrained. Under Kalai's proportional bargaining with bargaining parameter γ, the optimization problem is to choose the terms of trade q and the amount to produce x to maximize:

$$\begin{aligned} \max_{x,q} \quad & xu - q \\ \text{subject to} \quad & \tfrac{xu-q}{q-xc} = \tfrac{\gamma}{1-\gamma} \text{ and } x \leq 1. \end{aligned}$$

Solving for q from the constraint and plugging into the objective we see that objective is increasing in x. Hence, the solution without liquidity constraint is $x^* = 1$ and $q^* = (1-\gamma)\,u + \gamma c$.[6]

Now, suppose the real value of the money that the buyer brings to the DM is $\bar{q}$ and the buyer is liquidity constrained so that $\bar{q} \leq q^*$. Now the problem becomes:

$$\max_{x,q} \quad xu - q \tag{11}$$

$$\text{subject to} \quad \tfrac{xu-q}{q-xc} = \tfrac{\gamma}{1-\gamma} \text{ and } q \leq \bar{q}. \tag{12}$$

Solving for q and plugging into the objective, we can rewrite this problem as:

$$\max_{x} \quad x\,(u-c) \tag{13}$$

$$\text{subject to} \quad x((1-\gamma)u + \gamma c) \leq \bar{q}. \tag{14}$$

The solution is

$$x^{**} = \frac{\bar{q}}{(1-\gamma)\,u + \gamma c} \text{ and } q^{**} = \bar{q} \tag{15}$$

The buyer's utility under this solution is $(\gamma\,(u-c)\,/((1-\gamma)\,u+\gamma c))\,\bar{q}$. Now, let's step back and ask how much liquidity the buyer brings to the DM. Buyer's utility is increasing in $\bar{q}$ (up to q^*). Hence, the buyer's optimal liquidity, or the price of the DM good, is $q^* = (1-\gamma)\,u + \gamma c$. Consequently, the buyer's utility is $\gamma\,(u-c)$ and the seller's utility is

[5] Alternatively, we can assume that the seller has the production capacity of one unit.

[6] When buyers are not liquidity constrained, the Kalai and the Nash bargaining solutions are the same.

$(1-\gamma)(u-c)$. For expositional clarity, in the rest of the paper, we set $c=0$. Thus, the resulting real price for the good y is: $p^j\phi^j = u(1-\gamma)$.

2.4 Buyers in the DM

Given the matching probabilities, we obtain the DM value function for each individual buyer who chooses to trade on market $j \in \{P, L\}$ as

$$V^j_{b,t}(m^j_{b,t}, 0) = a_{jb}(n_{jt})[u + W_{b,t}(m^j_{b,t} - p^j_t, 0)] + (1 - a_{jb}(n_{jt}))W_{b,t}(m^j_{b,t}, 0). \tag{16}$$

The first term states that, conditional on being matched, the buyer gains utility u and carries the after-trade money balance $m^j_{b,t} - p^j_t$ into the centralized market (CM). The second term captures the case where the buyer simply carries over their money to the CM if not matched. By plugging for $W_{b,t}(\cdot)$, we simplify the value function as follows:

$$V^j_{b,t}(m^j_{b,t}, 0) = a_{jb}(n_{jt})[u - \phi^j_t p^j_t] + \phi^j_t m^j_{b,t} + W_{b,t}(0, 0). \tag{17}$$

In the steady state, the real price of the DM good is set by the bargaining rule: $\phi^j_t m^j_t = \phi^j_t p^j_t = u(1-\gamma), \forall t$ and buyers bring the exact amount of money to pay for it. Plugging the real price, the money holding, and (17) into (5), we obtain:

$$W^j_b(0,0) = -\underbrace{f_j}_{\text{Fee}} + \underbrace{\beta a_{jb}(n_j)\gamma u}_{\text{Utility from trade}} + \underbrace{(\beta - \mu_j)(1-\gamma)u}_{\text{Cost of holding money}} + \underbrace{\beta W_b(0,0)}_{\text{Continuation value}} \tag{18}$$

2.5 Buyers' marketplace choice

Next, we formalize the buyer's marketplace selection as a random discrete choice problem. In this framework, the buyer's decision is influenced not only by the anticipated value each marketplace offers but also by an idiosyncratic choice shock and a behavioural bias toward inflation. In our interpretation, a buyer's actual experienced payoff is $W^j_b(0,0)$ but at the choice stage each buyer l uses their perceived payoffs $\hat{W}^j_{l,b}$ plus a white noise η_{jl} to choose between the two marketplaces. The idiosyncratic noise term η_{jl} captures the randomness of the choice stage. Formally, the perceived payoff of buyer l is given by:

$$\hat{W}^j_{l,b}(\xi) = -f_j + \beta a_{jb}(n_j)\gamma u + (\beta - \xi\mu_j)(1-\gamma)u + \beta W(0,0) \tag{19}$$

where the parameter $\xi \in [0,1]$ captures the salience of inflation (from either outside money or platform money) to the buyer. That is, the buyer does not fully account for inflation costs when choosing between marketplaces. Thus, the perceived advantage of platform (P) over legacy (L) marketplace is

$$\Delta_b \equiv \underbrace{\beta\left(a_{Pb}(n_P) - a_{Lb}(n_L)\right)\gamma u}_{\text{Utility from trade difference}} + \underbrace{\xi(\mu_L - \mu_P)}_{\text{Inflation cost difference}} + \underbrace{(f_L - f_P)}_{\text{Fee difference}}. \tag{20}$$

It is important to note that the salience of inflation can either help or hinder platform P's ability to attract buyers. When the legacy marketplace faces higher inflation set by the central bank, any inflation cost savings offered by platform P become less compelling to buyers, who tend to discount these savings. Conversely, if the legacy marketplace experiences lower inflation, platform P can afford to impose a higher inflation rate, and buyers may not fully account for this increased inflation in their decision-making.

Hence, the probability of a buyer choosing market P in the CM can be determined by the following attraction function $\Pi_b(\cdot)$:

$$\Pi_b(\Delta_b) = \Pr\left\{l : \Delta_b \geq \eta_{Ll} - \eta_{Pl}\right\}. \tag{21}$$

This attraction function also yields the fraction of buyers choosing platform P.

2.5.1 Sellers

Sellers do not want to hold any additional money in the CM and would convert all the money they have received (if matched) to CM goods immediately to avoid facing inflation cost in the next period. A seller's value function in steady state if he chooses to enter market j to trade is then:

$$W^j_s(0,0) = -\underbrace{k_j}_{\text{Fee}} + \underbrace{\beta a_{js}(n_j)(1-\gamma)u}_{\text{Utility from trade}} + \underbrace{\beta W_s(0,0)}_{\text{Continuation value}} \tag{22}$$

The attraction function of a seller for platform P over legacy L can be similarly defined as $\Pi_s(\Delta_s)$ where

$$\Delta_s = \beta \left(a_{Ps}(n_P) - a_{Ls}(n_L)\right)(1-\gamma)u + (k_L - k_P), \tag{23}$$

and a seller also faces an idiosyncratic choice shock. The attraction function $\Pi_s(\Delta_s)$ yields the fraction of sellers who trade on P.

2.5.2 Legacy and Platform Owners

Marketplaces often use multi-channel marketing and set fees independently to customers who come to the marketplace through different channels. Although buyers and sellers enter the marketplace through various channels, they all face search frictions and find trading partners using the same matching technology. As a result, we assume that marketplace owners take matching probabilities (or equivalently equilibrium market tightness) as given and maximize their revenue by optimally setting the entry fees for sellers and buyers ($k_j \geq 0$, $f_j \geq 0$ where $j \in \{P, L\}$) and choosing the rate of money growth in the case of platform P.[7]

We first study platform P owner's optimization problem who chooses the sellers' entry fee k_P, money growth rate μ_P, and the buyers' entry fee f_P to maximize:

$$\underbrace{\bar{N}_s \Pi_s\left(\Delta_s\right) k_P}_{\text{Fee revenue from sellers}} \quad + \quad \underbrace{\bar{N}_b \Pi_b\left(\Delta_b\right) f_P}_{\text{Fee revenue from buyers}} \quad + \underbrace{\left(M_{t+1}^P - M_t^P\right) \phi_t^P}_{\text{seigniorage}}. \tag{24}$$

where M_t^P is the supply of platform money at t, and Δ_b and Δ_s are given by (20) and (23).

Using the market clearing condition for platform money ($m_{b,t}^P \phi_t^P = u\left(1-\gamma\right)$), we obtain

$$M_t^P = \bar{N}_b \Pi_b\left(\Delta_b\right) m_{b,t}^P = \bar{N}_b \Pi_b\left(\Delta_b\right) \frac{u\left(1-\gamma\right)}{\phi_t^P}. \tag{25}$$

Combining this expression with (24), the objective function of the platform P owner becomes:

$$\bar{N}_s \Pi_s\left(\Delta_s\right) k_P + \bar{N}_b \Pi_b\left(\Delta_b\right) \left[u\left(1-\gamma\right)\left(\mu_P - 1\right) + f_P\right]. \tag{26}$$

[7] Thinking of a marketplace as a multi-channel market that pools its revenues is reminiscent of the "Lucas family" (Lucas (1990)) and simplifies the equilibrium construction.

We are now ready to take first order conditions with respect to fees and rate of money growth. The first order condition with regard to the seller fee k_P gives:

$$k_P \;\; = \tfrac{\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)}. \tag{27}$$

That is, the fee is set so that the marginal increase in the seller fee revenue from the fee levied on all sellers who choose to enter is equal to the marginal loss from those choose not to enter.

