Mortgage Structure, Financial Stability, and Risk Sharing*

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Abstract

Adjustable-rate mortgages (ARMs) expose households to rising payments, increasing defaults, while fixed-rate mortgages (FRMs) expose lenders to greater interest rate risk. We evaluate these competing forces in a quantitative model with flexible mortgage contracts, liquidity-driven household default, and a banking sector with sticky deposits. We find financial stability risks are U-shaped in mortgage fixation length. While FRMs benefit from deposit rate stickiness, ARMs provide net worth hedging by concentrating defaults in states when intermediary net worth is high due to increases in mortgage income. An intermediate fixation length balances these effects, minimizing financial sector volatility and improving aggregate risk sharing.

JEL: E52, G21, G28, R31, E44.

Keywords: mortgages, financial stability, interest rate risk, credit risk, fixed-rate, adjustable-rate, risk sharing, intermediary asset pricing, household finance

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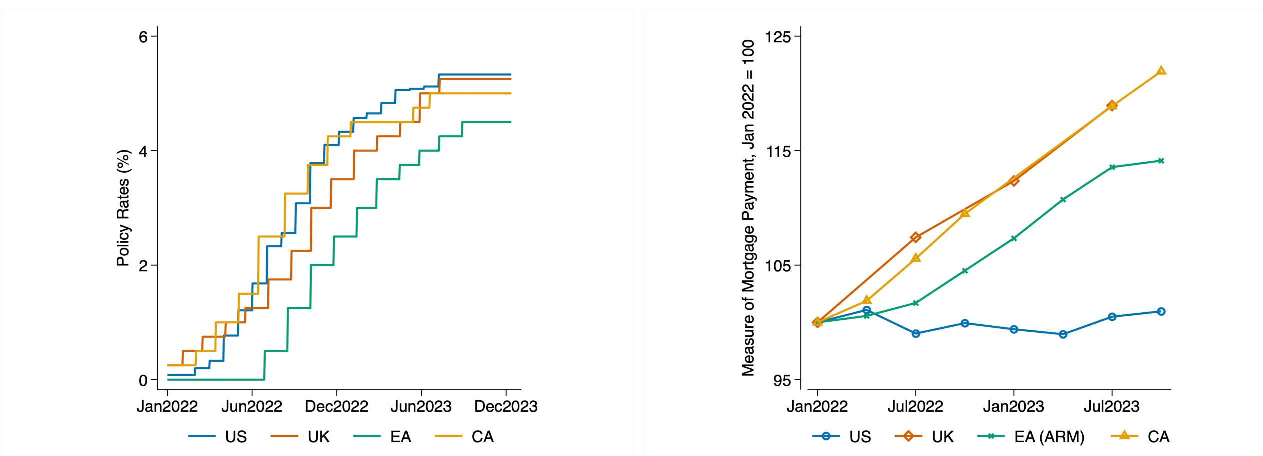
1 Introduction

Mortgage structure matters for macroeconomic outcomes. It affects the transmission of monetary policy, since adjustable-rate mortgages (ARMs) reset more immediately compared to fixed-rate mortgages (FRMs) (e.g. Calza et al., 2013; Di Maggio et al., 2017; Fuster and Willen, 2017; Garriga et al., 2017). In this paper, we show that mortgage structure also directly shapes financial stability risks. Differences in mortgage structures across countries were brought into sharp relief by the global monetary tightening cycle between 2022 to 2023. Despite similar policy rate increases of approximately 400 to 500 basis points across major economies, mortgage payments increased by 15 to 25% in countries with ARMs (U.K., Canada, and Euro Area), while remaining stable in the U.S., where 30-year FRMs predominate (Figure 1).

Figure 1: Comparison of Policy Rates and Mortgage Payments, 2022–2023

(a) Policy Rates

(b) Measure of Mortgage Payments



Notes: Panel (a) shows main monetary policy rates for the United States (US), United Kingdom (UK), Euro Area (EA), and Canada (CA). Panel (b) shows measures of average mortgage payments. EA ARM aggregates Finland, Italy and Portugal. Data sources: US: 2024Q2 revised mortgage debt service ratio (DSR) from FRED; UK: total expected (incl. agreed changes in payments e.g. due to forbearance) monthly mortgage payment from the Financial Conduct Authority (FCA); Euro Area: total DSR from BIS; Canada: average monthly scheduled outstanding mortgage payments from the Canada Mortgage and Housing Corporation (CMHC).

The contrasting mortgage payment sensitivity to rate changes highlights distinct financial stability risks and risk-sharing properties across mortgage structures. Rising interest rates in ARM economies directly increase household mortgage payments, thereby raising household defaults and bank credit losses. Conversely, FRMs shield households from rising payments but potentially expose banks to greater interest rate risk.

Given these competing forces, can we find a mortgage structure that is best for financial stability and risk sharing between households and financial intermediaries? A natural starting point might be to select a mortgage structure that offsets the cash flow sensitivity of bank liabilities, particularly deposits, achieving a "zero duration" financial system that fully hedges interest rate risk. However, such an approach overlooks several channels that likely arise in equilibrium.[1] First, interest rate changes also affect credit risk, as households make endogenous default decisions that differ across macroeconomic environments and mortgage structures (Campbell and Cocco, 2015). For instance, rising rates and ARM payments can trigger defaults among liquidity-constrained households, an effect absent under FRMs. Second, financial intermediaries' willingness to hold mortgages and their mortgage pricing, especially risk premia, depend on intermediary net worth. As a result, overall financial stability depends on both interest rate risk and credit risk, and the correlation of these risks with intermediary capital.

To embed both rich household behavior and intermediary capital constraints in equilibrium, we develop a quantitative macro-finance model with flexible mortgage contract structures, borrowers who endogenously default, and a financial sector that funds itself with sticky deposits and faces occasionally binding constraints. We calibrate the model to the U.S. FRM economy as a benchmark, and compare it to counterfactual economies with alternative mortgage structures.

The model yields three main results. First, rising interest rates affect households and financial intermediaries in opposite directions depending on mortgage structure: under FRMs, intermediary net worth deteriorates; under ARMs, borrower defaults increase but intermediary net worth improves due to higher mortgage payments. Second, financial stability risks exhibit a U-shaped relationship with mortgage fixation length. While the FRM economy is rendered more stable by sticky deposit rates, ARMs provide inherent net worth hedging, which is strengthened by deposit stickiness: defaults typically occur when intermediary net worth is high due to interest income rising relative to the deposit funding cost. This novel hedging force is evidenced by lower risk premia in constrained states relative to the FRM economy. Intermediate fixation lengths of 3 and 5 years minimize intermediary net worth volatility and optimize aggregate risk-sharing, respectively. Third, the optimal fixation length depends on

[1] We develop the intuition behind deviations from this interest rate "immunization" more formally in Appendix Section IV.

the correlation of interest rates with aggregate incomes. In a procyclical rate environment, the optimal fixation lengths are higher – rising to 3.5 and 5.5 years when calibrated to the 1987 to 2024 sample, for instance.

In the model, there are two types of households: borrowers who borrow to finance their housing purchases, and savers, who own intermediaries ("banks"). Households face idiosyncratic income shocks. Borrowers and banks trade in two financial markets: deposits and mortgages. We model flexible mortgage payment structures informed by variation in mortgage contracts across countries. FRMs have fixed payments. In the data, ARMs are typically issued with fixed payments in an initial stage lasting several years, and subsequently convert to floating payments (a fixed spread over the contemporaneous policy rate). Hence, payments that a lender receives on its mortgage portfolio consist of some fixed and some floating cash flows. In the model, we capture this by modeling ARM payments as being floating or fixed with some probability, and map this probability to varying fixed-rate lengths in typical adjustable-rate mortgages.[2]

Motivated by our focus on borrowers' default sensitivity to interest rate changes under ARMs, the model incorporates a realistic notion of liquidity-driven default (Gerardi et al., 2018; Ganong and Noel, 2022), where defaulting allows liquidity-constrained households to increase immediate consumption at the expense of future wealth, in addition to more traditional pure net-worth-driven default motives. To do so, we follow Diamond et al. (2022) and model household decision-making in two distinct stages with a cash-in-advance-type constraint. In the first stage ("consumption stage"), households must rely on liquid assets – income and deposits – to finance consumption, housing costs, and mortgage payments, and decide whether or not to default. They cannot access illiquid housing wealth at this stage. Default provides immediate liquidity but may reduce subsequent wealth. In the second ("trading") stage, households make portfolio decisions to allocate their wealth between deposits, housing, and stocks, and they can adjust their mortgage balance by taking out a new mortgage. Banks lend in the mortgage market subject to a leverage constraint, financing their loan portfolios with savers' equity and deposits, which are risk-free one-period bonds held by households and also elastically demanded by outside investors. ARMs are indexed to the policy rate, while the deposit rate does

[2] We cast the model in real terms to study the redistributive effects of real interest rate changes on borrowers and savers depending on mortgage structure. The effects of mortgage structure also operate through nominal (Fisherian) channels, as studied by Garriga et al. (2017).

not necessarily move one-for-one with the policy rate.[3]

To solve the model, we follow Diamond and Landvoigt (2022) and Diamond et al. (2022) and show that, despite idiosyncratic and undiversifiable risks, borrowers make identical choices per unit of wealth. This removes the borrower wealth distribution as an infinite-dimensional state variable, making the model tractable.

We evaluate the U.S. fixed-rate mortgage regime relative to counterfactual adjustable-rate mortgage economies with varying fixation lengths, starting with the main ARM counterfactual where mortgage rates reset every year. The benchmark FRM economy and ARM counterfactual produce empirically consistent responses to a rise in rates. In the FRM economy, mortgage payments remain stable, slightly reducing defaults since holding on to the current mortgage becomes more valuable, consistent with recent U.S. experience. In contrast, the ARM economy experiences sharply higher mortgage payments, elevated defaults, and a reduction in house values, similar to recent U.K. dynamics (illustrated in Appendix Figures IA.3-IA.4, with the caveat that the model is calibrated to the U.S.).

Under FRMs, banks face unchanged interest income and rising deposit expenses when policy rates increase, reducing net interest margins and profitability despite slightly offsetting decreases in credit losses. In addition, FRMs have a long duration. In response to higher rates, the market value of bank assets falls (Jiang et al., 2024). With both lower cash flows and lower asset values, the net worth of the banking sector declines. More constrained banks demand higher compensation to take on mortgage risk, a key implication of intermediary-based asset pricing models (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Elenev et al., 2016; Diamond and Landvoigt, 2022).

Note that we model intermediaries to reflect the financial sector as a whole. While U.S. banks have experienced a substantial reduction in on-balance sheet mortgage lending and substitution towards mortgage-backed securities (MBS) over recent decades (Buchak et al., 2018, 2024a,b), the banking sector remains the largest private holder of mortgage-backed securities as a whole. In Appendix Figure IA.1, we show that more than half of all non-government residential mortgages (MBS and portfolio loans) are held by the banking sector, and that share

[3] Our reduced-form model of imperfect pass-through is consistent with banks' market power in deposit markets (Drechsler et al., 2017) and time-varying liquidity premia due to the opportunity cost of holding money (Nagel, 2016; Krishnamurthy and Li, 2022).

has remained relatively stable over the past decade.[4] Conversely, in the ARM economy, the net interest margin of banks increases as mortgage payments rise faster than deposit costs, enhancing intermediary net worth despite higher defaults.[5]

We next evaluate how these dynamics translate into financial stability outcomes by evaluating counterfactual economies with mortgage fixation lengths ranging from pure ARMs (annual resets) to the benchmark fully fixed-rate economy. Our headline measure of financial stability is the volatility of intermediary net worth, since financial stability goals of central banks typically relate to the volatility and cost of credit provision,[6] with alternative measures providing similar results. The volatility of intermediaries' return on equity (ROE) exhibits a "U-shaped" relationship with mortgage structure. Volatility is highest in a pure ARM economy where intermediary net worth is very sensitive to interest changes, leading to large negative duration, i.e., net worth increasing in rates. Volatility is somewhat lower in an FRM economy as sticky deposits partially hedge the large positive duration of fixed-rate mortgages, reducing absolute duration.

However, because mortgages carry credit risk, bank net worth sensitivity to interest rates depends not only on policy rates but also on expected credit losses and time-varying risk premia. We find that credit risk hedges interest rate risk in both FRM and ARM economies: defaults rise when net worth is high – for FRMs when rates are low, for ARMs when rates are high, since mortgage payments increase relative to deposit funding cost. But defaults are more rate-sensitive in the ARM economy due to liquidity-driven motives. The stronger net worth hedging effect of ARMs manifests as lower risk premia in constrained states of the world compared to FRMs.

[4]We also conduct robustness exercises where we allow for greater pass-through of policy rates to deposit rates, capturing the notion that other financial intermediaries and holders of MBS may have less sticky sources of funding compared to banks.

[5]This is consistent with recent experiences in the Euro Area where policy rate rises led to net interest rate margin increases, e.g. the ECB Financial Stability Review (May 2024) states: "*In recent years, strong euro area bank profitability has primarily been driven by rising net interest margins. This is because bank funding costs adjusted more slowly than lending rates due to the automatic repricing of floating-rate loans.*"

[6]For instance, the Federal Reserve monitors risks to the financial system "*to help ensure the system supports a healthy economy for U.S. households, communities, and businesses*", and is "*resilient and able to function even following a bad shock*" (https://www.federalreserve.gov/financial-stability.htm). The European Central Bank aims to "*[mitigate] the prospect of disruptions in the financial intermediation process that are severe enough to adversely impact real economic activity*" (https://www.ecb.europa.eu/paym/financial-stability/html/index.en.html).

How general is net worth hedging under ARMs? We characterize the sensitivity of mortgage portfolio values to interest rates in closed form. We decompose the response to a rate rise into an *interest rate channel*, capturing the effect of higher rates on mortgage payments and prices, and a *credit channel*, reflecting higher rates of default and changes in recovery rates. We show analytically under what conditions the credit channel has the opposite sign to the interest rate channel, and a weaker absolute magnitude than the interest rate channel, generating net worth hedging. Under conservative assumptions, we find that the default rate sensitivity to rate rises for ARMs would have to be at least 14 to 15 times greater than generated by our baseline calibration, for the positive cash flow effect from an increase in mortgage income on performing loans to be offset by an increase in credit losses. Detailed derivations and calibration ranges are provided in Appendix II. These results are supported by state-dependent impulse responses evaluated across different points of the borrower wealth distribution, which do not switch sign even when borrowers have very high mortgage-to-income ratios and default more frequently. We thus show that net-worth hedging of ARMs arises robustly under a wide range of plausible paths for interest rates and default, and borrower conditions.

The strength of the net worth hedging channel of ARMs works against the absolute duration advantage of FRMs. An intermediate fixation length of around 3 years balances these opposing forces and minimizes intermediary net worth volatility, reducing the sensitivity of defaults and net worth to interest rate fluctuations.

We further assess how mortgage structure determines risk-sharing between households.[7] To quantify the degree of risk sharing, we measure borrower-saver aggregate risk-sharing by comparing borrower and saver aggregate consumption variance across fixation lengths. Borrower-saver sharing of interest rate risk is optimized at a fixation length of around 5 years – slightly longer than the volatility-minimizing fixation length, suggesting a modest trade-off between financial stability and risk-sharing. In this economy with low effective mortgage duration and default rates that respond little to interest rates, rate shocks have the weakest redistributive effect.[8] However, low exposure to aggregate risk leads borrowers to borrow more, yielding a higher exposure to idiosyncratic risk and highlighting a somewhat subtle downside in equilibrium.

[7] Our focus on interest rate risk sharing through mortgages of various fixation lengths is complementary to Greenwald et al. (2019)'s study of contracts that share house price risks.

[8] See e.g. Auclert (2019).

Lastly, we investigate how results vary with different macroeconomic scenarios, by introducing aggregate income shocks that are correlated with interest rate shocks. In the data, this correlation is time-varying and depends on the sample period, reflecting underlying demand or supply shocks (leading to a positive or negative correlation, respectively). We find that a positive correlation between aggregate income and interest rate shocks of 0.3 (its empirical value in the 1987 to 2024 sample) makes FRM economies relatively more stable and ARM economies relatively riskier.[9] Intuitively, higher rates in an FRM economy come with even lower default risks due to increased borrower incomes. Thus, a positive correlation between income and rate shocks increases the optimal mortgage fixation length. Quantitatively, the fixation length that minimizes intermediary net worth volatility rises from approximately 2.7 to 3.6 years as the correlation increases from -0.3 to 0.3. Overall, these effects are modest and reinforce the central finding that an intermediate fixation length (around 3 to 5 years) best balances financial stability and risk sharing.

Our work has implications for monetary policy and macroprudential regulation of financial stability risks. The paper provides a framework for quantifying how interest rate fluctuations differentially affect financial stability depending on mortgage structure. It thus helps formalize underlying mechanisms behind monetary policy and financial stability linkages. We propose a flexible modeling framework to study the effect of mortgage structure on financial stability, which takes into account endogenous household default decisions, interaction effects between interest rate and credit risk, and the capitalization of the banking system. Our findings highlight how intermediate fixation lengths, common in many countries, can balance sources of volatility in both pure ARM and FRM structures.

Related Literature Our paper makes several contributions to the existing literature. First, we assess macroeconomic implications of different mortgage contract designs, similar to Garriga et al. (2017); Greenwald et al. (2019); Campbell et al. (2021); Guren et al. (2021), but focusing on the novel channel of interest rate and credit risk sharing between households and banks. Conceptually, we thus integrate features of existing quantitative macro-models with financial

[9] In the data, this correlation varies over time, taking positive or negative values in demand or supply-shock driven macroeconomic contexts, respectively. In a finance context, Campbell et al. (2009, 2017, 2020) show that inflation and monetary policy can explain this time variation and variation in the sign of stock-bond return correlation.

intermediaries (e.g. Elenev et al., 2016; Diamond et al., 2022; Sanchez Sanchez, 2023)[10] into a framework with flexible mortgage structures and liquidity-driven default, matching empirical evidence (Gerardi et al., 2018; Ganong and Noel, 2022). Our mechanism is closely related to Campbell and Cocco (2015) who show that fixed- and adjustable-rate mortgages default in different macroeconomic states of the world, and we integrate this intuition into a macroeconomic framework with a banking sector.

Both Campbell et al. (2021) and Guren et al. (2021) focus on the role that mortgage structure can play in providing liquidity to households in downturns when interest rates are low while default rates are high, given the context of the 2008–2009 financial crisis. In contrast, we study how mortgage structure affects household and intermediary outcomes in response to isolated rate shocks given a low historical correlation of income with real rates overall and also given the 2022–2023 rate hike cycle, where both rates and defaults rose in ARM but not in FRM countries. Like Campbell et al. (2021), we study how different mortgage structures expose not just borrowers but lenders to risk. These exposures not only affect the ex-ante prices of mortgages but have implications for the stability of financial intermediary balance sheets, a particular focus of our paper.

We uncover a novel net-worth-hedging channel due to the interaction of credit and interest rate risk. Our paper is thus related to a large body of work that has studied interest rate exposure of intermediaries,[11] and our approach allows us to quantify the role of different mechanisms and equilibrium effects across mortgage structures.

We contribute to existing work on mortgage choice[12] as well as optimal mortgage contract design.[13] Liu (2022) studies household mortgage choice with intermediate fixation lengths

[10]These papers also study the effect of the Government-Sponsored Enterprises (GSEs). In case of default, they guarantee to an MBS trust the "timely payments of principal and interest", but typically repurchase a defaulted mortgage loan within 24 months, meaning that default leads to missed interest payments akin to prepayment (e.g. Fannie Mae, 2023). As a result, GSEs only partially protect intermediaries from cash flow shortfalls in our framework. For FRMs, defaults are higher when rates are low, making prepayment costly. For ARMs, defaults are higher when rates are high (i.e. when mortgage payments are high), also making prepayment costly.

[11]E.g. Hanson et al. (2015); Drechsler et al. (2017); Haddad and Sraer (2020); Drechsler et al. (2021); Gomez et al. (2021); Jiang et al. (2024); Greenwald et al. (2024); DeMarzo et al. (2024); Drechsler et al. (2024); Begenau et al. (2025).

[12]E.g.Campbell and Cocco (2003); Koijen et al. (2009); Badarinza et al. (2018); Liu (2022); Albertazzi et al. (2024); Boutros et al. (2025).

[13]E.g. Piskorski and Tchistyi (2010); Campbell (2012); Eberly and Krishnamurthy (2014); Mian and Sufi (2015); Piskorski and Seru (2018).

where households trade off insurance benefits of longer fixation lengths against declining credit spreads over time. We evaluate the macroeconomic implications of varying fixation length, where rate rises lead to payment increases and household default. Boutros et al. (2025) study the benefits of introducing a contract common in Canada which avoids this feature, increasing contract maturity instead of payment amounts.

Our work is further related to papers that emphasize the role of the mortgage market[14] and financial intermediaries[15] on monetary policy transmission. The paper offers a novel lens to interpret linkages between monetary policy and financial stability,[16] highlighting that mortgage structure mediates how changes in interest rates affect financial stability. Lastly, we contribute to a growing body of work on the financial stability implications (Jiang et al., 2024; Drechsler et al., 2023; Haas, 2023; Varraso, 2023; Begenau et al., 2024) and transmission mechanism (Fonseca and Liu, 2024; Bracke et al., 2024; De Stefani and Mano, 2025) of recent rate rises.

2 Motivating Facts on Mortgage Structure

This section illustrates variation in mortgage structure across a range of different countries. This variation motivates the counterfactual mortgage structures that we study using our model.

2.1 Mortgage Structure Across Countries

There is substantial variation in mortgage market systems and contract structures across countries (Campbell, 2012; Badarinza et al., 2016).[17] Figure 2 shows the average fixed-rate length across countries from different data sources.

[14]E.g. Scharfstein and Sunderam (2016); Di Maggio et al. (2017); Fuster and Willen (2017); Greenwald (2018); Chen et al. (2020); Di Maggio et al. (2020); Berger et al. (2021); Garriga et al. (2021); Eichenbaum et al. (2022); Altunok et al. (2024).

[15]E.g. Wang (2018); Di Tella and Kurlat (2021); Wang et al. (2022); Diamond et al. (2024).

[16]E.g. Adrian and Shin (2008); Hanson et al. (2011); Stein (2012); Borio (2014); Jiménez et al. (2014); Garriga and Hedlund (2018); Smets (2018); Caballero and Simsek (2019); Martinez-Miera and Repullo (2019); Ajello et al. (2022); Boyarchenko et al. (2022); Gomes and Sarkisyan (2023).

[17]In this paper, we do not evaluate the drivers of underlying mortgage structure and take prevalent contract structures as given. Reasons that have been put forward to explain cross-country heterogeneity in mortgage structure include historical path dependence, the availability of long-term mortgage funding, historical inflation experiences (Badarinza et al., 2018), as well as variation in underwriting standards and the role of credit risk (Liu, 2022).

A striking fact noted by Campbell (2012) is that the US appears as an outlier in international comparison, with an average fixed-rate length of almost 25 years, driven by the reliance on 30-year and 15-year FRMs.[18] The U.S. is followed by a group of countries including Denmark, Germany, Belgium, and the Netherlands, which offer mortgages with fixation lengths of up to 30 years, but where the average mortgage outstanding has a length closer to 10 years. For Belgium, data is available only for new mortgage originations, whose maturity is close to 20 years. The vast majority of all other mortgage markets have fixed-rate lengths between two to five years on average, including Australia, Canada, the U.K., Ireland, Portugal, Greece, and Spain. Aside from Denmark, Scandinavian countries (Finland, Sweden, and Norway, the latter with no data on average fixed-rate lengths) are typically considered to originate many pure adjustable-rate mortgages, with rates resetting at least every year.