The first order condition with regards to the buyer fee f_P gives:

$$\Pi_b\left(\Delta_b\right) - \Pi_b'\left(\Delta_b\right)\left[u\left(1-\gamma\right)\left(\mu_P - 1\right) + f_P\right] \leq 0, \text{ with equality if } f_P > 0. \tag{28}$$

The first order condition with regards to the rate of money growth μ_P gives:

$$\Pi_b\left(\Delta_b\right) - \Pi_b'\left(\Delta_b\right)\left[u\left(1-\gamma\right)\left(\mu_P - 1\right) + f_P\right]\xi \leq 0, \text{with equality if } \mu_P > 1. \tag{29}$$

Notice that for $\xi < 1$, we have the optimal solution as:

$$f_P \;\; = \;\; 0 \tag{30}$$

$$\mu_P \;\; = \;\; 1 + \frac{1}{\xi\left(1-\gamma\right)u}\frac{\Pi_b\left(\Delta_b\right)}{\Pi_b'\left(\Delta_b\right)} \tag{31}$$

That is, when buyers do not fully account for inflation costs, the owner of platform P prefers to charge buyers via an inflation mechanism rather than by imposing a direct fee. Conversely, if buyers fully internalize the inflation cost, the owner becomes indifferent between the two methods. In this case, his primary concern is to extract an optimal combined charge from buyers, which is given by $u\left(1-\gamma\right)\left(\mu_P - 1\right) + f_P$, where μ_P represents the inflation rate and f_P is the buyer fee. This leads to the following lemma.

Lemma 1 *Platform P prefers to charge buyers via inflation rather than by setting a direct fee when $\xi < 1$ and is indifferent between the two methods when $\xi = 1$.*

It is important to note that the owner of platform P strictly prefers using platform money over outside money if either $\mu_L > 1$ or $\xi < 1$ or both. Using platform money instead of outside money gives the platform two advantages. First, the platform can set a different inflation rate than the central bank to control the cost of inflation experienced by the buyers on the platform. Hence, the platform strictly prefers to use platform money as long as $\xi < 1$ since it is more effective to charge buyers through inflation than fees. This advantage is demonstrated by the perceived advantage of platform over the legacy marketplace for buyers expressed in eq. (20). Second, it earns seigniorage from issuing new money when $\mu_P > 1$. When $\mu_L > 1$, adapting outside money would mean losing this seigniorage income and the platform strictly prefers adopting its own money on its marketplace. Only when both $\mu_L = 1$ and $\xi = 1$, the platform would be indifferent between using outside money and platform money since both advantages of issuing its own money disappear. The following lemma summarizes this result.

Lemma 2 *Platform P prefers to adopt its own money if either $\mu_L > 1$, or $\xi < 1$, or both.*

We next study legacy market L owner's optimization problem which is simpler since legacy market L's owner does not have control over the outside money supply. Legacy market chooses f_L and k_L to maximize:

$$\bar{N}_s \left(1 - \Pi_s \left(\Delta_s\right)\right) k_L + \bar{N}_b \left(1 - \Pi_b \left(\Delta_b\right)\right) f_L \tag{32}$$

The two first order conditions are:

$$k_L = \frac{1 - \Pi_s \left(\Delta_s\right)}{\Pi_s' \left(\Delta_s\right)} \tag{33}$$

$$f_L = \frac{1 - \Pi_b \left(\Delta_b\right)}{\Pi_b' \left(\Delta_b\right)} \tag{34}$$

where Δ_b and Δ_s are given by (20) and (23).

2.6 Equilibrium Definition

Definition 1 *A stationary equilibrium consists of market tightness measures on the two platforms (n_P^*, n_L^*), platform entry fees for the buyers and the sellers (f_P^*, k_P^*), platform's money growth policy μ_P^*, and legacy market entry fees for the buyers and the sellers (f_L^*, k_L^*) such that*

1. *In the CM buyers and sellers optimally choose which market to enter (and hold money of that market for trade)*

2. *Given* $(n_P^*, n_L^*, f_L^*, k_L^*)$ *and buyers' and sellers' entry decisions, platform's profit maximizing fees are* $f_P = f_P^*$ *and* $k_P = k_P^*$ *and its optimal money growth policy is* $\frac{M_{t+1}^P}{M_t^P} = \mu_P^*$.

3. *Given* $(n_P^*, n_L^*, f_P^*, k_P^*, \mu_P^*)$ *and buyers' and sellers' entry decisions, legacy market's profit maximizing fees are* $f_L = f_L^*$ *and* $k_L = k_L^*$.

4. *Market tightness on the two markets are given by*

$$
\begin{aligned}
\frac{\overline{N}_s \Pi_s(\Delta_s)}{\overline{N}_b \Pi_b(\Delta_b)} &= n_P^* \\
\frac{\overline{N}_s(1-\Pi_s(\Delta_s))}{\overline{N}_b(1-\Pi_b(\Delta_b))} &= n_L^*
\end{aligned}
$$

3 Equilibrium Properties

In the remainder of the paper we assume that the attraction functions take the following form: $\Pi_b(\Delta_b) = \left[1+\exp\left(-\frac{\Delta_b}{\sigma_b}\right)\right]^{-1}$ and $\Pi_s(\Delta_s) = \left[1+\exp\left(-\frac{\Delta_s}{\sigma_s}\right)\right]^{-1}$. These functional forms are standard in discrete choice where shocks to payoffs follow Gumbel distribution with scale parameters σ_b for buyers and σ_s for sellers.[8]

We first establish a lemma that links the comparison of market tightness on the platform versus the legacy market to the relative attractiveness of the platform to the sellers versus buyers.

Lemma 3 *The platform has lower market tightnesses than the legacy market (i.e. seller buyer ratio is lower on the platform) iff its relative attractiveness is lower for sellers than for buyers, i.e.,*

$$
n_P \leq n_L \Leftrightarrow \frac{\Delta_s}{\sigma_s} \leq \frac{\Delta_b}{\sigma_b}. \tag{35}
$$

[8]It becomes difficult to attract buyers (sellers) as the scale parameter increases. When σ_b (σ_s) approaches ∞, the likelihood of buyers (sellers) to enter the platform versus the legacy market approaches half and half regardless of Δ_b (Δ_s). At the other extreme, when σ_b (σ_s) approaches 0, all buyers (sellers) go to the platform if $\Delta_b > 0$ ($\Delta_s > 0$) and to the legacy if $\Delta_b < 0$ ($\Delta_s < 0$).

This lemma allows us to show that platform P has a unique advantage in attracting buyers by controlling its own money supply and hence offers sellers a higher matching probability.

Proposition 1 *In equilibrium, sellers are more likely to be matched on the platform than on legacy market, i.e.* $a_{Ps}(n_P) > a_{Ls}(n_L)$*.*

Note that this result holds even when matching technology is symmetric across two marketplaces and $\xi = 1$. The key insight is that platform P has a distinct advantage: it can control its own inflation and collect seigniorage income, whereas the legacy market cannot manage inflation, does not benefit from seigniorage, and passes the full inflation cost onto its buyers. Under the condition $\xi = 1$, platform P can choose to regulate either inflation or buyer fees. For example, if platform P opts to control fees, it can set $\mu_P = 1$ (i.e., maintain zero inflation) and adjust its fee structure to attract more buyers. When $\xi < 1$, Lemma 1 implies that platform P optimally sets a zero fee to buyers and can charge them a higher "fee" via the inflation mechanism and attract the same amount of (if not more) buyers since inflation is less salient to buyers.

A corollary of this result is that if two marketplaces have similar matching technologies, platform P has lower seller to buyer ratio than the legacy marketplace.

Corollary 1 *If* $a_{Ps}(n) - a_{Ls}(n) \geq 0$ *is small enough for all* n *then* $n_P < n_L$*.*

This follows directly from the previous proposition: if $a_{Ps} = a_{Ls}$ then $a_{Ps}(n_P) > a_{Ls}(n_L) \Rightarrow n_P < n_L$. By continuity this must also hold if a_{Ps} is close to a_{Ls}.

In search and matching models, cross-group positive network externalities are common: an increase in the number of buyers improves the matching probabilities for sellers, and vice versa. Since platform P has an advantage in attracting buyers from legacy market, it initiates the positive externalities from the buyer side that improves the seller matching probability. This dynamic enables platform P to leverage its advantage by drawing more sellers into its market while also charging sellers a higher fee. This finding is summarized in the next proposition.

Proposition 2 *Platform P charges a higher seller fee than the legacy market, i.e.,* $k_P > k_L$*, and attracts more sellers than the legacy market, i.e.* $\Delta_s > 0$*.*

The following proposition demonstrates the effect of the feedback loop of cross-group positive externalities in search and matching models. As more sellers are drawn in by the better matching probability on platform P, the buyer's matching probability is improved and more buyers choose to move from legacy market to platform P, creating a reinforced feedback loop.

Proposition 3 *There are more buyers on the platform.*

The next proposition shows that when legacy inflation is below a threshold platform inflation is below the legacy inflation and otherwise it is above.

Proposition 4 *Suppose $\xi < 1$. There is a threshold value $\hat{\mu}_L > 1$ such that if $\mu_L \lesseqqgtr \hat{\mu}_L$ then $\mu_P \gtreqqless \mu_L$.*

It is clear that $\mu_L = 1$, platform sets $\mu_P > 1$ to generate seigniorage. This proposition follows because as legacy inflation μ_L goes up, platform inflation μ_P increases at a rate less than one and eventually falls below the legacy inflation. To see why, note that as legacy inflation rises, the legacy market lowers its buyer fee to retain customers. Consequently, the platform owner must take into account both the higher legacy inflation and the reduced legacy buyer fee, resulting in less than on a one-for-one increase in its inflation.