Even within the common currency euro area, countries vary from longer-term fixed-rate mortgage systems (such as Germany and France) to largely adjustable-rate mortgage systems such as Finland, Greece, Ireland and Portugal, consistent with the divergence in mortgage payments in 2022 in Figure 1.[19]

In sum, mortgages exist on a spectrum from fully adjustable-rate mortgages common in countries such as Finland and Sweden to intermediate fixation periods of two to five years common in many countries, to the 30-year fixed-rate mortgage common in the U.S. We think of mortgages with intermediate fixation periods as sitting between pure ARM and FRM structures from an interest rate risk perspective, as these allow households to fix their mortgage rate for some, but often not all, of the term over which the mortgage is repaid.[20]

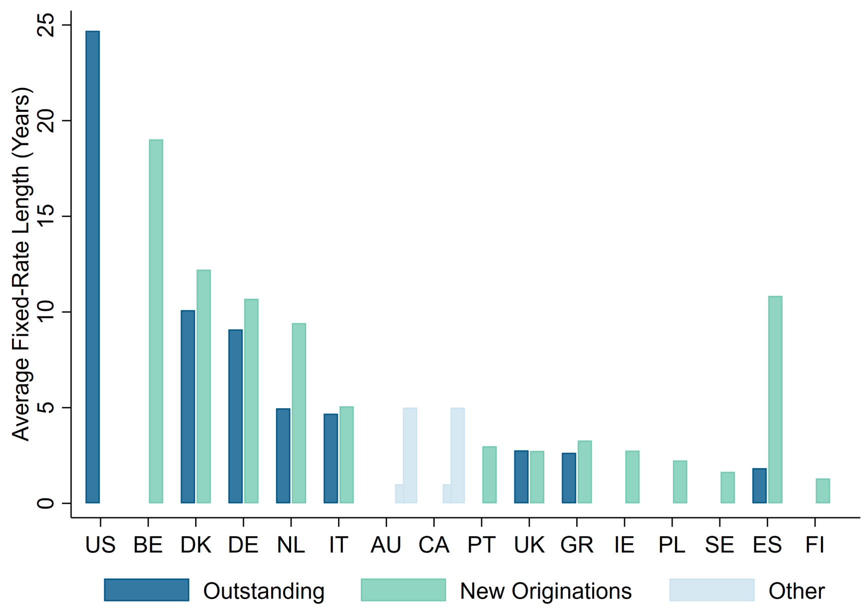
Of course, mortgage structure is not the only economic fundamental that differs across countries. To assess how differential mortgage structures lead to differences in economic outcomes, financial stability, and risk-sharing properties more formally, we develop and calibrate a quantitative model of an FRM economy in the next section, and evaluate counterfactual economies

[18] France is typically considered the only country with a comparable average fixed-rate term. Although precise data on France's average fixation lengths aren't available, the European Mortgage Federation notes that the standard French mortgage carries a 30-year fixed rate.

[19] Spain has seen much longer fixation lengths in recently originated mortgages compared to past originations, likely a result of government interventions in 2022 that allow conversion of adjustable to fixed-rate mortgages, aimed at protecting vulnerable borrowers from interest rate rises, see e.g. Financial Times, November 2022.

[20] Thus fixation length is a distinct feature and different from the choice of the loan repayment window, which is generally 30 years on average in most countries (see Liu (2022) for a more detailed discussion).

Figure 2: Average Mortgage Fixed-Rate Lengths Across Countries



Notes: This figure shows the average mortgage fixation length in years across countries. "Outstanding" reflect data on outstanding mortgages from Badarinza et al. (2016) as of 2013, while "New Originations" reflect data from the European Mortgage Federation (EMF) on new mortgage originations in 2023Q1, from the EMF Quarterly Review of European Mortgage Markets 2023 Q2. "Other" reflects general official data sources that indicate a range of fixation lengths for Australia and Canada. EMF data for the Netherlands reflects information as of 2022 Q4. Average fixation length from EMF data is computed using the binned frequency distribution reported by EMF multiplied by the mid-point of 1-year, 1 to 5-year, 5 to 10 year, and 10-year or greater (assumed to be 10 to 30 year) bins. For Sweden and Finland, only pooled bins from 5 to 10 and 10-year or greater are reported, and thus an average of both bin mid-points is used. For Greece, the unreported remaining distribution is assumed to be 1 to 5 years. Countries are sorted in descending order by "Outstanding" fixation length, or, if missing, by "New Originations".

with a pure ARM structure as well as varying intermediate fixation lengths.

3 Model

In this section, we develop a rich quantitative dynamic model of lending and borrowing in the mortgage market.

Time is infinite and discrete with $t = 0, 1, \ldots$. The economy is populated by continua of two types of households with preferences over housing and non-durables – borrowers labeled B

indexed by $i \in [0, \ell]$ and savers labeled S indexed by $i \in (\ell, 1]$.

Households' utility function is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i, h_{t-1}^i)$$

$$u(c_t^i, h_{t-1}^i) = \frac{\left((c_t^i)^{1-\theta}(h_{t-1}^i)^{\theta}\right)^{1-\gamma} - 1}{1-\gamma}$$

where β is the discount factor, θ governs the share of housing in the utility function, and γ is the coefficient of relative risk aversion.

The aggregate supply of houses is exogenous and fixed at $\bar{H}$, with a fraction α_H owned by borrowers while the remaining fraction $1 - \alpha_H$ belongs to savers. Only borrowers trade houses. Each unit of housing requires a maintenance payment of δ_h every period to prevent its full depreciation.

Non-durable goods are produced by a continuum $k \in [0, 1]$ of Lucas trees, whose aggregate yield each period is given by Y_t. Borrowers own a total of α trees, while savers own the remaining $1 - \alpha$. Each type of agent can trade trees within their type, but not across types. The yield of borrower-owned trees is subject to an idiosyncratic shock ε_t^i, which is i.i.d. across borrowers and time. Saver-owned trees are not subject to idiosyncratic shocks. Therefore, each household's income is given by

$$y_t^i = s_{t-1}^i (Y_t + \varepsilon_t^i) \qquad \forall i \in [0, \ell]$$

$$y_t^i = s_{t-1}^i Y_t \qquad \forall i \in (\ell, 1]$$

where s_{t-1}^i is the share of trees owned by household i at the start of period t, so that markets clear when $\int_0^{\ell} s_{t-1}^i di = \alpha$ and $\int_{\ell}^1 s_{t-1}^i di = 1 - \alpha$.

In addition to trading houses, borrowers trade in two financial markets – deposits and mortgages. Deposits are one-period risk-free bonds, while mortgages are long-term, defaultable, and may have fixed or adjustable payments.

Their counterparties in these markets are banks labeled I (short for "intermediaries"). Banks are firms who issue equity to saver households.

3.1 Borrowers

Following Diamond et al. (2022), we split each period into two stages – *consumption* and *trading*. In the consumption stage, shocks are realized, and borrowers make mortgage payments or default, subject to a cash-in-advance type constraint, which allows them to only access their liquid wealth, not their illiquid housing wealth. In the trading stage, all households make portfolio choices.

Borrowers enter the period with trees s^i_{t-1}, housing h^i_{t-1}, a mortgage with outstanding balance m^i_{t-1}, and deposits d^i_{t-1}. Their trees yield income y^i_t after the realization of aggregate and idiosyncratic income shocks.

Mortgage Structures To flexibly characterize payment structures under different mortgage regimes, we introduce π_τ as the probability of a mortgage being in the floating stage, with $1 - \pi_\tau$ the probability of a mortgage being in the fixed stage. The implied duration of the fixed stage, or "fixation length," is $\frac{1}{\pi_\tau}$. To avoid tracking the fixed vs. floating status of every borrower's mortgage, in the numerical solution of the model we let the floating indicator $\mathbb{1}^i_\tau$ be an i.i.d. Bernoulli random variable, which takes a value of 1 (floating) with probability π_τ for each borrower in each period. Thus, a pure fixed-rate mortgage (FRM) would have $\pi_\tau = 0$, and a pure adjustable-rate mortgage (ARM) that resets annually would have $\pi_\tau = 1$.

In the fixed stage, the outstanding balance of the mortgage implies a fixed mortgage payment $x^i_t = \iota_f + \delta_m \bar{q}^m$ per unit of mortgage m^i_{t-1}, where ι_f denotes the interest component and the principal component is normalized to fraction δ_m of the steady-state mortgage price $\bar{q}^m$.

In the floating stage, the mortgage payment is determined by the policy rate r^p_t plus the spread ι_a on adjustable-rate mortgages. To summarize:

$$x^i_t = \begin{cases} \iota_f + \delta_m \bar{q}^m, & \mathbb{1}^i_\tau = 0 \\ r^p_t + \iota_a + \delta_m \bar{q}^m, & \mathbb{1}^i_\tau = 1. \end{cases}$$

After payments are made, the mortgage balance decreases by δ_m, such that the remaining balance is $(1 - \delta_m) m^i_{t-1}$.

Consumption Stage In the consumption stage, households use income y_t^i and their deposits holdings d_{t-1}^i to make mortgage payments $x_t^i m_{t-1}^i$, housing maintenance payments $\delta_h h_{t-1}^i$, and to consume.

Households can choose to default by failing to make the mortgage payment. If they default, they lose their house and their mortgage balance is written off. They also lose a fraction λ of their endowment of Lucas trees and face a continuous idiosyncratic shock to their post-default value function. In other words, default carries both a pecuniary and a non-pecuniary cost.

A household that repays the mortgage faces a consumption-stage budget constraint given by:

$$c_t^{i,nd} + x_t^i m_{t-1}^i + \delta_h h_{t-1}^i + a_t^i = y_t^i + d_{t-1}^i$$

where $a_t^i \geq 0$ denotes holdings of intra-period deposits that a household can bring into the trading stage in lieu of consuming. It enters the trading stage with wealth:

$$w_t^{i,nd} = a_t^i - (1 - \delta_m) m_{t-1} q_t^m + p_t^h h_{t-1}^i + p_t^s s_{t-1}^i$$

where q_t^m is the price of the mortgage, p_t^h is the price of housing, and p_t^s is the price of the Lucas trees. The nonnegativity constraint $a_t^i \geq 0$ operates similarly to cash-in-advance or working capital constraints, requiring borrowers to have enough liquidity to finance their consumption before being able to rebalance their portfolios by selling assets or borrowing.

A household that defaults faces a budget constraint given by:

$$c_t^{i,d} = y_t^i + d_{t-1}^i$$

Having expunged their mortgage, lost their house, and given up a fraction of future income, it enters the trading stage with wealth:

$$w_t^{i,d} = (1 - \lambda) p_t^s s_{t-1}^i$$

The default decision depends on the utility of consumption plus the continuation value as represented by the trading stage value function $V_t^i(w_t^i, \mathcal{Z}_t)$, where $\mathcal{Z}_t$ denotes state variables exogenous to an individual borrower – aggregate borrower wealth W_t^B, saver wealth W_t^S, and the exogenous process states Y_t and r_t^p.

Denote the value of default by $V^{i,d}$ and the value of repayment by $V^{i,nd}$. The value of making the mortgage payment is given by:

$$V_t^{i,nd}(d_{t-1}^i, m_{t-1}^i, \mathbb{1}_\tau^i, h_{t-1}^i, \epsilon_t^i, \mathcal{Z}_t) = \max_{a_t^i \geq 0} u(c_t^{i,nd}, h_{t-1}^i) + V(w_t^{i,nd}, \mathcal{Z}_t)$$

while the value of default is given by:

$$V_t^{i,d}(d_{t-1}^i, m_{t-1}^i, \mathbb{1}_\tau^i, h_{t-1}^i, \epsilon_t^i, \mathcal{Z}_t) = u(c_t^{i,d}, h_{t-1}^i) + V(w_t^{i,d}, \mathcal{Z}_t)$$

subject to the budget constraints and wealth evolution equations above. Households default iff:

$$\eta_t^i V_t^d(\cdot) > V_t^{nd}(\cdot)$$

where η_t^i is the household's idiosyncratic default shock.

Trading Stage In the trading stage households make portfolio decisions. They allocate their wealth w_t^i between deposits d_t^i, housing h_t^i, and Lucas trees s_t^i. They can also revise their mortgage balance from $(1-\delta_m)m_{t-1}^i$ to m_t^i at current price q_t^m.[21]

Borrowers are subject to a cost of deviating from a target loan-to-value ratio, given by $\Phi\left(\frac{q_t^m m_t^i}{p_t^h h_t^i} - \overline{LTV}\right)$. This cost, rebated as $\mathcal{R}_t^i$ to the household in proportion to wealth to neutralize income effects, captures the notion of a mortgage rate schedule in reduced form and rules out equilibria in which borrowers take on LTV ratios >> 1 at very high rates in the expectation that they will likely default.

The trading stage budget constraint is given by:

$$w_t^i + \mathcal{R}_t^i = \frac{d_t^i}{1+r_t^d} + q_t^m m_t^i + p_t^h h_t^i + p_t^s s_t^i + \Phi\left(\frac{q_t^m m_t^i}{p_t^h h_t^i} - \overline{LTV}\right)$$

[21]This formulation does not allow households to prepay the mortgage at par, i.e., refinance. Granting households an option to prepay would make FRMs less attractive to intermediaries by limiting intermediary gains from rate cuts in the FRM economy (e.g. Hanson, 2014; Diep et al., 2021).

and the value function is:

$$V(w_t^i, \mathcal{Z}_t) = \max_{d_t^i, h_t^i, s_t^i, m_t^i} \beta \mathrm{E}_t \left[\max \left\{ \max_{a_t^i \geq 0} u(c_{t+1}^{i,nd}, h_t^i) + V(w_{t+1}^{i,nd}, \mathcal{Z}_t), \eta_t^i \left(u(c_{t+1}^{i,d}, h_t^i) + V(w_{t+1}^{i,d}, \mathcal{Z}_t) \right) \right\} \right]$$

where the innermost maximization indicates the optimal consumption-savings choice in next period's consumption stage, the middle maximization indicates the default decision, and the outermost maximization indicates portfolio choices in the current period.

The model characterizes two types of default: when households have ample liquid assets, they equalize the marginal utility of consumption and wealth. As a result, default is net-worth driven and occurs to increase net wealth.[22] In contrast, when households have low liquid assets, they may prefer to give up wealth in the trading stage to consume in the consumption stage. Given the constraint, the only way to increase liquid assets at that point is to default, and thus household default can also be liquidity-driven even if at a cost to future wealth. Liquidity-driven default is particularly relevant for ARMs, where mortgage payments can increase together with policy rates.

3.2 Banks

Banks frictionlessly issue equity to savers and so maximize the stream of dividends discounted at the saver's stochastic discount factor.

They lend in the mortgage market, financing their loan portfolios with equity and deposits. Deposits are risk-free one-period bonds held by borrowers and outside investors. Outside investors have perfectly elastic demand for deposits at a price of $\frac{1}{1+r_t^d}$. The deposit rate r_t^d differs from the policy rate r_t^p to which adjustable mortgages are indexed. Recent work has shown that changes to policy rates do not pass through one-for-one to deposits, complicating banks' exposure to interest rate risks.[23] We model the relationship in reduced form as

$$r_t^d = (\bar{r} - \alpha_d) + \beta_d(r_t^p - \bar{r})$$

[22] This has been referred to as "strategic" default in the literature (Gerardi et al., 2018; Ganong and Noel, 2022), but in our framework both types of default are optimal decisions by households.

[23] E.g., Nagel (2016), Drechsler et al. (2017), and Krishnamurthy and Li (2022)

with $\alpha_d \geq 0$ and $\beta_d \in (0, 1]$. The parameter α_d captures the average spread between policy and deposit rates, while β_d capture the degree of deposit rate sensitivity to policy rate deviations from its mean. When $\alpha_d = 0$ and $\beta_d = 1$, the two rates are always equal.[24]

Bank portfolios are perfectly diversified and hence identical across banks, so we can write the bank's problem without i subscripts. They enter a period with a stock of outstanding mortgages m^I_{t-1}, of which a fraction F^η_t default. On mortgages that do not default, banks receive a payment x_t per unit of mortgage m^I_{t-1} and have an ex-payment value $(1 - \delta_m)q^m_t$.

Mortgage defaults lead lenders to seize the house, on which they must make a maintenance payment before selling it in foreclosure at a price $p_t(1 - \zeta)$ per unit, where ζ represents a foreclosure cost. The total foreclosure proceeds are

$$\int_0^\ell \mathbb{1}^i_{\text{default}} h^i_{t-1} p_t((1 - \zeta) - \delta_h) di$$

The payoff per unit of mortgage is therefore:

$$\mathcal{X}_t = (1 - F^\eta_t)(x_t + (1 - \delta_m)q^m_t) + \int_0^\ell \mathbb{1}^i_{\text{default}} \frac{h^i_{t-1}}{m^I_{t-1}} p_t((1 - \zeta) - \delta_h) di$$

Running the intermediation technology is costly. Banks must pay a fraction ν of the value of their mortgage portfolio as intermediation costs. Their net worth is then given by:

$$w^I_t = (1 - \nu)\mathcal{X}_t m^I_{t-1} + d^I_{t-1}$$

where negative values of d^I_t represent borrowing by the lender.

Banks use their equity deposits to finance dividends and mortgage purchases, maximizing

[24]Because savers own banks, when banks are unconstrained $r^d_t = 1/\mathrm{E}_t[\mathcal{M}^S_{t,t+1}] - 1 \equiv r^f_t$, i.e. the deposit rate emerges as the risk-free rate, the rate at which the saver discounts risk-free cash flows in the next period. However, when banks are constrained, the savers' risk-free rate r^f_t would be greater than r^d_t, capturing the shadow cost of relaxing bank constraints. As a result, the risk-free rate is not generally equal to the policy rate r^p_t. Note that the discounted present value of all future "deposit spreads" $r^f_t - r^d_t$ has been referred to as (gross) franchise value in the literature (e.g. Drechsler et al., 2017, 2023; Haddad et al., 2023; DeMarzo et al., 2024; Jiang et al., 2024). In our framework, r^p_t governs the "loan spread" (mortgage rates less deposit rates) in the ARM economy by serving as the indexation rate for floating mortgage payments, but is not generally equivalent to r^f_t as explained above. Instead, the value of $r^p_t - r^d_t$ is priced when pricing the mortgage with the saver's SDF.

$$\max_{m_t^I, d_t^I} \mathrm{E}_t \left[\sum_{s=t}^{\infty} \mathcal{M}_{t,s}^S \mathrm{Div}_t \right]$$

subject to a budget constraint:

$$w_t^I = \frac{d_t^I}{1 + r_t^d} + q_t^m m_t^I + \mathrm{Div}_t$$

and a capital requirement:

$$-d_t \leq \xi(\kappa \bar{q}^m + (1 - \kappa) q_t^m) m_t^I$$

where ξ represents the maximum leverage ratio and κ represents the fraction of the mortgage portfolio that is carried at book value on the lender's balance sheet. A value of $\kappa = 1$ indicates that mark-to-market losses on the mortgage portfolio do not tighten leverage constraints, while $\kappa = 0$ indicates a fully mark-to-market regime.

3.3 Savers

Saver households have the same preferences as borrowers, but receive income from their shares of Lucas trees free from idiosyncratic risk. As owners of bank equity, they also receive net dividends from the banks. Finally, they are rebated lump-sum the costs associated with mortgage default – both the pecuniary cost of default faced by borrowers and the foreclosure cost faced by banks – as well as the cost of intermediation. Their budget constraint is simply:

$$c_t^s = \mathrm{Div}_t + \frac{\alpha}{\ell} Y_t + \mathrm{Rebates}_t.$$

3.4 Equilibrium

Given the exogenous processes for aggregate income Y_t and policy rate r_t^p and given the idiosyncratic income shocks ε_t^i, ARM reset shocks $\mathbb{1}_\tau^i$, and idiosyncratic default shocks η_t^i, an equilibrium is a set of borrower household allocations $\{c_t^i, h_t^i, s_t^i, m_t^i, d_t^i, a_t^i\}_{t=0}^\infty$, borrower default decisions $\{\mathbb{1}_{\text{default}^i}\}_{t=0}^\infty$ bank allocations $\{\text{Div}_t, m_t^I, d_t^I\}_{t=0}^\infty$, saver allocations $\{c_t^S\}_0^\infty$, and prices $\{p_t^h, p_t^s, q_t^m\}_{t=0}^\infty$ such that each agent maximizes their value function subject to their constraints, and the following market-clearing conditions hold:

1. The mortgage market clears:

$$(1-\ell)m_t^I = M_t^B \equiv \int_0^\ell m_t^i di$$

2. The borrower housing market clears:

$$\alpha_H \bar{H} = H_t^B \equiv \int_0^\ell h_t^i di$$

3. The market for borrower Lucas tree shares clears:

$$\alpha = \int_0^\ell s_t^i di$$

Note that the elastic demand for deposits by outside investors at rate r_t^d implies that the deposit market within the model does not need to clear.

Appendix III contains the derivation of the equilibrium conditions and the solution to the model.

4 Calibration

We calibrate the model at an annual frequency in two steps. Table 1 displays parameters whose values we choose outside the model based on external sources. Table 2 displays "internally" calibrated parameters, whose values are chosen so that the model with fixed-rate mortgages ($\pi_\tau = 0$) matches moments estimated in the data. We discuss each set of parameters in turn.

Stochastic Environment Aggregate dynamics of the model are governed by shocks to aggregate income Y_t and the policy rate r_t^p. In our baseline calibration, we abstract away from income shocks, setting $Y_t = 1$. The mean policy rate $\bar{r}$ is set to the average 1-year Treasury Constant Maturity rate from 1987 to 2024. Its deviations from the mean are parameterized by an AR(1) process with standard deviation σ_r, and persistence ρ_r, calibrated to match the dynamics of the 1-year real rate, from the Cleveland Fed, over the same sample. We estimate a mean rate of 0.034, an unconditional standard deviation of 0.014, and a persistence of 0.724. The standard deviation and persistence parameters imply the standard deviation of interest rate shocks.

We normalize the idiosyncratic income shocks to have a mean of 0, which means that they are governed by two parameters. The probability of a low income realization π_L is set to 0.058, which is the average post-war unemployment rate. The magnitude of the low income shock ϵ_L is set based on the Ganong and Noel (2019) estimates of the income loss from unemployment. They find that income loss occurs gradually over the first year as unemployment insurance expires. Since our model is annual, we average the income loss in months after UI kicks in as reported in Figure 2, Panel A of that paper, producing a value of -0.456. The high income shock ϵ_H is set to ensure that the expected value of the idiosyncratic income shock is zero.

Deposit Rates Bank deposit rates are lower than policy rates, such as T-Bill and Fed Funds, on average and adjust less than one for one with those rates. We estimate deposit rates using quarterly Call Reports data from 1987 to 2024 as the ratio of interest expense to previous quarter's balance on all non-time deposits. The main role deposits play in our model is that they provide liquidity – they are the only asset that can be liquidated to finance consumption in the consumption stage. Time deposits incur penalties for liquidation before maturity, motivating their exclusion. We set α_d to the average spread between the Fed Funds rate and the deposit rate of 0.018.

It often takes multiple quarters for deposit rates to adjust after a change in the Fed Funds rate. Our specification of r_t^d as a linear function of r_t^p does not allow for such inertia, and contemporaneous responses of deposit rates may understate the sensitivity of the deposit rate to policy rate. We estimate a VAR(1) of Fed Funds and deposit rates and set $\beta_d = 0.340$, the

Table 1: Externally Calibrated Parameters

Parameter		**Value**
Panel A: Stochastic Processes		
Mean of policy rate process	μ_r	0.034
Std. dev. of policy rate process	σ_r	0.014
Persistence of policy rate process	ρ_r	0.724
Probability of low idiosyncratic income shock (ϵ_L)	π_L	0.058
Idiosyncratic income drop in low state	ϵ_L	-0.456
Idiosyncratic income increase in high state	ϵ_H	Set such that E[ϵ] = 0
Panel B: Deposit Rates		
Deposit spread w.r.t. policy rate	α_d	0.018
Deposit sensitivity w.r.t. policy rate	β_d	0.340
Panel C: Borrowers and Savers		
Borrower population share	ℓ	0.400
Borrower income share	α	0.600
Borrower housing share	α_h	0.500
Risk aversion	γ	1.5
Panel D: Housing, Mortgages and Banks		
Housing maintenance payment	δ_h	0.020
Mortgage rate reset probability	π_τ	0.000
Deviation from target LTV cost	ϕ	0.050
Max. leverage ratio	ξ	0.920
Share at book value	κ	0.000

peak of the deposit rate impulse response to a one-unit shock to the Fed Funds rate.

Population, Income, and Housing Shares Using 2023 SCF data, we set $\ell = 0.400$ to the approximate share of homeowners that have a mortgage LTV of at least 30%. Given this definition of borrowers, $\alpha = 0.600$ and and $\alpha_h = 0.500$ are set to the approximate shares of income and housing, respectively, held by borrowers in the SCF data.

Banks Banks are subject to a capital requirement that limits their leverage. We set the maximum leverage ratio ξ to 0.920, which is the maximum Tier 2 capital ratio for banks under Basel III. This calibration effectively assumes a mortgage risk weight of 100%, which is the standard risk weight for residential mortgages. In the baseline calibration, we set the book value share κ to zero,meaning that mortgages are held at market value.