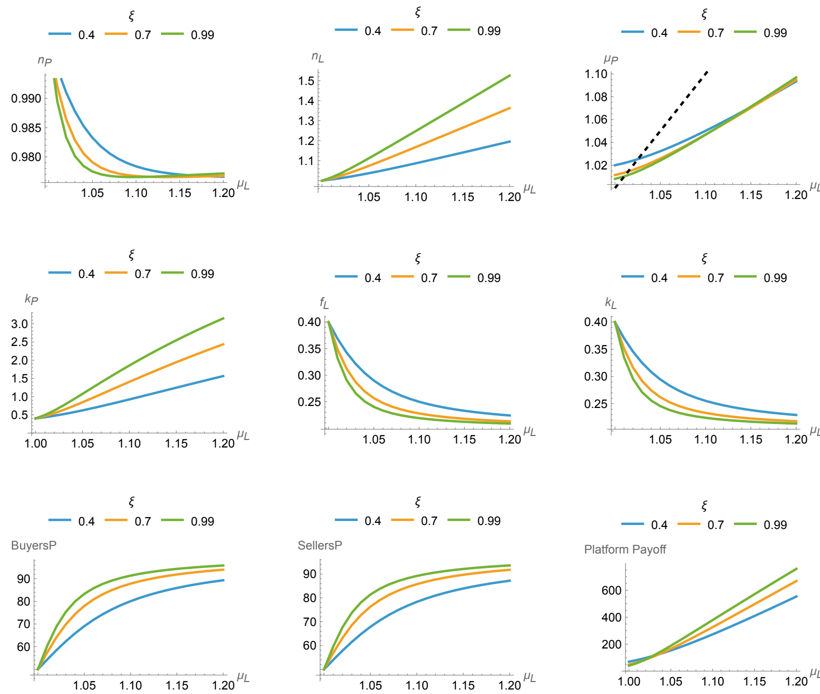
4 Comparative Statics: Numerical Exercises

4.1 Identical Matching Technology

In our initial set of numerical analysis, we assume that the platform and legacy market share the same matching technology. Hence, the numerical findings in this subsection focus exclusively on the platform's unique advantage over the legacy market - its ability to control its money supply and collect seigniorage.

Figure 2 summarizes our key findings regarding buyers' inflation salience by illustrating how variations in inflation salience affect equilibrium outcomes. Each graph plots legacy inflation on the x-axis against one outcome variable on the y-axis (e.g. market tightness, inflation rates, seller fees, the number of buyers and sellers, the platform owner's payoff, and buyer and seller fees in the legacy market). In every graph, three lines represent different levels of inflation salience (e.g., 0.4, 0.7, and 0.99).

Figure 2: Legacy Inflation: Salience



The graphs have μ_L on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\xi = 0.4$ in the blue line, $\xi = 0.7$ in the orange line, and $\xi = 0.99$ in the green line. Legacy inflation μ_L in the black dashed line. Parameters: $\alpha_P = 0.1$; $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 100$; $\sigma_b = 0.2$; $\gamma = 0.5$; $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 100$.

In the third graph on the top row, we observe that platform inflation rises as legacy inflation increases. This response is less than one-to-one, as its slope is smaller than the 45-degree dashed line. Proposition 4 has explained that this subdued response occurs because, as fiat inflation rises, the legacy marketplace lowers its buyer fee to retain customers (see the middle graph in the middle row). Consequently, the platform owner must raise its own inflation in reaction to both the higher legacy inflation and the reduced buyer fee but not on a one-for-one basis since the platform does not charge a buyer fee. Therefore, for low legacy inflation, the platform inflation exceeds the legacy inflation, and for high legacy inflation, the opposite holds.

The first and the second graphs on the bottom row demonstrate that, as legacy inflation rises, the platform attracts more buyers and sellers. This attraction is the strongest when buyers fully account for inflation costs (i.e., with a higher ξ). In other words, if buyers are less sensitive to inflation, it becomes more challenging for the platform to attract them (as evidenced by the green lines lying above the blue lines in these graphs). Nonetheless, the platform's ability to control its own inflation and collect seigniorage income enables it to reduce buyer's participation costs, thereby drawing a large fraction of buyers and sellers away from the legacy market.

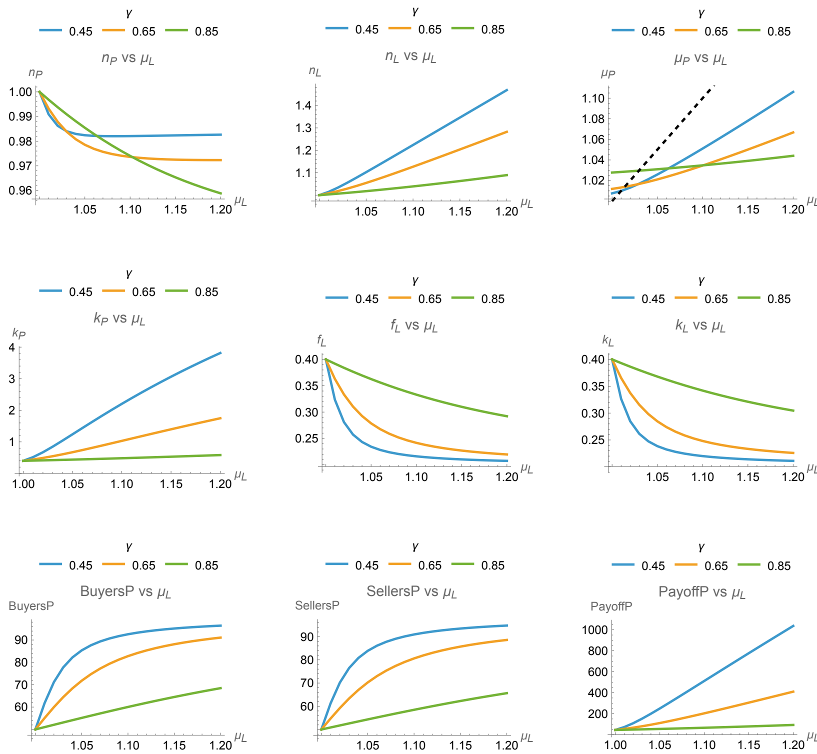
This advantage, in turn, allows the platform to charge a higher seller fee - since its increased buyer base improves matching probabilities for sellers (see k_P in the first graph, middle row) - while the legacy market is forced to lower its seller fee (k_L in the third graph, middle row). As a result, the market tightness (measured by the seller-buyer ratio) is lower on the platform (n_P, first graph, top row) and higher in the legacy market (n_L in the second graph, top row) as legacy inflation increases.

Finally, the third graph on the bottom row illustrates the double-edged effect of inflation salience. On the one hand, low inflation salience allows the platform to set higher inflation and generate more seigniorage. On the other hand, it makes the legacy inflation appear to be less costly to buyers. When legacy inflation is low, the first effect dominates and the platform's payoff is higher under low inflation salience (the blue line lies above the green line). As the legacy inflation rises, eventually the second effect dominates and the platform earns more under high inflation salience (the green line then lies above the blue).

Using the same set of graphs and parameters (with $\xi = 0.8$), Figure 3 summarizes how variations in buyer's bargaining power affect equilibrium outcomes. In each graph, three lines represent different levels of buyer's bargaining power (for example, 0.45, 0.65, and 0.85). The figure shows that as buyers bargaining power decreases, the platform's profit increases, for instance, in the third graph (bottom row), the blue line (indicating the lowest bargaining power) lies above the others.

Intuitively, when buyers have low bargaining power, they must pay higher prices for DM goods from sellers. As a result, they are required to hold more platform money for their transactions, which in turn gives the platform a greater advantage over the legacy market.

Figure 3: Legacy Inflation: Bargaining Power



The graphs have μ_L on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\gamma = 0.45$ in the blue line, $\gamma = 0.65$ in the orange line, and $\gamma = 0.85$ in the green line. Legacy inflation μ_L in the black dashed line. Parameters: $\alpha_P = 0.1$; $\alpha_L = 0.1$; $\xi = 0.8$; $\rho = 0.5$; $\beta = 0.9$; $u = 120$; $\sigma_b = 0.2$; $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 100$.

We see this advantage in the third graph on the third row where platform's payoff is higher when the buyers' bargaining power is lower. However, as the third graph on the first row show bargaining power has a subtle impact on platform inflation. When legacy inflation is low, platform faces stronger competition and sets lower inflation when buyers hold more money (i.e., the blue line is below all other lines). As legacy inflation increases, this competitive pressure eases and the platform charges higher inflation when buyers hold more money to generate more seigniorage revenue (i.e., the blue line is above all other lines).

Platform's enhanced competitive advantage under low buyer bargaining power means that the platform is able to attract more buyers and, in turn, more sellers (see the first and the second graph on the third row). This advantage also allows the platform to charge sellers higher fees. As a result, the legacy market faces more competition for both buyers and sellers and must set lower fees for buyers and, surprisingly also for sellers (In both the second and third graphs on the second row, the blue line is the lowest and the green line is the highest). Finally, the market tightness is higher for legacy under low bargaining power but the effect on market tightness on the platform is more complex due to the competing effects.

4.2 Better Platform Matching Technology

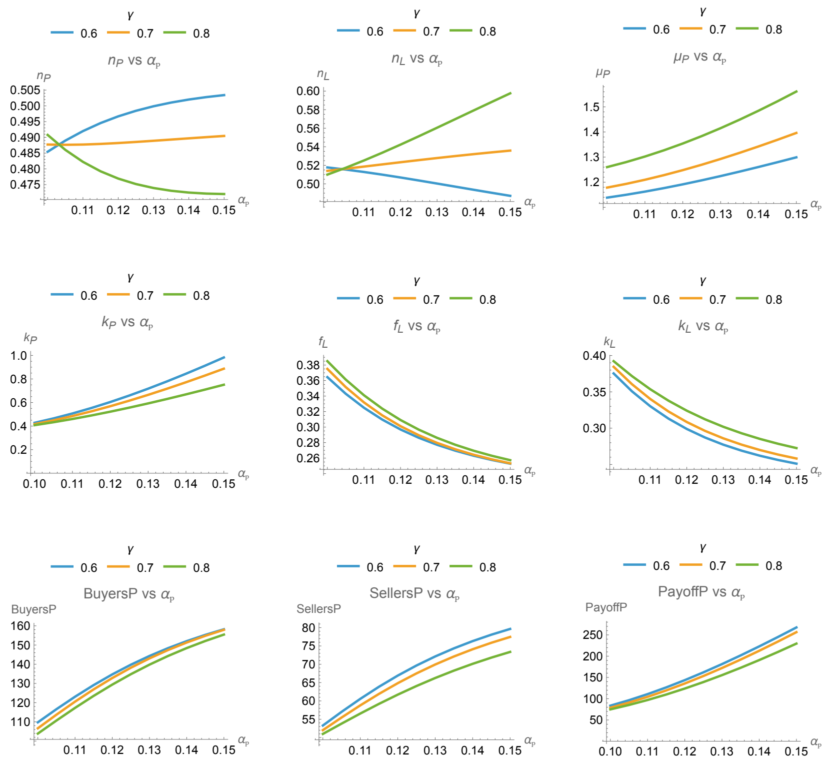
Next, we study the impact of better matching technology. As the platform matching technology improves, the likelihood of matches and the expected trade surplus generated on the platform both increase, introducing additional tradeoffs relative to the identical technology case. We use either buyer bargaining power γ or inflation salience ξ to measure the extent of the advantage of platform over legacy by having private money and α_P to measure the superiority of platform matching technology.

Figure 4 examines how variations in buyer bargaining power affect equilibrium outcomes. In these graphs, α_P is plotted on the x-axis while the y-axis represents a specific outcome variable. Each graph includes three lines corresponding to different levels of γ (e.g., 0.6, 0.7, and 0.8).

Our findings indicate that as the platform's matching technology improves, several key variables increase, including the fraction of buyers on the platform, the number of sellers attracted, the seller fee, platform inflation, and the platform owner's payoff. At the same time, the seller-to-buyer ratio (market tightness) might increase or decreases with further technological improvements. This pattern suggests that while the platform earns additional seigniorage income by attracting more buyers through its private money system, superior technology also boosts the number of matches and enables higher seller fee extraction. The combination of these effects makes it more profitable for the platform to adjust the balance between buyers and sellers.

Moreover, buyer bargaining power plays a crucial role in these dynamics. We see in the third graph on the first row that the sensitivity of platform inflation to improvements in technology is lower when buyer bargaining power is lower. This is because as the matching technology improves, most of the increase in expected gains from trade accrue to the sellers

Figure 4: Bargaining Power with Better Platform Technology



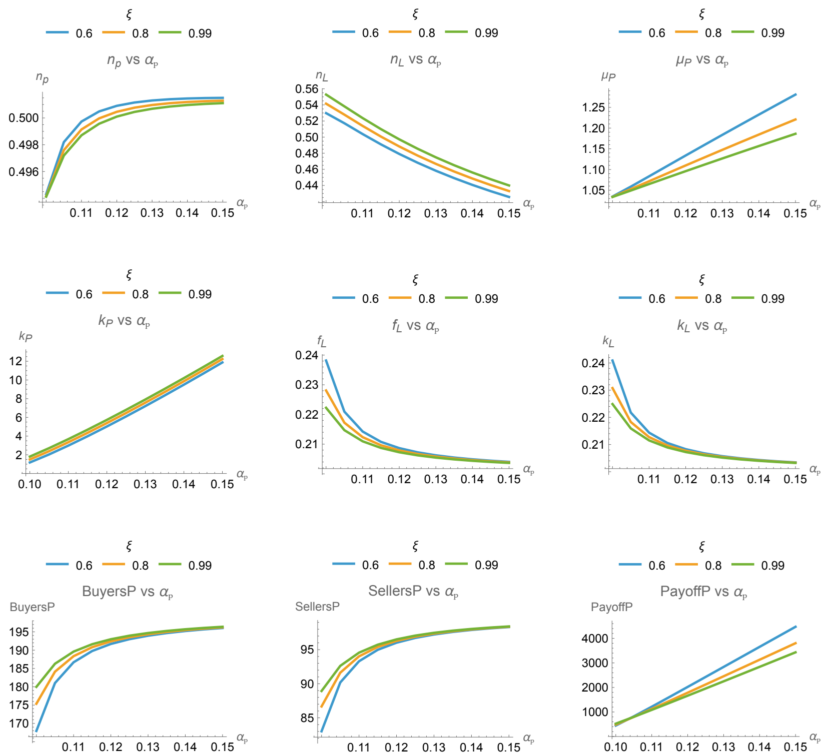
The graphs have α_P on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\gamma = 0.6$ in the blue line, $\gamma = 0.7$ in the orange line, and $\gamma = 0.8$ in the green line. Parameters: $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 10$; $\sigma_b = 0.2$; $\xi = 0.8$; $\mu_L = 1.05$, $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 200$.

when the buyers bargaining power is lower. As a result, the platform raises its inflation at a lower rate.

Next, we examine how variations in buyer's inflation salience affect equilibrium outcomes in Figure 5. Each graph plots bargaining power on the x-axis against one outcome variable on the y-axis. In every graph, three lines represent different levels of ξ (e.g., 0.6, 0.8, and 0.99).

Figure 5 offers an additional insight: improved matching technology amplifies the platform's advantage in controlling its own money supply. This effect is most evident in the third graph on the first row, which shows that as matching technology improves, the platform's inflation rate rises more rapidly when inflation salience is low. Consequently, even when

Figure 5: Inflation Salience with Better Platform Technology



The graphs have α_P on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\xi = 0.6$ in the blue line, $\xi = 0.8$ in the orange line, and $\xi = 0.99$ in the green line. Parameters: $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 120$; $\sigma_b = 0.2$; $\gamma = 0.5$; $\mu_L = 1.05$, $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 200$.

legacy inflation is high, the platform earns greater revenue under low inflation salience if it has superior matching technology (see the third graph on the last row).

This result contrasts with the case of identical matching technology (refer to the third graph on the bottom row in Figure 2). In that scenario, high legacy inflation combined with low inflation salience disadvantages the platform, because legacy buyers do not fully internalize the true cost of inflation. In the case of superior matching technology, however, the platform can offset this drawback by increasing its inflation rate more aggressively, thereby collecting more seigniorage income.

5 The Planner's Problem

In this section, we analyze social welfare by studying the planner's solution. The planner's objective is to maximize the total utility for all buyers, sellers, and the owners of marketplaces in this economy. Given that all the transactions in the CM as well as the payments from buyers to sellers are transfers between agents, maximization of total utility is equivalent to maximization of the total surplus from trade in DM.[9] Hence planner's problem can be stated as:

$$\begin{aligned}\max_{\Pi_b,\Pi_s} \quad & \overline{N}_b \gamma u\left[\Pi_b a_{Pb}(n_P) + (1-\Pi_b) a_{bL}(n_L)\right] \\ & + \overline{N}_s \left(1-\gamma\right) u\left[\Pi_s a_{Ps}(n_P) + (1-\Pi_s) a_{Ls}(n_L)\right] \end{aligned} \tag{36}$$

and subject to the market clearing conditions:

$$n_P = \frac{\overline{N}_s \Pi_s}{\overline{N}_b \Pi_b} \text{ and } n_L = \frac{\overline{N}_s \left(1-\Pi_s\right)}{\overline{N}_b \left(1-\Pi_b\right)}. \tag{37}$$

Note that we allow the planner to allocate the shares of buyers (Π_b) and of sellers (Π_s) to each marketplace directly. Clearly, any allocation that the planner can achieve by choosing fees and money growth rates, she can also achieve by directly allocating buyers and sellers. In fact, the opposite is also true. The planner can achieve any allocation of buyers and sellers by choosing the fees to buyers and sellers appropriately.

We can simplify the above objective function using $a_{js}(n_j) = a_{jb}(n_j)/n_j$ and plugging in for the expressions of n_P and n_L. The objective becomes:

$$\max_{\Pi_b,\Pi_s} \Pi_b a_{Pb}(n_P) + (1-\Pi_b) a_{Lb}(n_L). \tag{38}$$

That is, maximizing the trading surplus is equivalent to maximizing the combined matching probabilities for buyers on the two marketplaces.

Proposition 5 *When the matching technology is symmetric across marketplaces, the planner's solution is $n_P = n_L = \overline{N}_s/\overline{N}_b$. When platform P has superior matching technology, the planner's solution is $\Pi_s = \Pi_b = 1$.*

[9] See the appendix for a formal derivation.

Due to the concavity of the matching function, in any marketplace where there is trade, it is optimal to set market tightness equal to $\overline{N}_s/\overline{N}_b$. With symmetric technology any allocation of buyers and sellers to the two marketplaces that preserves the optimal tightness is socially optimal. When platform P has superior matching technology, it is optimal to have all sellers and buyers on the platform which automatically preserves the optimal tightness.

Recall from Corollary 1 that $n_P < n_L$. This leads to the next corollary.

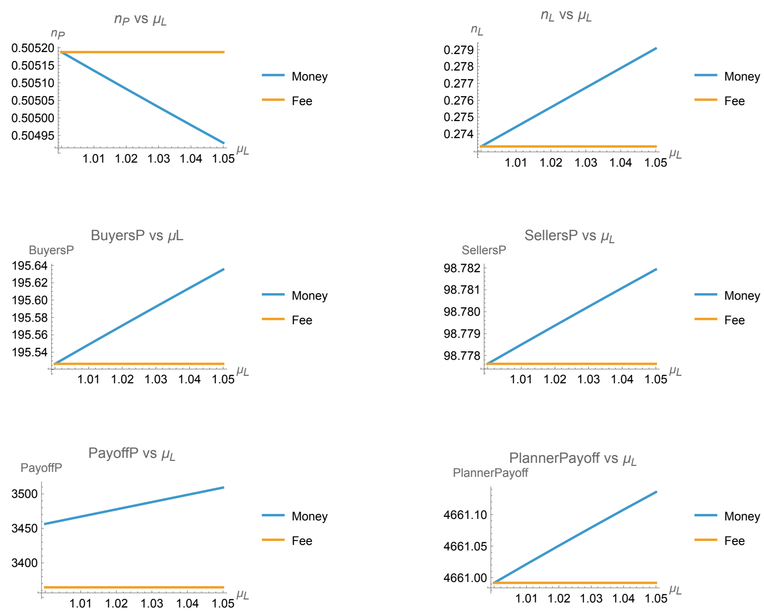
Corollary 2 *When the matching technology is the same cross two marketplaces, competitive equilibrium with private money does not achieve the social optimum because the seller-buyer ratio on platform P is too low.*

When platform P has superior matching technology, the social optimum is for all buyers and sellers to be on the platform. There are two important outcome differences between the competitive equilibrium and the social optimum: the market tightness is generically not equal to $\overline{N}_s/\overline{N}_b$ and the number of buyers or sellers are too low on the platform. Controlling platform money helps platform P to attract buyers, moving the competitive equilibrium towards the social optimum. However it also has a cost, which is that as more buyers come to platform, there are not enough sellers to enter the platform due to the high entry cost set by the platform's owner. Because of the concavity of the matching function, the probability of buyers being matched is not increasing fast enough, causing the competitive outcome deviating from the social optimum. The following proposition states this result formally.