Borrower Preferences, Housing, and Defaults Housing maintenance payments as a fraction of housing are set to 0.020 based on the post-war average residential housing depreciation

Table 2: Internally Calibrated Parameters

Parameter		**Value**	**Target**	**Model (FRM)**	**Data**
Panel A: Borrowers					
Patience	β	0.969	Mortgage/income	150.37	151
Housing utility weight	θ	0.183	Housing/income	255.56	253
Std. dev of idiosyncratic default shock	σ_η	0.045	Default rate	2.32	2.45
Income loss upon default	λ	0.148	Deposits/income	24.49	28
Panel B: Intermediaries					
Foreclosure cost	ζ	0.530	LGD	21.00	20.8
Intermediation cost	ν	0.034	Mortgage rates	5.90	5.88
Principal payment share	δ_m	0.085	Mortgage duration	6.92	6.91

rate. Our model does not include housing investment, so the maintenance payment can be thought of as investment needed to offset depreciation and maintain housing stock at its steady-state value.

We set household risk aversion γ to 1.5, a standard value in the literature.

The remaining set of borrower preference and default-related parameters are calibrated internally. Panel A of Table 2 displays four parameters that must be calibrated jointly. We set patience β to 0.969, which yields a mortgage/income ratio of 150.37% given the values of other parameters, matching its value in the 2023 SCF. The value of housing to income is determined in equilibrium by the present value of user costs parameterized by the utility weight on housing θ, discounted at the rate implied by β and the probability of losing the house in foreclosure (i.e, default rate). We set θ to 0.183 such that, at the target default rate and given the calibrated value of β, the value of housing/income 255.56% matches 253% in the SCF.

Housing- and mortgage-to-income ratios imply a LTV ratio of approximately 60%. The mapping of this ratio into default rates depends on two parameters – the standard deviation of the idiosyncratic default shock σ_η and the share of future income lost in default λ. The pecuniary cost of default motivates agents to hold deposits so that they can decrease their default probability in the event of a low income realization. We set σ_η to 0.045 and λ to 0.148 to match the average 2003-2023 flow into 90-day delinquency in the New York Fed's Quarterly Report on Household Debt and Credit (QRHDC) of 2.45%, and the deposits-to-income ratio of 28% in the SCF.

Mortgages In our model, there are no idiosyncratic shocks to home values, so in the cross-section defaulting households have the same LTV ratios as non-defaulting households. Given the LTV ratio implied by the calibration of housing and mortgage-to-income ratios, we set foreclosure cost ζ to 0.530, which implies a loss given default (LGD) of 21.00%. This is consistent with the average LGD in the data, computed as average charge-off rate on mortgages held by depository institutions from the St. Louis Fed FRED database, divided by the average default rate from the NY Fed QRHDC.

The mortgage interest payment in the FRM economy ι_f is set so that the steady-state mortgage price $\bar{q}^m$ is equal to 1, and thus ι_f can be interpreted as the steady-state mortgage yield, or par rate. The historical average rate is 0.059. In the model, the mortgage yield, defined as the discount rate which discounts expected future cash flows to par, depends on (1) the intermediary's cost of funding, a leverage-weighted average of the equity cost of capital implied by β and the deposit cost of capital $\bar{r} - \alpha_d$, (2) expected losses, a function of the default rate and LGD, and (3) the cost of intermediation parameterized by ν. Given a calibration that matches target default rates and LGD, we set ν to 0.034 so that $\bar{q}^m = 1$ at $\iota_f = 0.059$. In counterfactual exercises with adjustable rate mortgages, we set $\iota_a = \iota_f - \bar{r}$, making payments the same on average in both stages.

Borrowers in our model do not endogenize the effect of their demand on *their*, rather than the equilibrium, mortgage rate.[25] As a result, at low equilibrium rates, they may face an incentive to take on a large mortgage that implies a high default probability and hence a low expected cost of borrowing. One way to address this issue is to set a maximum LTV constraint that would be slack in steady state but bind in some states of the dynamic model. To simplify model solution, we follow a different approach and impose a per-housing-dollar quadratic cost of deviating from the steady-state book LTV ratio $\frac{\phi}{2}\left(\frac{q_t^m}{p_t h_t^i} - \overline{LTV}\right)^2$. We set ϕ to a small positive value of 0.050. It has negligibly small effects on equilibrium dynamics but improves our ability to solve the model by ruling out equilibria with counterfactually high LTV ratios.

The last mortgage contract feature is the fraction of the principal paid in each period, δ_m. This parameter determines the duration of the mortgage, which we set to match the duration

[25] Models with an endogenous debt schedule and long-term debt must tackle dilution incentives and the optimal contract can be difficult to solve. In our framework, such a model would be intractable.

of a 30-year fixed rate mortgage in the data. Our model generates an endogenous reduction in duration relative to its contractual value that occurs because of default, but we do not capture the reduction due to moving-induced prepayments. To calculate the correct target duration in the data, we compute an amortization schedule for a 30-year fixed rate mortgage with a rate of ι_f and an annual prepayment probability of 6%, close to the unconditional annual moving probability of mortgage borrowers reported by Fonseca and Liu (2024). This procedure yields δ_m equal to 0.085, which implies a duration of 6.9 years.[26]

4.1 Model Solution

The model is solved numerically using the global Transition Function Iteration method of Elenev et al. (2021). Our main experiments compare the performance of the economy across a range of mortgage fixation lengths parameterized by π_τ. When this parameter is equal to 0, the economy is in a fully fixed-rate mortgage (FRM) regime. At the other extreme when π_τ is equal to 1, the economy is in an adjustable-rate mortgage (ARM) regime where mortgage payments reset every year. For each economy considered below, we simulate 16 paths of 5,000 periods each after discarding the first 1,000 and report unconditional moments of the long simulation. We also consider impulse responses to interest rate shocks. To compute impulse responses, we initialize the economy at the stochastic steady state of a long simulation at $t = 0$ and compute its $t = 1$ transition given a particular realization of exogenous variables. Subsequently, we let the economy evolve stochastically, simulating 5,000 paths of 25 years each. The average path constitutes the reported impulse response.

5 Results

We first show that rising interest rates affect households and financial intermediaries in opposite directions depending on mortgage structure, using impulse responses to an increase in rates. Under FRMs, intermediary net worth deteriorates; under ARMs, borrower defaults increase but intermediary net worth improves due to higher mortgage payments. Second, we

[26] Appendix II shows the duration of a FRM to be $1/(\iota_f + \delta_m)$, implying the mapping between δ_m and duration for a given ι_f.

show outcomes for financial stability and risk-sharing based on unconditional moments of long simulations across a range of counterfactual mortgage structures. We find that mortgages with intermediate fixation lengths balance sources of volatility in pure ARM and FRM structures, minimizing intermediary net worth volatility and optimizing aggregate risk sharing. Lastly, we show that the optimal fixation length depends on the macroeconomic environment, captured by the correlation of interest rate risk with aggregate income risk.

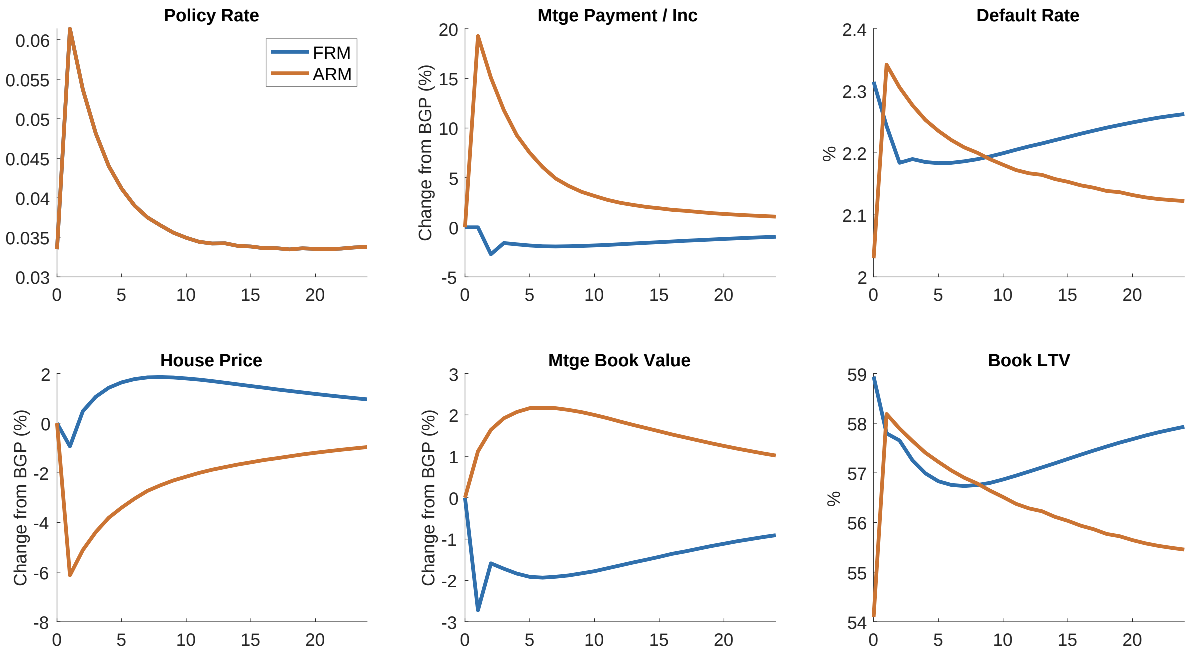
5.1 FRM vs. ARM Economies Respond Differently to Rate Shocks: Impulse Responses and Mechanisms

First, to understand how mortgage structure modulates interest rate shocks, we analyze impulse responses of the pure FRM and ARM economies to a positive shock to the policy rate r_t^p, which raises it from 3.1% to 6%. Figure 3 displays the results for borrower variables, while Figure 4 displays the results for banks.

Borrowers When rates are fixed ("FRM"), total mortgage payments remain unchanged on impact and borrower liquidity is unaffected. Mortgage rates go up, but existing borrowers are shielded from the increase. In contrast, when mortgage payments reset every year ("ARM"), borrowers face higher payments immediately. The liquidity burden of higher payments causes a spike in default rates with higher defaults persisting as long as rates and hence payments remain higher. With FRMs, higher rates raise the opportunity cost of default because holding on to the current mortgage with a lower rate becomes more valuable, as captured by the decline in mortgage value and hence borrower LTV. As a result, borrowers are less likely to default for strategic reasons. At the same time, new borrowers face higher mortgage rates and are less likely to take out a loan, decreasing the aggregate mortgage balance and driving down demand for housing, leading to a slight decrease in house prices. However, the persistent decrease in default rates due to lower LTVs raises house prices subsequently.[27] For ARMs, the need to spend a larger share of their liquid assets on mortgage payments disproportionately reduces borrower consumption relative to wealth. The desire to smooth consumption raises demand for

[27] We do not model explicit mortgage lock-in effects (Fonseca and Liu, 2024) and their impact on house prices in an FRM economy, see e.g. Fonseca et al. (2024).

Figure 3: Impulse Responses to a Positive Interest Rate Shock: Borrowers

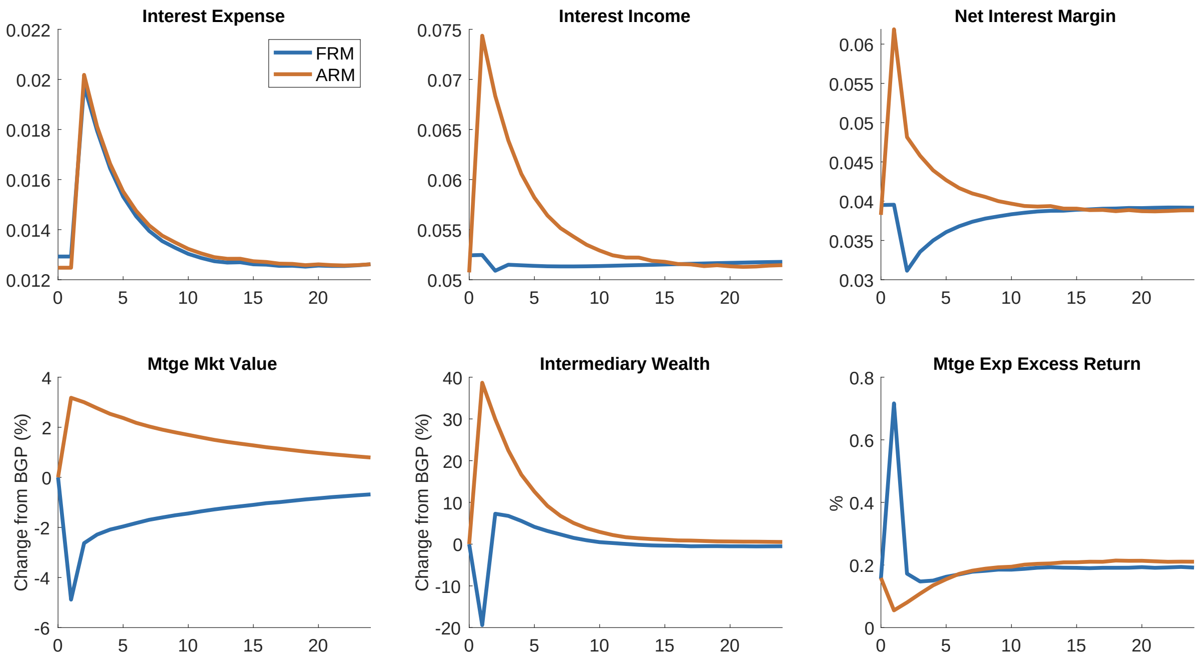


Notes: Impulse Response Functions for a positive shock to the interest rate r_t^p. "FRM" (blue) denotes an economy in which mortgage payments remain fixed at $\iota_f + \delta_m \bar{q}^m$. "ARM" (orange) denotes an economy with a rate fixation length of 1 year ($\pi_\tau = 1.0$) in which mortgage payments mortgages increase with rates $r_t^p + \iota_a + \delta_m \bar{q}^m$.

credit, resulting in larger mortgage balances relative to the FRM economy.