Proposition 6 *When platform P has strictly better matching technology, allowing private money in the decentralized equilibrium can achieve a better outcome from the planner's perspective than the case where private money is not allowed.*

Proposition 6 is demonstrated in an example shown in Figure 6. In this example, we compare how the planner's payoff and each of the other equilibrium outcomes (n_P, n_L, number of buyers (BuyerP), number of sellers (SellerP), and platform's payoff) varies with the legacy inflation for the following two cases: the case when the platform is allowed to use private money (labeled as money in the blue line) and the case where the private platform money is not allowed (labeled as fee in the orange line). In these graphs, the legacy inflation is plotted on the x-axis while the y-axis represents a specific outcome variable. We observe

Figure 6: Money vs Fiat-Fee



The graphs have μ_L on x-axis and an outcome variable (n_p, n_L, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff, planner's payoff) on y-axis. Platform money in the blue line and platform fee (denominated in outside money) only in the orange line. Parameters: $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 10$; $\sigma_b = 0.2$; $\gamma = 0.5$; $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 200$; $\alpha_P = 0.8$; $\xi = 0.95$.

that when the platform is not allowed to use private money and charges a fee denominated in outside money, the equilibrium outcomes do not vary with the legacy inflation. This is because in this case legacy inflation does not affect how platform competes with the legacy marketplace as both experience the same inflation rate. The second graph on the bottom row shows that allowing private money in the equilibrium the planner achieves a higher payoff than the fee only outcome. In the equilibrium where the platform money is allowed, the market tightness on the platform is lower (further away from the social optimum), but the number of buyer on the platform is larger (closer to the social optimum), than the case when platform is only allowed to charge a fixed fee.

6 Extension: Should the platform accept outside money?

In the baseline model, the platform strictly prefers to use platform money and has no incentive to accept fiat central bank money. In practice, however, many platforms accept both platform money and fiat. To capture this, we extend the model by allowing a fraction of buyers to prefer holding fiat over platform money. In this extended setup, we show that the platform may sometimes benefit from accepting both types of money.

Suppose there are two groups of buyers. A fraction ϕ are flexible, meaning their preferences between platform and outside money are as in the baseline model. The remaining $1-\phi$ are biased toward fiat: if they use platform money, they suffer a utility loss of κ. In this environment, we compare two policies available to the platform:

(i) accept only platform money, as in the baseline model; or

(ii) accept both platform money and fiat.

Under the first policy, the perceived advantage of the platform over the legacy market for the flexible buyers is exactly as in the baseline model and given by Δ_b defined in (20). The perceived advantage for the biased consumers is $\Delta_b - \kappa$. Under the first policy platform's objective function is given by:

$$\bar{N}_s \Pi_s(\Delta_s) k_P + \bar{N}_b (\phi \Pi_b(\Delta_b) + (1-\phi)\Pi_b(\Delta_b - \kappa)) [u(1-\gamma)(\mu_P - 1)]. \tag{39}$$

Platform's optimal inflation rate under the first policy is:

$$\mu_P = 1 + \frac{1}{\xi(1-\gamma)u} \frac{\phi \Pi_b(\Delta_b) + (1-\phi)\Pi_b(\Delta_b - \kappa)}{\phi \Pi_b'(\Delta_b) + (1-\phi)\Pi_b'(\Delta_b - \kappa)}. \tag{40}$$

Similarly, the legacy market's objective under the first policy is:

$$\bar{N}_s (1 - \Pi_s(\Delta_s)) k_L + \bar{N}_b (1 - \phi \Pi_b(\Delta_b) - (1-\phi)\Pi_b(\Delta_b - \kappa)) f_L \tag{41}$$

and the legacy market's optimal buyer fee is:

$$f_L = \frac{(1 - \phi \Pi_b(\Delta_b) - (1-\phi)\Pi_b(\Delta_b - \kappa))}{\phi \Pi_b'(\Delta_b) - (1-\phi)\Pi_b'(\Delta_b - \kappa)}. \tag{42}$$

Under the second policy, the perceived advantage of the platform over the legacy market for buyers who use platform money is still given by (20) but for buyers who use outside money it is given by:

$$\Delta_b^g = \beta\left(a_{Pb}(n_P) - a_{Lb}(n_L)\right)\gamma u + \left(f_L - f_P^g\right) \tag{43}$$

where f_P^g is the fee that a buyer pays for using outside money on the platform.

We can assume without loss of generality that $\Delta_b \geq \Delta_b^g$ and the flexible buyers always use platform money. To see this note that if the inequality is reversed then all buyers on the platform use outside money. In this case platform can always set μ_P such that $u(1-\gamma)(\mu_P - 1) = f_P^g$. Flexible buyers are weakly better off by switching to platform money and the platform makes the same profit. At the same time we must have $\Delta_b - \kappa \leq \Delta_b^g$ and the biased buyers use outside money on the platform. Otherwise, the platform can reject outside money without any profit loss. Given these observations, under the second policy platform's objective function is:

$$\bar{N}_s \Pi_s\left(\Delta_s\right) k_P + \bar{N}_b\left(\phi \Pi_b\left(\Delta_b\right)\left[u\left(1-\gamma\right)\left(\mu_P - 1\right)\right] + \left(1-\phi\right)\Pi_b\left(\Delta_b^g\right) f_P^g\right) \tag{44}$$

Platform's optimal inflation rate under the second policy is given by:

$$\mu_P = 1 + \frac{1}{\xi\left(1-\gamma\right)u}\frac{\Pi_b\left(\Delta_b\right)}{\Pi_b'\left(\Delta_b\right)}. \tag{45}$$

The platform charges a surcharge fee for using outside money which is given by:

$$f_P^g = \frac{\Pi_b\left(\Delta_b^g\right)}{\Pi_b'\left(\Delta_b^g\right)}. \tag{46}$$

The legacy market's objective under the second policy is:

$$\bar{N}_s\left(1 - \Pi_s\left(\Delta_s\right)\right)k_L + \bar{N}_b\left(1 - \phi\Pi_b\left(\Delta_b\right) - \left(1-\phi\right)\Pi_b\left(\Delta_b^g\right)\right) f_L \tag{47}$$

and the legacy market's optimal buyer fee is:

$$f_L = \frac{\left(1 - \phi\Pi_b\left(\Delta_b\right) - \left(1-\phi\right)\Pi_b\left(\Delta_b^g\right)\right)}{\phi\Pi_b'\left(\Delta_b\right) - \left(1-\phi\right)\Pi_b'\left(\Delta_b^g\right)}. \tag{48}$$

For the analytical results in this section we fix the fraction of sellers on the platform and focus only on the buyer side of the market. Specifically, we assume that the number of sellers on the platform is $\bar{N}_s\Pi_s$ and on the legacy market is is $\bar{N}_s(1-\Pi_s)$. We also assume that $\mu_L > 1$.

The next result proves that, as long as there are some flexible buyers, the platform never uses only outside money. This result holds even if most buyers are biased and they have strong preference for outside money.

Proposition 7 *As long as $\phi > 0$, platform either uses only platform money or accepts both types of money but never uses only outside money.*

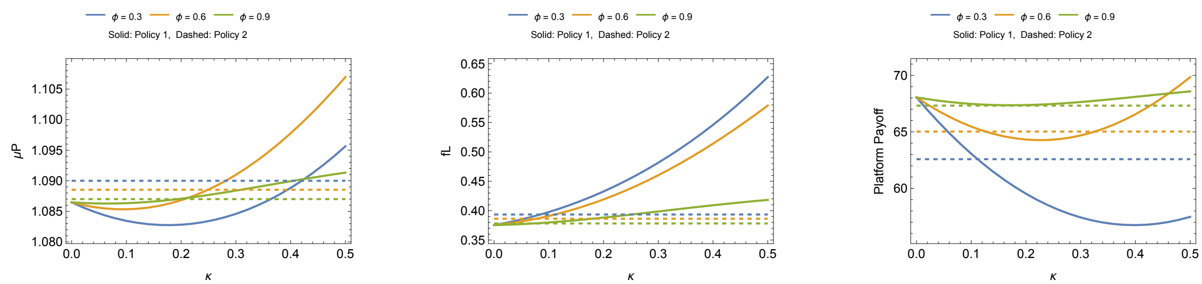
Next we consider two edge cases. As the first edge case, suppose that biased buyers never hold platform money, i.e., $\kappa = \infty$ and almost all buyers are biased, i.e, ϕ is close to zero. In this case, if the platform follows the first policy, its profit is almost zero since almost all buyers go to the legacy market. If it follows the second policy, although it competes with the legacy market for the biased consumers, its profit is strictly positive. As a result, the platform strictly prefers to adopt the second policy and accept both types of money.

As the second edge case, suppose that $\phi \in (0,1)$ and disutility of using platform money for biased buyers, κ, is small. Intuitively, since the bias is very small, if the platform follows the first policy and accepts only platform money then it loses a very small fraction of biased buyers to the legacy market. When $\mu_L > 1$, the platform's profit from the remaining biased buyers that it captures is higher when it charges them through platform money as opposed to charging a fee that is paid in outside money. Hence, when the bias is small enough the platform strictly prefers the first policy. The next proposition formalizes this result.

Proposition 8 *When $\kappa < \xi\left(\mu_L - 1\right)\left(1-\gamma\right)u$, the platform adopts the first policy (accepts only platform money).*

We next examine the economic mechanism underlying payment choice in the intermediate cases where the disutility of using platform money for biased buyers, κ, is moderate, and the fraction of flexible buyers, $\phi \in (0,1)$. In these settings, interesting effects emerge. To illustrate these dynamics, Figure 7 presents an example where κ ranges from 0 to 0.5 and ϕ takes values 0.3, 0.6, and 0.9.

Figure 7: Comparison of the two policies



The graphs have κ on the x-axis and μ_p, f_L, and the platform's payoff under the first (accept only platform money) versus the second policy (accept both forms of money) on the y-axis. Parameters: $\alpha_P = 0.1$, $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 10$; $\gamma = 0.5$; $\sigma_b = 0.2$; $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 200$; $\xi = 0.99$.

To dis-entangle various strategic effects, we first investigate the impact of increasing κ on the platform's inflation under policy 1 where the platform accepts only its own money (the solid lines). We can observe from the first panel of Figure 7 that the platform's inflation is, interestingly, U-shaped (first decreasing and then increasing) in κ. This pattern arises because κ has two effects on the platform's inflation rate. The first is a direct effect: holding the legacy market's buyer fee constant, as the disutility of holding platform money, κ, increases, the platform has to lower its inflation to retain the biased consumers. There is also a second strategic effect: as κ increases, the legacy market increases its buyer fee and the platform increases its inflation in response. Overall, whether the platform's inflation increases or decreases as κ goes up depends on which of these effects dominate. When κ is small the direct effect is stronger and the platform lowers its inflation. In this case, the platform's inflation and the legacy market's buyer fees are strategic substitutes. However, as κ increases the second strategic effect dominates and the platform increases its inflation. In this case, the platform's inflation and the legacy market's buyer fees are strategic complements.

To understand why the platform's inflation and the legacy market's buyer fees become strategic complements, note that as κ becomes large, the market becomes increasingly segmented. Legacy market captures most of the biased buyers and the platform captures most of the flexible buyers. Hence, when the legacy market increases its buyer's fee, the platform increases its inflation and its revenues from its large base of flexible buyers goes up overwhelming the loss from the biased ones.

Next, consider the impact of increasing κ on the platform's payoff under policy 1. We can observe from the third panel of Figure 7 that the platform's payoff is also U-shaped in κ. When κ is small, the platform's payoff decreases in κ for two reasons. First, it loses some biased consumers to the legacy market, and second (given that the platform's inflation and the legacy market's buyer fees are strategic substitutes) it reduces its inflation and loses its seigniorage income from the buyers it retains. As κ increases, the platform keeps losing biased consumers to the legacy market. But the second effect reverses. For high enough κ, the platform's inflation and the legacy market's buyer fees become strategic complements, and the platform increases its inflation, and consequently its seigniorage income from the buyers it retains. This gain in seigniorage income from retained buyers eventually dominates the loss of income from biased buyers and the platform's payoff increases in κ.

Under policy 2, when the platform accepts both types of money, its payoff does not depend on κ. This happens because under policy 2 the biased consumers hold outside money whether they trade on the platform or the legacy market. Since they do not experience the loss κ varying κ does not effect the inflation on the platform and the buyer fees on either market. We can observe this in Figure 7—the dashed lines that correspond to policy 2 do not vary with κ. We also observe from the figure that the platform's inflation and payoff are both larger for larger ϕ.

Finally, we turn to the comparison of the policies. The example shows that when ϕ is large, the platform prefers policy 1 for all values of κ. This is shown in the third panel of Figure 7 where the solid green line is above the dashed green line ($\phi = 0.9$).

This panel also shows that when the fraction of flexible buyers in the population is small, the payment choice is monotone in κ (which can be observed by comparing blue solid and dashed lines: $\phi = 0.3$). The platform only accepts platform money as the payment option when disutility κ is small and accepts both types of money for payment when disutility is above a threshold.

When the fraction of flexible buyers in the population is in the intermediate range, however, the payment choice might be not monotone in κ (which can be observed by comparing yellow solid and dashed lines: $\phi = 0.6$). The platform prefers policy 1 (only platform money) for lower values of κ, switches to policy 2 (both types of money) for intermediate values of

κ, but gives up accepting outside money for large values of κ.

Overall, the example suggests that the platform accepts outside money when either (i) ϕ is large and κ exceeds a threshold, or (ii) both ϕ and κ take intermediate values. Conversely, when ϕ is moderate and κ is large, the platform accepts only platform money, servicing mostly the flexible buyers, while the legacy market captures the biased ones. In this case, the platform sets a high inflation rate for flexible consumers, and the legacy market charges high fees to biased consumers.

7 Conclusion

Our analysis demonstrates that when a platform possesses the ability to issue its own money, it can strategically control its money supply to attract buyers. This increased buyer participation subsequently draws more sellers, thereby launching and amplifying network externalities between buyers and sellers - especially when combined with enhanced matching technologies. Moreover, our results indicate that such platforms exercise considerable market power - not only over the money supply but also over seller entry - by imposing relatively high seller fees. Importantly, the resulting equilibrium may deviate from social efficiency, highlighting potential welfare implications.

This study raises a critical policy question regarding the social welfare consequences of allowing platforms to maintain private payment systems. Currently, regulated financial institutions, regarded as trustworthy third parties, dominate payment systems. However, digital platforms are increasingly equipped with advanced data processing and machine learning capabilities that not only improve buyer-seller matching but also secure transactions. With the growing prevalence of platform-based economies, it is essential to examine whether these digital marketplaces, through their intrinsic economic synergy with payment systems, should be entitled to the seigniorage income traditionally captured by financial institutions.

Empirical policy experiences further underscore the relevance of this inquiry. For instance, following the easing of COVID-19 restrictions, cities and regional governments in China deployed e-coupons and e-voucher disbursed directly to resident's WeChat or Alipay wallet to boost consumption. A study of 42 Chinese cities reveals that the most adversely affected sectors, including dining, retail, and tourism, received significant support (with 81% of vouchers allocated to eating out, 73% for retail, and 48% for tourism).[10] In September 2024, the Shanghai government further injected 500 million yuan (approximately US $71.2 million) in consumption vouchers into digital wallets as part of a broader strategy to revive the economy. By contrast, the U.S. response to the pandemic involved dispersing Economic Impact Payments (or Stimulus checks) via direct deposits and bank-issued cards under the $1.9 billion American Rescue Plan Act, without spending restrictions. Future work could investigate the ultimate beneficiaries of these distinct policy approaches and evaluate their social welfare impacts.

[10]https://www.caixinglobal.com/2020-05-20/in-depth-who-really-benefits-from-consumption-vouchers-101556620.html

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A Appendix

A.1 Proof of Lemma 3

By the equilibrium condition:

$$\frac{\overline{N}_s \Pi_s(\Delta_s)}{\overline{N}_b \Pi_b(\Delta_b)} = n_P, \text{ and} \tag{A.1}$$

$$\frac{\overline{N}_s(1-\Pi_s(\Delta_s))}{\overline{N}_b(1-\Pi_b(\Delta_b))} = n_L. \tag{A.2}$$

Hence:

$$n_p \leq n_L \Leftrightarrow \Pi_s(\Delta_s) \leq \Pi_b(\Delta_b) \Leftrightarrow \frac{1}{1+\exp\left(-\frac{\Delta_s}{\sigma_s}\right)} \leq \frac{1}{1+\exp\left(-\frac{\Delta_b}{\sigma_b}\right)} \Leftrightarrow \frac{\Delta_s}{\sigma_s} \leq \frac{\Delta_b}{\sigma_b}.$$

A.2 Proof of Proposition 1

Suppose towards a contradiction that $a_{Ps}(n_P) \leq a_{Ls}(n_L)$. Then $n_P \geq n_L$ and by Lemma 3 $\frac{\Delta_s}{\sigma_s} \geq \frac{\Delta_b}{\sigma_b}$. Unpacking the expression of Δ_b using (20) and Δ_s using (23) and plugging optimal $f_L, f_P, k_P, k_L, \mu_P$ as in (27), (33), (34), (30), and (31), we obtain

$$\frac{1}{\sigma_s}\beta\left(a_{Ps}(n_P)-a_{Ls}(n_L)\right)(1-\gamma)u+\frac{1}{\sigma_s}\left(\frac{1-2\Pi_s\left(\Delta_s\right)}{\Pi_s'\left(\Delta_s\right)}\right)\geq$$
$$\frac{1}{\sigma_b}\left[\beta\left(\underbrace{a_{Pb}(n_P)-a_{Lb}(n_L)}_{\text{due to } n_P\geq n_L}\right)\gamma u+\underbrace{\xi\left(1-\gamma\right)u\left(\mu_L-1\right)}_{\text{Control money supply}}\right]+\frac{1}{\sigma_b}\left(\frac{1-2\Pi_b\left(\Delta_b\right)}{\Pi_b'\left(\Delta_b\right)}\right). \quad \text{(A.3)}$$

Since $\mu_L > 1$, $a_{Ps}(n_P) \leq a_{Ps}(n_L)$, and $a_{Pb}(n_P) \geq a_{Lb}(n_L)$, (A.3) implies:

$$\frac{1}{\sigma_s}\left[\left(\frac{1-2\Pi_s\left(\Delta_s\right)}{\Pi_s'\left(\Delta_s\right)}\right)\right]\geq\frac{1}{\sigma_b}\left[\left(\frac{1-2\Pi_b\left(\Delta_b\right)}{\Pi_b'\left(\Delta_b\right)}\right)\right]. \quad \text{(A.4)}$$

By Gumble distribution, we know that:

$$\Pi_i\left(x\right) = \left[1+\exp\left(-\frac{\Delta_i}{\sigma_i}\right)\right]^{-1} \text{ and} \quad \text{(A.5)}$$
$$\Pi_i'\left(x\right) = \frac{1}{\sigma_i}\left[1+\exp\left(-\frac{\Delta_i}{\sigma_i}\right)\right]^{-2}\exp\left(-\frac{\Delta_i}{\sigma_i}\right), i\in\{b,s\}. \quad \text{(A.6)}$$

Plugging in for $\Pi_i(\cdot)$ and $\Pi_i'(\cdot)$, (A.4) implies:

$$\exp\left(-\frac{\Delta_b}{\sigma_b}\right)\exp\left(-\frac{\Delta_s}{\sigma_s}\right)\left[\left(\exp\left(-\frac{\Delta_s}{\sigma_s}\right)\right)-\left(\exp\left(-\frac{\Delta_b}{\sigma_b}\right)\right)\right]>\exp\left(-\frac{\Delta_b}{\sigma_b}\right)-\exp\left(-\frac{\Delta_s}{\sigma_s}\right).$$

Note that for this inequality to hold we must have:

$$\frac{\Delta_s}{\sigma_s}<\frac{\Delta_b}{\sigma_b}, \quad \text{(A.7)}$$

which is a contradiction.