Banks The different dynamics of default and credit demand have consequences for the financial sector. The top row of Figure 4 plots banks' net interest margin and its components in output units, to aid comparison. When rates go up, the cost of deposit funding – the banks' interest expenses – also increases, though less than one for one. When mortgage rates are fixed, interest income remains unchanged, leading to a drop in banks' net interest margin. Banking becomes less profitable, despite the slight offsetting decrease in credit losses discussed above (borrowers default less because their low-rate mortgage becomes more attractive). Moreover, fixed-rate mortgages have a long duration. The bottom row of Figure 4 plots asset pricing moments. In response to higher rates, the price and market value of long-dated bank assets falls. With both lower cash flows due to smaller net interest margins, and lower asset values due to higher discount rates, the net worth of the banking sector declines. More constrained banks demand higher compensation to take on mortgage risk, a result common to intermediary-based

Figure 4: Impulse Responses to a Positive Interest Rate Shock: Banks



Notes: Impulse Response Functions for a positive shock to the interest rate r_t^p. "FRM" (blue) denotes an economy in which mortgage payments remain fixed at $\iota_f + \delta_m \bar{q}^m$. "ARM" (orange) denotes an economy with a rate fixation length of 1 year ($\pi_\tau = 1.0$) in which mortgage payments mortgages increase with rates $r_t^p + \delta_m \bar{q}^m$.

asset pricing models. The spike in risk premia, i.e. expected excess returns on mortgages, amplifies mortgage duration, further contributing to market value losses of banks as it increases discount rates.

In contrast, in the ARM economy, higher rates lead to higher mortgage payments. Since mortgages are indexed to the policy rather than the deposit rate, the net interest margin of banks (cash flows) increases as mortgage income received rises by more than deposit expense paid. Banks become more profitable even though credit losses rise due to an increase in defaults. Intuitively, banks' credit losses in the ARM economy partially offset higher cash flows from mortgage payments, which act as a hedge. Stronger cash flow news than discount rate news raise mortgage values. This implies that adjustable-rate mortgages effectively have negative duration. With higher cash flows and higher asset values, the net worth of the banking sector increases. The increase in intermediary net worth lowers mortgage risk premia. But risk premia are nonlinear in intermediary net worth. An improved capital position of already healthy banks in the ARM economy does not reduce risk premia much, but a deterioration in bank balance sheets in the FRM economy leads to a sharp spike in risk premia.

Cross-Country Evidence Do these model predictions have empirical support? We show illustrative evidence consistent with predicted differences in FRM and ARM economies using differential developments in US and UK delinquencies, house prices, and bank equities over 2022 to 2024, as well as in other ARM economies. Appendix Figure IA.3 shows that delinquencies in the UK rose by more than 60% from their 2022 levels by the beginning of 2024, whereas US delinquencies actually declined by almost 20% (albeit from a higher level). Figure IA.6 shows that US real house prices outperformed house prices in ARM economies by 10 to 15% between 2022 and 2024. Similarly, UK, Australia, and Euro Area bank equities outperformed bank equity indices in the US (and also Canada) by almost 40 per cent.

While merely suggestive since the model ARM economy is a U.S. counterfactual, not a calibration, e.g., to the U.K., we consider the differential cross-country evidence in outcomes following the 2022 to 2023 rate tightening cycle as highly consistent with our model's predictions.

5.2 Financial Stability

We start by building intuition for mechanisms in the full FRM and ARM economies, before evaluating financial stability across a broader range of mortgage structures. We measure financial stability as the volatility of intermediary net worth, reflecting that financial stability goals by central banks relate to the volatility and cost of credit provision.

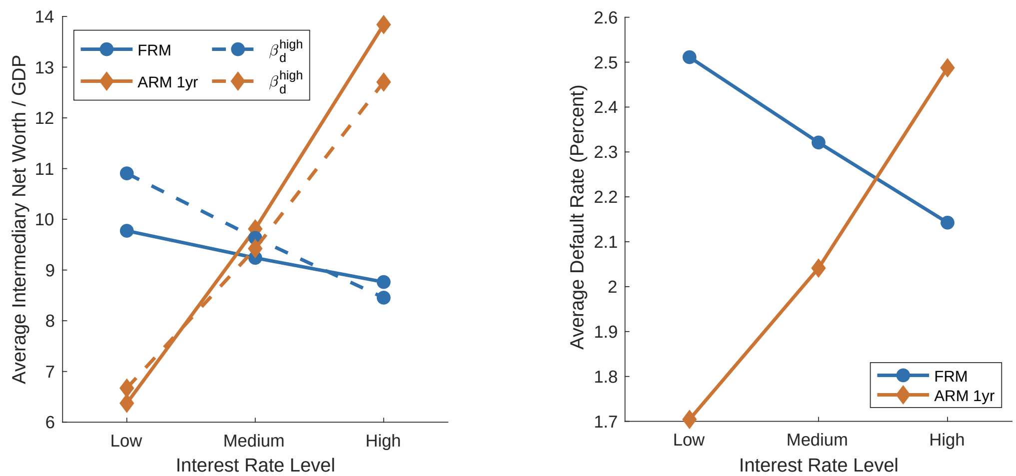
5.2.1 Interest Rate Levels, Intermediary Net Worth, and Default

The impulse responses suggest that intermediary net worth is differentially correlated with interest rates depending on the underlying mortgage structure. Figure 5 confirms this intuition, showing average levels of intermediary net worth to GDP by interest rate levels from long simulations of each economy (Panel a). Intermediary net worth is strongly increasing in interest rates for ARM economies, meaning it has large negative net worth duration. Interest income rises by more than deposit funding cost increases, which outweighs the rise in defaults (Panel

Figure 5: Intermediary Net Worth and Default By Level of Interest Rate

(a) Intermediary Net Worth

(b) Default Rates



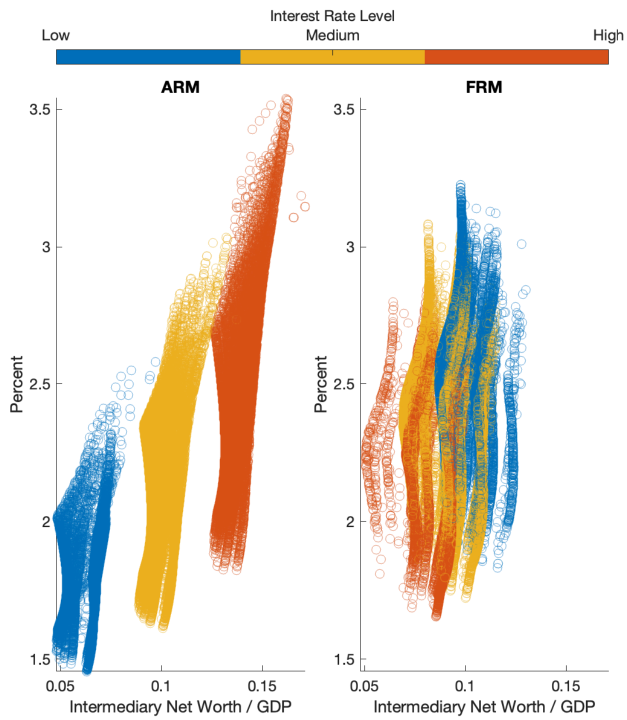
b). Conversely, net worth is somewhat decreasing in interest rates for FRM economies, meaning it has positive net worth duration.

Deposit Sensitivity Figure 5a also shows that the level of intermediary net worth is more sensitive to interest rate changes in the pure ARM economy, i.e., its absolute duration is larger. The lower sensitivity in the FRM economy is due to the calibrated degree of deposit stickiness with $\beta_d = 0.34$. $\beta_d \leq 1$ governs the pass-through of interest rate changes to deposit rates and thus interest expense. In a counterfactual where deposits are more responsive to policy rates ($\beta_d^{High} = 0.67$, shown in dashed lines in Figure 5a), FRM intermediary net worth becomes more sensitive as well. Intuitively, higher deposit sensitivity aligns the duration of liabilities more with the duration of assets under an ARM structure, while the reverse is true in an FRM regime, consistent with findings by Drechsler et al. (2017, 2021).

Mortgage Default and Risk Premia Figure 5b, which shows average default rates by interest rate levels, suggests a hedging mechanism in both economies: defaults are high when intermediary net worth tends to be high. Figure 6 illustrates this further by comparing default rates across simulations by interest rate level (color) and intermediary net worth (x-axis). In both ARM and FRM economies, defaults are positively correlated with net worth.

However, Figure 6 also reveals substantial differences between FRM and ARM economies.

Figure 6: Default Rates By Level of Interest Rate and Intermediary Net Worth



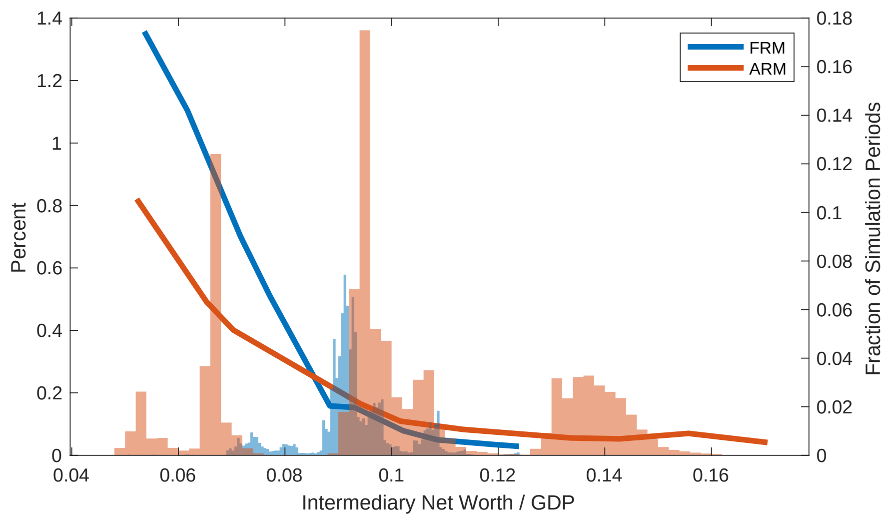
Notes: This figure shows simulation scatter plots of default rates by the level of intermediary net worth (x-axis) and interest rates (color: blue is low, yellow is medium, orange is high). The left plot shows the ARM economy, while the right plot shows the FRM economy.

Consistent with Campbell and Cocco (2015), default occurs in different macroeconomic states across mortgage structures, and is more rate-sensitive in the ARM economy. In the ARM economy (orange), defaults are high when interest rates are high – precisely when intermediaries' net worth is bolstered by wider net interest margins. Thus, ARM defaults are *net-worth hedged*: interest rate gains offset credit losses. In the FRM economy, defaults rise when rates fall, another period of strong intermediary balance sheets, but the overall sensitivity of defaults to rate swings is much lower. Consequently, the net worth hedging channel is weaker under fixed-rate mortgages.

The relative strength of net worth hedging forces manifests in risk premia. Figure 7 shows that the weaker FRM net-worth hedging channel makes risk premia (left y-axis) more sensitive to intermediary net worth (x-axis) at low values, when intermediaries are constrained. In the FRM economy (blue), impaired intermediary balance sheets lead to high risk premia, while ARM economy (orange) risk premia are only moderately elevated in the same region. However,

Figure 7 also shows that pure ARMs make intermediary net worth more volatile on average given the large absolute duration exposure, with a higher probability of being in the low intermediary net worth region compared to the FRM economy (shown as frequency distribution of simulation periods on the right-hand y-axis). As a result, risk premia in the pure ARM economy are not necessarily lower on average.

Figure 7: Mortgage Risk Premia Decrease with Intermediary Net Worth



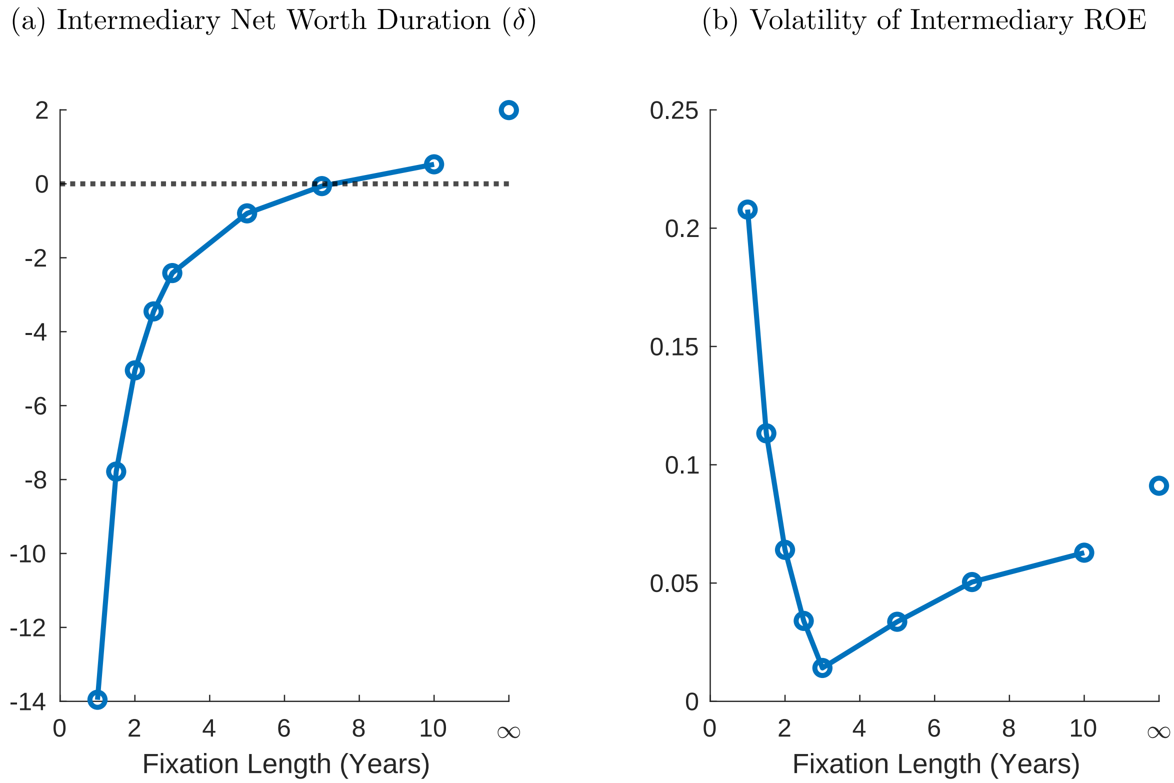
Notes: This figure shows conditional means of mortgage excess returns by the level of intermediary net worth, for ARM and FRM economies.

5.2.2 Net Worth Duration and Volatility Across Mortgage Structures

So far, we have examined the pure FRM and ARM economies at opposite ends of the mortgage-structure spectrum. To explore the full range of mortgage structures, we now compare financial-stability outcomes in our benchmark FRM economy with a series of counterfactual economies featuring varying mortgage fixation lengths. To do so, we solve the model and simulate outcomes for economies with values of $\pi_\tau \in [0, 1]$ where π_τ reflects the annual probability of the loan being in the floating stage, and $1/\pi_\tau$ the (implied) fixation length. For instance, a full ARM economy has a rate that resets every year with $\pi_\tau = 1.0$, a 10-year fixed-rate mortgage economy has $\pi_\tau = 0.1$, while the full FRM economy has $\pi_\tau = 0$.

Figure 8a plots the duration of intermediary net worth δ across mortgage structures, where duration is measured as the negative of the regression coefficient of log wealth on interest rates,

Figure 8: Measures of Financial Stability Across Mortgage Structures



Notes: This figure shows the duration of intermediary net worth as the negative regression coefficient δ from a regression of log wealth on interest rates: $\log W_t^I = \text{const.} - \delta r_t^p + \epsilon_t^w$ (Panel (a)) and the volatility of intermediary return on equity, measured as the standard deviation of net income over net worth (Panel (b)). The x-axis reflects an annual rate reset probability of $\pi_\tau \in \{1, 2/3, 0.5, 0.4, 1/3, 0.2, 1/7, 0.1, 0\}$, which corresponds to fixed-rate lengths of 1, 1.5, 2, 2.5, 3, 5, 7, 10 years and ∞, respectively.

i.e. the OLS estimate of $\log W_t^I = \text{const.} - \delta r_t^p + \epsilon_t^w$, run for each economy. Net worth duration measures the percent decline in intermediary net worth declines in response to a 1 percentage point increase in rates. The pure ARM economy with a fixation length of one year has large negative duration, meaning that net worth increases substantially when interest rates go up, as shown earlier. Likewise, the pure FRM economy with an infinite fixation length has moderate positive duration, meaning net worth declines somewhat when rates go up. Net worth duration increases in fixation length and is zero at an intermediate fixed-rate length of seven years.

However, net worth duration is an incomplete measure of risk as the regression above only measures the contemporaneous effect of interest rates. Its R^2 in the benchmark FRM economy is only 0.164, suggesting that there are dynamic and persistent effects of rate changes on net worth that are not captured by duration. In other words, other state variables, and their correlation with interest rates reflecting, e.g., the effect of credit risk and risk premia, must be accounted for to fully capture interest rate exposure. We expand on this intuition more formally in Section IV in the Appendix.

As a result, our preferred measure of financial stability is the volatility of banks' return on equity (ROE), shown in Figure 8b. This measure captures the combined equilibrium effects of asset and liability-side volatility as well as leverage on the volatility of intermediary net worth.[28] The volatility of banks' ROE has a "U-shape" pattern, higher at both extremes of mortgage structure, fully adjustable or fully fixed, than at intermediate values. Banks' ROE volatility is minimized at a fixation length of approximately 3 years, which is shorter than the zero-duration fixation length of 7 years. This discrepancy highlights the importance of evaluating financial stability in equilibrium, taking into account endogenous default and risk premia.

The findings suggest that *an intermediate fixation length balances sources of volatility in both ARM and FRM structures*, trading off a lower absolute duration in the FRM economy against better net worth hedging in the ARM economy. Figure IA.10 in the Appendix adds the 3-year fixed-rate economy ("ARM 3yr") to the plot that shows intermediary net worth and default by interest rate levels. Compared to both the pure FRM and ARM economies, both intermediary net worth and default are relatively stable across interest rate states in the 3-year fixed-rate economy. The results suggest that an intermediate fixation length broadly balances the contrasting mechanisms in both extremes of mortgage structure, making the intermediary sector more stable across states of the world with different interest rate levels.

5.3 Robustness, Analytical Results on Net Worth Hedging, and Further Evidence

State-Dependent Impulse Responses We evaluate robustness of our baseline results across the state space. Since we solved the model globally, we can show one-period impulse responses to a rate rise, starting at different borrower mortgage-to-income ratios (Appendix Figure IA.8). Underlying these different mortgage-to-income ratios are different percentiles ($p \in \{1, 5, 10, 30, 50, 70, 90, 95, 99\}$) of aggregate borrower wealth state variable W_t^B in a long

[28] This is consistent with the intuition in Meiselman et al. (2023), who show that banks' ROE is a strong predictor for systematic tail risks.

simulation of the model, conditional on the average ("medium") level of interest rates. Since the other state variable – intermediary wealth – is correlated with W_t^B, for each level of W_t^B we select its initial IRF value as the simulation average, conditional on average interest rates and W_t^B being in a 5 percentile neighborhood of the given starting borrower wealth. The signs for impulse responses for intermediary net worth and default (measured as % and p.p. deviations from the unconditional path, respectively) remain unchanged across the entire borrower wealth distribution, and magnitudes remain relatively stable. As borrowers become more indebted, defaults increase, but magnitudes remain within ranges that below we develop analytically.

Analytical Results: Net Worth Hedging - Interest Rate vs. Credit Risk Channel How general is net worth hedging under ARMs? To evaluate under what conditions net worth hedging arises, we characterize the sensitivity of mortgage portfolio values to interest rates in closed form. Mortgage portfolio values change (1) because payments and prices of surviving mortgages change, and (2) because default rates and recovery rates change. We term (1) the *interest rate channel*, and (2) the *credit channel*. As an upper bound on how much credit effects can offset interest rate effects, we consider the extreme case where loss-given-default (LGD) is 100%, that is, there are no recoveries. We develop conditions under which the credit channel could offset the positive mortgage cash flow gains of the interest rate channel. Intuitively, default rates must be increasing enough in rates, default rate levels must be high enough to begin with, and these effects must not be offset too much by potentially lower discount rates (which would raise the present value of remaining principal on surviving loans), to overwhelm the positive effect of adjustable-rate loan coupons rising. Detailed derivations are provided in Appendix Section II. We find that the ARM default rate sensitivity to rate rises would have to be at least 14-15 times greater than generated by our baseline calibration, for higher cash flows from an increase in mortgage payments on performing loans to be offset by an increase in credit losses. We thus show that net-worth hedging of ARMs arises robustly under a wide range of plausible paths for interest rates and default, and borrower conditions.

Bank Decision-Making and Further Empirical Evidence We evaluate financial stability outcomes for a given mortgage structure, which determines the duration of household debt. This does not imply that bank equities merely inherit resulting risks from their assets (mortgages): in the model, banks can manage equity duration endogenously by adjusting their leverage. Leverage affects the duration of equity in two ways: (1) it governs how much of the asset duration is hedged/immunized by the duration of liabilities, and (2) it translates dollar gains/losses from interest rate fluctuations into percentage gains/losses, with higher leverage resulting in greater amplification. We expand on this more formally in Appendix Section III.6. We find that banks slightly reduce leverage when facing higher volatility (see Table 3, "Leverage"), consistent with a precautionary savings motive.

In principle, banks can also manage interest rate exposure by adjusting their holdings of Treasuries. If mortgage fixation lengths decline, they can buy more long-term government bonds. We investigate the extent to which shorter fixation lengths are associated with larger and longer-duration government bond portfolios by comparing bank holdings across countries with different mortgage structure using BIS data. We decompose the total level of non-financial credit (in USD) into credit to general government, households (including non-profit institutions), and non-financial corporations for the U.S., Euro Area (EA), Canada, Australia, and the U.K., shown in Appendix Figure IA.9. While the share of non-financial credit going to households in the U.S. is relatively small, with only the euro area having a smaller share, the share going to government is largest in the U.S., despite it having the highest average fixation length of about 25 years. Moreover, while governments issue debt of varying maturities across countries, according to data by De Graeve and Mazzolini (2023), variation in weighted-average government debt maturities across countries is limited.[29] These patterns suggest that variation in government debt maturities and non-financial credit shares does not offset the variation in average maturity of non-financial credit driven by varying mortgage structures across countries, which serves as a proxy for household debt maturity.[30] In other words, in the aggregate, banks do not "undo" the effect of mortgage structure on their interest rate exposure by adjusting government bond holdings.

To further support the idea that in the data mortgage structure is correlated with differential bank performance in response to interest rate fluctuations, we exploit variation across countries within the euro area, where banks faced common banking regulation and a common policy rate

[29] U.S. government debt has average maturity of 5.3 years, the Euro Area average at 6.3 years, Australia at 5.5 years, and Canada at 6.2 years. Only the UK has a markedly longer average maturity of government debt, at 12.0 years.