A.3 Proof of Proposition 2

Using FOCs for k_P and k_L, we have $k_P > k_L$ if and only if $\frac{\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)} > \frac{1-\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)}$. This is true if and only if $\Pi_s\left(\Delta_s\right) > \frac{1}{2}$ if and only if $\Delta_s > 0$. Note $\Delta_s = \beta\left(a_{Ps}(n_P) - a_{Ls}(n_L)\right)(1-\gamma)u + (k_L - k_P)$. The first term is strictly positive. Suppose that $k_P \leq k_L$, the second term is also positive and hence, $\Delta_s > 0$, which implies $k_P > k_L$. Thus, we must have $k_P > k_L$. We must also have $\Delta_s > 0$.

A.4 Proof of Proposition 3

More buyers on platform P if

$$\Delta_b = \beta \left(a_{Pb}(n_P) - a_{Lb}(n_L) \right) \gamma u + \xi \left(1 - \gamma \right) u \left(\mu_L - 1 \right) + \left(\frac{1 - 2\Pi_b \left(\Delta_b \right)}{\Pi_b' \left(\Delta_b \right)} \right) > 0. \qquad \text{(A.8)}$$

Towards a contradiction, let us suppose that $\Delta_b < 0$ or equivalently $\Pi_b \left(\Delta_b \right) < 0.5$. In this case, it must be that $a_{Pb}(n_P) < a_{Lb}(n_L)$ since otherwise all the terms on the right side of the above equation are positive.

However, if $a_{Pb}(n_P) < a_{Lb}(n_L)$ then $n_P < n_L$, which by Lemma 1 implies $\frac{\Delta_s}{\sigma_s} < \frac{\Delta_b}{\sigma_b} < 0$.

But then $\Delta_s < 0$, which is a contradiction since we already established $\Delta_s > 0$ in Proposition 2. Then, we must have $\Delta_b > 0$.

A.5 Proof of Proposition 4

Taking total derivatives of n_P and n_L, we obtain:

$$\begin{aligned}
\frac{\partial n_P}{\partial \mu_L} &= n_P \left(\frac{\Pi_s' \left(\Delta_s \right)}{\Pi_s \left(\Delta_s \right)} \frac{\partial \Delta_s}{\partial \mu_L} - \frac{\Pi_b' \left(\Delta_b \right)}{\Pi_b \left(\Delta_b \right)} \frac{\partial \Delta_b}{\partial \mu_L} \right) \\
\frac{\partial n_L}{\partial \mu_L} &= n_L \left(\frac{\Pi_b' \left(\Delta_b \right)}{\left(1 - \Pi_b \left(\Delta_b \right) \right)} \frac{\partial \Delta_b}{\partial \mu_L} - \frac{\Pi_s' \left(\Delta_s \right)}{\left(1 - \Pi_s \left(\Delta_s \right) \right)} \frac{\partial \Delta_s}{\partial \mu_L} \right).
\end{aligned}$$

Taking total derivatives of Δ_b and Δ_s, we obtain:

$$\begin{aligned}
\frac{\partial \Delta_b}{\partial \mu_L} &= \beta \left(a_{Pb}'(n_P) \frac{\partial n_P}{\partial \mu_L} - a_{Lb}'(n_L) \frac{\partial n_L}{\partial \mu_L} \right) \gamma u + \xi \left(1 - \gamma \right) u - \xi \frac{\partial \mu_P}{\partial \mu_L} \left(1 - \gamma \right) u \\
&\quad - \left(1 + \frac{\Pi_b'' \left(\Delta_b \right) \left(1 - \Pi_b \left(\Delta_b \right) \right)}{\left(\Pi_b' \left(\Delta_b \right) \right)^2} \right) \frac{\partial \Delta_b}{\partial \mu_L}, \text{ and} \\
\frac{\partial \Delta_s}{\partial \mu_L} &= \beta \left(a_{Ps}'(n_P) \frac{\partial n_P}{\partial \mu_L} - a_{Ls}'(n_L) \frac{\partial n_L}{\partial \mu_L} \right) \left(1 - \gamma \right) u \\
&\quad - \left(2 + \frac{\Pi_s'' \left(\Delta_s \right) \left(1 - 2\Pi_s \left(\Delta_s \right) \right)}{\left(\Pi_s' \left(\Delta_s \right) \right)^2} \right) \frac{\partial \Delta_s}{\partial \mu_L}.
\end{aligned}$$

Substituting for $\partial n_P/\partial\mu_L$ and $\partial n_L/\partial\mu_L$, we can rewrite these expressions as:

$$\frac{\partial\Delta_b}{\partial\mu_L}\left(2+\frac{\Pi_b''\left(\Delta_b\right)\left(1-\Pi_b\left(\Delta_b\right)\right)}{\left(\Pi_b'\left(\Delta_b\right)\right)^2}+\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_b'\left(\Delta_b\right)}{\Pi_b\left(\Delta_b\right)}+a_{Lb}'(n_L)n_L\frac{\Pi_b'\left(\Delta_b\right)}{\left(1-\Pi_b\left(\Delta_b\right)\right)}\right)\gamma u\right)=$$
$$\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_s'\left(\Delta_s\right)}{\Pi_s\left(\Delta_s\right)}+a_{Lb}'(n_L)n_L\frac{\Pi_s'\left(\Delta_s\right)}{\left(1-\Pi_s\left(\Delta_s\right)\right)}\right)\frac{\partial\Delta_s}{\partial\mu_L}\gamma u+\xi\left(1-\gamma\right)u\left(1-\frac{\partial\mu_P}{\partial\mu_L}\right),\ \text{and}$$
$$\frac{\partial\Delta_s}{\partial\mu_L}\left(3+\frac{\Pi_s''\left(\Delta_s\right)\left(1-2\Pi_s\left(\Delta_s\right)\right)}{\left(\Pi_s'\left(\Delta_s\right)\right)^2}-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_s'\left(\Delta_s\right)}{\Pi_s\left(\Delta_s\right)}+a_{Ls}'(n_L)n_L\frac{\Pi_s'\left(\Delta_s\right)}{\left(1-\Pi_s\left(\Delta_s\right)\right)}\right)\left(1-\gamma\right)u\right)$$
$$=-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_b'\left(\Delta_b\right)}{\Pi_b\left(\Delta_b\right)}+a_{Ls}'(n_L)n_L\frac{\Pi_b'\left(\Delta_b\right)}{\left(1-\Pi_b\left(\Delta_b\right)\right)}\right)\frac{\partial\Delta_b}{\partial\mu_L}\left(1-\gamma\right)u.$$

From these two equations we obtain:

$$\frac{\partial\Delta_b}{\partial\mu_L}\left(2+\frac{\Pi_b''\left(\Delta_b\right)\left(1-\Pi_b\left(\Delta_b\right)\right)}{\left(\Pi_b'\left(\Delta_b\right)\right)^2}+\frac{\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_b'(\Delta_b)}{\Pi_b(\Delta_b)}+a_{Lb}'(n_L)n_L\frac{\Pi_b'(\Delta_b)}{(1-\Pi_b(\Delta_b))}\right)\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{\left(\Pi_s'(\Delta_s)\right)^2}\right)\gamma u}{\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{\left(\Pi_s'(\Delta_s)\right)^2}-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_s'(\Delta_s)}{\Pi_s(\Delta_s)}+a_{Ls}'(n_L)n_L\frac{\Pi_s'(\Delta_s)}{(1-\Pi_s(\Delta_s))}\right)\left(1-\gamma\right)u\right)}\right)$$
$$=\xi\left(1-\gamma\right)u\left(1-\frac{\partial\mu_P}{\partial\mu_L}\right).$$

Taking total derivative of $\partial\mu_P/\partial\mu_L$, we get:

$$\frac{\partial\mu_P}{\partial\mu_L}=\frac{1}{\xi\left(1-\gamma\right)u}\left(1-\frac{\Pi_b''\left(\Delta_b\right)\Pi_b\left(\Delta_b\right)}{\left(\Pi_b'\left(\Delta_b\right)\right)^2}\right)\frac{\partial\Delta_b}{\partial\mu_L}.$$

Solving these equations gives us:

$$\frac{\partial\mu_P}{\partial\mu_L}=\frac{\left(1-\frac{\Pi_b''(\Delta_b)\Pi_b(\Delta_b)}{\left(\Pi_b'(\Delta_b)\right)^2}\right)}{\left(1-\frac{\Pi_b''(\Delta_b)\Pi_b(\Delta_b)}{\left(\Pi_b'(\Delta_b)\right)^2}\right)+2+\frac{\Pi_b''(\Delta_b)(1-\Pi_b(\Delta_b))}{\left(\Pi_b'(\Delta_b)\right)^2}+\frac{\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_b'(\Delta_b)}{\Pi_b(\Delta_b)}+a_{Lb}'(n_L)n_L\frac{\Pi_b'(\Delta_b)}{(1-\Pi_b(\Delta_b))}\right)\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{(\Pi_s'(\Delta_s))^2}\right)\gamma}{\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{(\Pi_s'(\Delta_s))^2}-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_s'(\Delta_s)}{\Pi_s(\Delta_s)}+a_{Ls}'(n_L)n_L\frac{\Pi_s'(\Delta_s)}{(1-\Pi_s(\Delta_s))}\right)(1-\gamma)\right)}}. \tag{A.9}$$

Observe that the numerator and the first term of the denominator of Equation (A.9) are positive since

$$1-\frac{\Pi_b''\left(\Delta_b\right)\Pi_b\left(\Delta_b\right)}{\left(\Pi_b'\left(\Delta_b\right)\right)^2}=-1+\frac{1}{\left(\Pi_b\left(x\right)\right)\left(\exp\left(-x\right)\right)}>0.$$

Moreover,

$$
\begin{aligned}
2+\frac{\Pi_b''\left(\Delta_b\right)\left(1-\Pi_b\left(\Delta_b\right)\right)}{\left(\Pi_b'\left(\Delta_b\right)\right)^2} &= 2+\frac{\left(2\left(\Pi_b\left(x\right)\right)^3\left(\exp\left(-x\right)\right)^2-\left(\Pi_b\left(x\right)\right)^2\exp\left(-x\right)\right)\left(1-\Pi_b\left(\Delta_b\right)\right)}{\left(\Pi_b\left(x\right)\right)^4\exp\left(-x\right)^2} \\
&= \frac{2}{\Pi_b\left(x\right)}-\frac{1}{\left(\Pi_b\left(x\right)\right)^2\exp\left(-x\right)}+\frac{1}{\Pi_b\left(x\right)\exp\left(-x\right)}>0.
\end{aligned}
$$

Finally, both the numerator and the denominator of the third term in the denominator of Equation (A.9) are positive. Hence, $\frac{\partial\mu_P}{\partial\mu_L}>1$.

A.6 Proof of Proposition 5

The first order condition with respect to $\Pi_b<1$ is:

$$
a_{Pb}(n_P)-a'_{Pb}(n_P)n_P \quad \geq \quad a_{Lb}(n_L)-a'_{Lb}(n_L)n_L, \tag{A.10}
$$

where the above condition holds with equality if $\Pi_b<1$.

The first order condition with respect to $\Pi_s<1$ is:

$$
a'_{Pb}(n_P) \quad \geq \quad a'_{Lb}(n_L), \tag{A.11}
$$

where the above condition holds with equality if $\Pi_s<1$.

We observe $a_{Pb}\left(0\right)=a_{Lb}\left(0\right)=0$. We assume that $a'_{Pb}(n)>a'_{Lb}(n), \forall n>0$. That is, the concave matching function for for the platform has a steeper slope for the same tightness than the legacy marketplace.

Let us suppose that $\Pi_s<1$. Then FOC gives $a'_{Pb}(n_P)=a'_{Lb}(n_L)$. By concavity of a_{Pb} and a_{Lb}, $n_P>n_L$. However, we find that

$$
\begin{aligned}
a_{Pb}(n_P)-a'_{Pb}(n_P)n_P &= \\
a_{Pb}(n_L)-a'_{Pb}(n_P)n_L+\int_{n_L}^{n_P}a'_{Pb}(n)dn-a'_{Pb}(n_P)\left(n_P-n_L\right) &> \\
a_{Pb}(n_L)-a'_{Pb}(n_P)n_L+\int_{n_L}^{n_P}a'_{Pb}(n_P)dn-a'_{Pb}(n_P)\left(n_P-n_L\right) &= \\
a_{Pb}(n_L)-a'_{Pb}(n_P)n_L &> \\
a_{Lb}(n_L)-a'_{Lb}(n_L)n_L. &
\end{aligned}
$$

The second inequality holds because $a'_{Pb}(n) > a'_{Pb}(n_P)$ for all $n \in (n_L, n_P)$. Since the first order condition with respect to Π_b holds in strict inequality, therefore $\Pi_b = 1$. Plugging $\Pi_b = 1$ into the objective function, we get:

$$\max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s\Pi_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}\left(\frac{\overline{N}_s(1-\Pi_s)}{\overline{N}_b(1-\Pi_b)}\right) = \max_{\Pi_s,\Pi_b} a_{Pb}\left(\frac{\overline{N}_s\Pi_s}{\overline{N}_b}\right).$$

This maximization problem implies that $\Pi_s = 1$ since a_{Pb} is increasing. Hence we must have $\Pi_s = 1$.

After proving that $\Pi_s = 1$, we now turn to show that $\Pi_b = 1$. Plugging $\Pi_s = 1$ into the planner's objective function, we obtain

$$\begin{aligned} \max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s\Pi_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}\left(\frac{\overline{N}_s(1-\Pi_s)}{\overline{N}_b(1-\Pi_b)}\right) &= \\ \max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}(0) &= \max_{\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right). \end{aligned}$$

The first order condition with respect to Π_b becomes:

$$a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right) - a'_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right)\frac{\overline{N}_s}{\overline{N}_b\Pi_b} \geq a_{Lb}(0) - a'_{Lb}(0)0 = 0. \tag{A.12}$$

Or

$$\frac{a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right)}{\frac{\overline{N}_s}{\overline{N}_b\Pi_b}} \geq a'_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right). \tag{A.13}$$

Since a_{Pb} is concave, the above cannot hold with equality. So we must have $\Pi_b = 1$.

A.7 Other proofs

Claim 1 *The social planner's objective function can be written as:*

$$\max \overline{N}_b\gamma u\left[\Pi_b a_{Pb}(n_P) + (1-\Pi_b)a_{Lb}(n_L)\right] + \overline{N}_s(1-\gamma)u\left[\Pi_s a_{Ps}(n_P) + (1-\Pi_s)a_{Ls}(n_L)\right] \tag{A.14}$$

By summing all agents' utility:

$$
\begin{aligned}
\max \underbrace{\overline{N}_b[\Pi_b\left(\Delta_b\right) W_b^P(0,0) + [1-\Pi_b\left(\Delta_b\right)]W_b^L(0,0)]}_{\text{Buyers' Utility}} \\
+ \underbrace{\overline{N}_s[\Pi_s\left(\Delta_s\right) W_s^P(0,0) + \left(1-\Pi_s\left(\Delta_s\right)\right) W_s^L(0,0)]}_{\text{Sellers' Utility}} \\
+ \underbrace{\overline{N}_s\Pi_s\left(\Delta_s\right) k_P + \overline{N}_b\Pi_b\left(\Delta_b\right)\left[f_P + (\mu_P - 1)u(1-\gamma)\right]}_{P\text{ Owner's Profit}} \\
+ \underbrace{\overline{N}_s\left(1-\Pi_s\left(\Delta_s\right)\right) k_L + \overline{N}_b\left(1-\Pi_b\left(\Delta_b\right)\right) f_L}_{L\text{ Owner's Profit}} \\
+ \underbrace{\overline{N}_b\left(1-\Pi_b\left(\Delta_b\right)\right)\left(\mu_L - 1\right)u(1-\gamma)}_{\text{Tax rebate for outside money seigniorage}},
\end{aligned}
$$

which is subject to

$$
\begin{aligned}
n_P[\bar{N}_b\Pi_b\left(\Delta_b\right)] &= \bar{N}_s\Pi_s\left(\Delta_s\right) \\
n_L[\bar{N}_b(1-\Pi_b\left(\Delta_b\right))] &= \bar{N}_s(1-\Pi_s\left(\Delta_s\right)).
\end{aligned}
$$

Next, we plug in (5) and (22) and after some algebra we get to equation (A.14).

A.8 Nash Bargaining with Liquidity Constraints

We assume that the seller's cost of producing x units of the good is xc where $0 \leq c < u$. We also assume that a buyer obtains utility xu if the seller produces $0 \leq x \leq 1$ and does not value more than one unit. (Alternatively we can assume the seller has the production capacity of one unit instead.) To begin, suppose the buyer is not liquidity constrained. The (generalized) Nash bargaining problem is :

$$\max_{x,q}\left(xu-q\right)^{\gamma}\left(q-xc\right)^{1-\gamma} \tag{A.15}$$

where q is the terms of trade for exchanging the consumption goods. The solution is: $x^* = 1$ and $q^* = \left(1-\gamma\right)u + \gamma c$. Hence the buyer's surplus is: $\gamma\left(u-c\right)$ and the seller's surplus is $\left(1-\gamma\right)\left(u-c\right)$.

Now suppose the buyer is liquidity constrained so that they will not be able to bring enough money to pay for the consumption good price from the above Nash solution. That is, $q \leq q^*$. The liquidity constraint is binding so the Nash bargaining problem becomes:

$$\max_{x,q} (xu - q)^{\gamma} (y - xc)^{1-\gamma} \tag{A.16}$$

subject to $q \leq q^*$. The first order condition is

$$\frac{q - xc}{xu - q} = \frac{1-\gamma}{\gamma}\frac{c}{u} \Rightarrow x = \frac{(\gamma u + (1-\gamma)\, c)}{cu} q.$$

Therefore, the solution is

$$x^* = \begin{cases} 1 & \text{if } \frac{(\gamma u + (1-\gamma) c)}{cu} q \geq 1 \\ \frac{(\gamma u + (1-\gamma) c)}{cu} q & \text{otherwise} \end{cases}$$

Hence, the term of trade is:

$$q^{**} = \frac{cu}{\gamma u + (1-\gamma)\, c}. \tag{A.17}$$

. The seller's surplus is:

$$\frac{cu}{\gamma u + (1-\gamma)\, c} - c = \frac{(1-\gamma)\, c\, (u-c)}{\gamma u + (1-\gamma)\, c}. \tag{A.18}$$

The buyer's surplus is:

$$u - \frac{cu}{\gamma u + (1-\gamma)\, c} = \frac{\gamma u\, (u-c)}{\gamma u + (1-\gamma)\, c}. \tag{A.19}$$

The Nash bargaining solution indicates that the buyer when liquidity constrained chooses to bring q^{**} specified in eq. (A.17) from the CM to the DM in order to trade the consumption goods. That is, Thus, the resulting real price for the good y is: $p^j \phi^j = cu/(\gamma u + (1-\gamma)\, c)$. The welfare analysis will be based on the split of the total surplus $u - c$ specified in eq. (A.18) and eq. (A.19). The qualitative results on the platform use of platform money to gain competitive advantage remain the same but the quantitative implications are different – since the liquidity constrained buyer potentially would bring less money, platform money in this case give the platform owner less advantages than the Kalai bargaining solution.

However, note that there is one peculiarity in this Nash bargaining solution that we consider undesirable. The derivative of the seller's surplus with respect to c has same sign as: $\gamma u^2 - 2\gamma cu - (1-\gamma)\,c^2$. It is negative quadratic in c and at $c = 0$ this is $\gamma u^2 > 0$ and at $c = u$ this is $-u^2 < 0$. Hence, if the seller has a low cost, he has incentive to exaggerate cost or engage in wasteful expenditure.