[30] Under the assumption that lending to non-financial corporations is of comparable maturity across countries.

shock from the ECB between 2022 to 2023. We use bank-level equity data from Bloomberg to construct and plot bespoke ARM (Spain, Finland, Greece, Italy, Poland, Ireland, Portugal) and FRM (Belgium, France, Germany, Netherlands) bank equity indices in Appendix Figure IA.5. The figure shows that within-euro area ARM bank equities outperformed FRM bank equities by about 67% (69%), using market-capitalization weights (equal weights) to aggregate across countries, between 2022 and 2024. The indices start to diverge especially starting in July 2022, when the ECB started to raise policy rates. This empirical evidence is consistent with our model's prediction that different mortgage structures leave banks with different exposures to interest rate risk.

Lastly, ours is a model in which banks are the marginal investors in duration, and hence price interest rate risk. This is consistent with the findings of Haddad and Sraer (2020), who show that when banks duration exposures increase, so do bond risk premia.

5.4 Risk Sharing

Our analysis of financial stability thus far highlights the risks borne by savers, who hold bank equity. To better understand risk sharing across mortgage structures, we compare outcomes for both borrowers and banks in Table 3. The top panel reports bank-related metrics, and the bottom panel shows borrower-related metrics. The first three columns present results for our baseline scenario with low calibrated deposit sensitivity: pure-ARM (annual resets), intermediate fixation length (3-year) at which net worth volatility is minimized, and pure-FRM economies, respectively.

Borrowers and Consumption Mortgage structure shapes both the extent and the nature of borrowers' interest rate exposure, affecting default behavior and portfolio decisions. In ARM economies, mortgage payment-to-income (PTI) ratios increase in rates, exposing borrowers to liquidity risks. The "PTI (OLS coef.)" row reports the coefficient of a regression of PTI on interest rates. A 1 percentage point increase in rates corresponds to a 1.61 percentage point increase in PTI in the ARM (1yr) economy, but only a 0.45 percentage point increase in the ARM (3yr) economy, and a -0.17 percentage point change in the FRM economy as households delever. But rate shocks also have wealth effects, which determine borrowers' strategic default

Table 3: Measures of Financial Stability

Deposit Sensitivity:	Low ($\beta_d = 0.34$)			High ($\beta_d = 0.67$)		
Mortgage Structure:	ARM (1yr)	ARM (3yr)	FRM	ARM (1yr)	ARM (3yr)	FRM
Excess ROE (mean)	2.26	1.50	1.70	1.65	1.70	2.29
ROE (st. dev.)	20.79	1.41	9.11	12.74	8.05	16.89
Excess ROA (mean)	0.22	0.16	0.19	0.16	0.19	0.25
ROA (st. dev.)	2.20	0.66	1.17	1.73	1.58	2.25
Leverage	90.49	91.83	91.46	91.04	91.36	90.86
Fraction of constraint binding	27.45	86.50	49.99	24.61	45.31	36.10
Duration of bank net worth	-13.96	-2.41	1.99	-11.51	-0.70	4.82
PTI (OLS coef.)	1.61	0.45	-0.17	1.48	0.28	-0.33
LTV (OLS coef.)	2.47	0.17	-1.19	1.52	-0.99	-2.19
Default Rate (mean)	2.07	2.36	2.32	2.21	2.30	2.19
Default Rate (std. dev.)	0.36	0.07	0.26	0.19	0.29	0.48
Default Rate (OLS coef.)	0.14	0.01	-0.07	0.09	-0.05	-0.12
DTI (mean)	145.16	151.15	150.37	148.16	149.80	147.38
LTV (mean)	55.15	59.68	59.08	57.48	58.68	56.86
Deposits / Income (mean)	25.74	24.36	24.49	25.16	24.76	25.24

Notes: Unconditional moments from a long simulation of the model. Except for the duration of bank net worth, all quantities are reported in percent. Rows marked "OLS coef." report the coefficient of a regression of the variable on the policy rate r_t^p.

behavior. Higher interest rates always lower house prices on impact, but the extent to which they affect the value of the mortgage – and, hence, LTV ratios – depends on the fixation length. In the FRM economy, high rates lead to low mortgage values. This creates LTV ratios that are mildly countercyclical in the interest rate, as reflected in a negative "LTV (OLS coef.)" (defined analogously to the PTI regression coefficient). Together with stable payments, it yields countercyclical default rates, or a negative "Default Rate (OLS coef.)," which is consistent with the impulse responses showing a decrease in default rates when rates go up. As fixation length shortens, mortgage duration drops and eventually flips sign. In the pure-ARM economy, rate increases lead not only to higher house prices but higher mortgage values, which implies LTV ratios being strongly procyclical in rates. Together with procyclical payments, this leads to procyclical default rates, which are more volatile than in the FRM economy. Conversely, default rates are mildly countercyclical in the FRM economy. At intermediate fixation lengths, default rates are close to acyclical with respect to interest rates, and are least volatile.

Higher exposure to interest rate risk in ARM economies lowers both the supply and the

Table 4: Consumption Measures

Deposit Sensitivity:	Low ($\beta_d = 0.34$)			High ($\beta_d = 0.67$)		
Mortgage Structure:	ARM (1yr)	ARM (3yr)	FRM	ARM (1yr)	ARM (3yr)	FRM
Panel A: Savers						
Cons. (mean)	49.15	49.93	49.84	49.54	49.78	49.50
Cons. gr. (st. dev.)	2.35	0.52	2.29	0.74	2.74	3.97
Cond. vol of cons. gr.	2.05	0.29	1.83	0.55	2.09	3.22
Panel B: Borrowers						
Cons. (mean)	47.79	47.01	47.10	47.43	47.20	47.49
Cons. gr. (st. dev.)	16.17	17.17	17.12	16.69	16.99	16.70
Cond. vol of cons. gr.	10.46	11.05	10.92	10.89	10.94	10.63

Notes: Unconditional moments from a long simulation of the model.

demand for credit. Together with more expensive mortgages due to higher risk premia ("Excess ROA"), volatile default rates cause households to reduce their demand for credit and expand precautionary saving. Relative to the FRM economy, in the pure-ARM economy, average mortgage debt falls both relative to income (DTI) and relative to house prices (LTV), while deposits to income increase. As a result, less indebted borrowers default less often on average. The opposite is true for the safer ARM (3yr) economy.

Differences in risk exposures and indebtedness have implications for consumption (Table 4). Fewer mortgages mean a smaller banking sector, with reduced dividends lowering saver consumption (Panel A) in the ARM economy. The volatility of consumption growth – both unconditional, and conditional, which determines the price of risk in asset pricing models – goes up, consistent with the higher intermediary volatility and risk premia in the ARM economy discussed above.

The effect on borrowers is the opposite in the ARM economy. With less debt, their interest burden is smaller, and they suffer the pecuniary consequences of default less often. This results in higher average consumption.[31] Lower debt makes borrower consumption less exposed to idiosyncratic shocks, the main source of their consumption volatility. As a result, both measures of consumption volatility are lower in the ARM economy.

[31] Because deposit markets do not clear, changes in each household type's *levels* of consumption, and consequently changes in welfare, are also due to changes in the economy's net deposit position and hence available resources. We therefore focus the normative analysis on higher order moments of consumption, namely, risk-sharing.

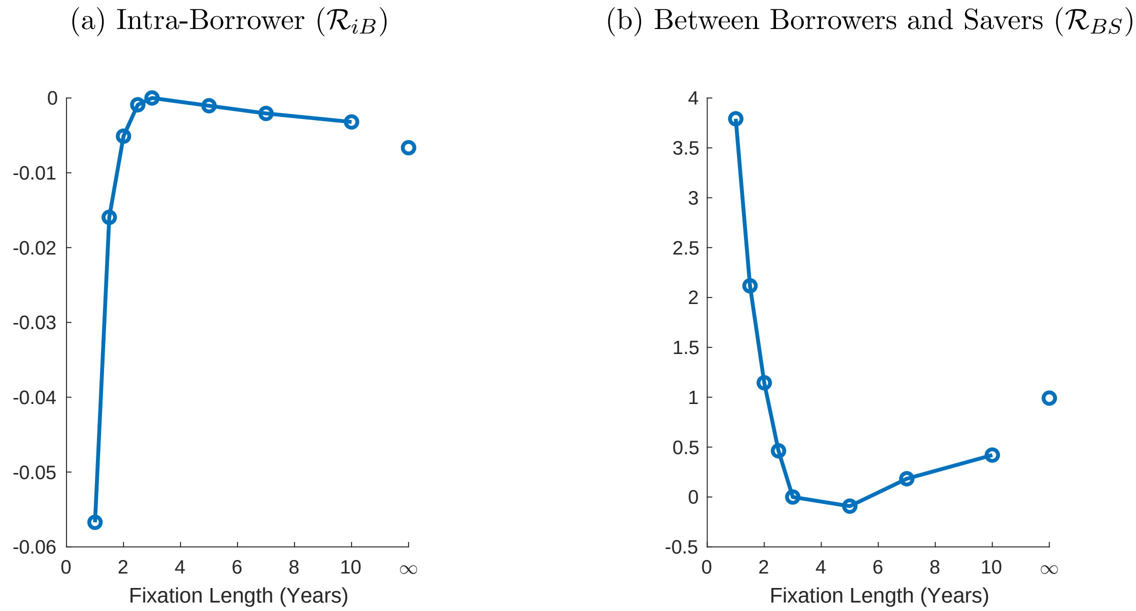
Robustness: Deposit Sensitivity An important source of financial stability risk in the ARM economy is the large difference between high sensitivity of mortgage payments to policy rates and the low sensitivity of deposit rates, calibrated to match the empirical evidence. We thus consider a counterfactual in which we double the calibrated benchmark sensitivity of $\beta_d = 0.34$ to $\beta_d = 0.67$ in the fourth through sixth columns of Tables 3 (Financial Stability) and 4 (Consumption).

With more volatile deposit rates at which banks fund themselves, the FRM economy becomes substantially riskier (third vs. sixth columns). Bank equity duration more than doubles, the volatility of both asset and equity returns increases considerably, and banks demand a larger compensation for the risk of holding mortgages. As before, a more volatile economy and more expensive mortgages lead to lower borrower indebtedness, lower default rates, and higher consumption. The effect of switching from FRMs to ARMs in the high deposit sensitivity counterfactual is opposite to that in the baseline experiment. When policy rates substantially pass through to deposit rates, a mortgage structure in which payments are indexed to the policy rate improves financial stability, reducing the volatility of bank balance sheets and the risk premia associated with them and stimulating mortgage credit. Intuitively, the asset and liability side of bank balance sheets are better aligned with ARMs when deposit rates fluctuate more strongly with interest rates. Hence, a banking sector that faces less sticky deposit rates is rendered most stable by an even shorter fixation length than 3 years, the level for the baseline calibrated economy.

Measuring Risk Sharing Ultimately, we investigate the way in which mortgage structure determines how risks are shared between households. To quantify the degree of risk sharing, it is instructive to consider a hypothetical complete markets benchmark. A social planner subject to rate shocks but not to any of the economy's frictions would insure households fully against idiosyncratic shocks and award each household a constant fraction of overall consumption. In other words, the difference $\Delta \log c_t^i - \Delta \log c_t^j$ between consumption growth rates of any two households i and j would be zero in all periods.[32]

[32] See Appendix III.5 for derivations. Moreover, the planner would optimize the overall economy's exposure to rate shocks. The planner would choose a net deposit position of the economy with respect to the rest of the world to satisfy the consumption-savings Euler equation of the representative agent, whose consumption would be equal to the aggregate consumption of the economy. We also derive these results in Appendix Appendix III.5,

Figure 9: Measures of Risk Sharing across Mortgage Structures



Notes: $\mathcal{R}_{iB}$ measures the variance of individual borrowers' consumption growth relative to aggregate borrower consumption growth, and $\mathcal{R}_{BS}$ measures the variance of aggregate consumption growth of borrowers relative to savers. In each panel, $\mathcal{R}$ is reported in deviations from the level in the ROE volatility-minimizing economy.

We can then measure the quality of risk sharing by the unconditional variance of differences in consumption growth rates between households. Recall that borrower households are subject to undiversifiable idiosyncratic risk, while saver households are not. We can define two scale-free measures of risk-sharing:

1. Higher values of $\mathcal{R}_{iB} = \text{Var}_0[\Delta \log c_t^i - \Delta \log C_t^B]$, where $i \in [0, \ell]$ and C_t^B is aggregate consumption of borrowers, indicate worse *intra-borrower* risk-sharing;

2. Higher values of $\mathcal{R}_{BS} = \text{Var}_0[\Delta \log C_t^B - \Delta \log C_t^S]$, where C_t^S is aggregate consumption of borrowers, indicate worse risk-sharing *between borrowers and savers*;

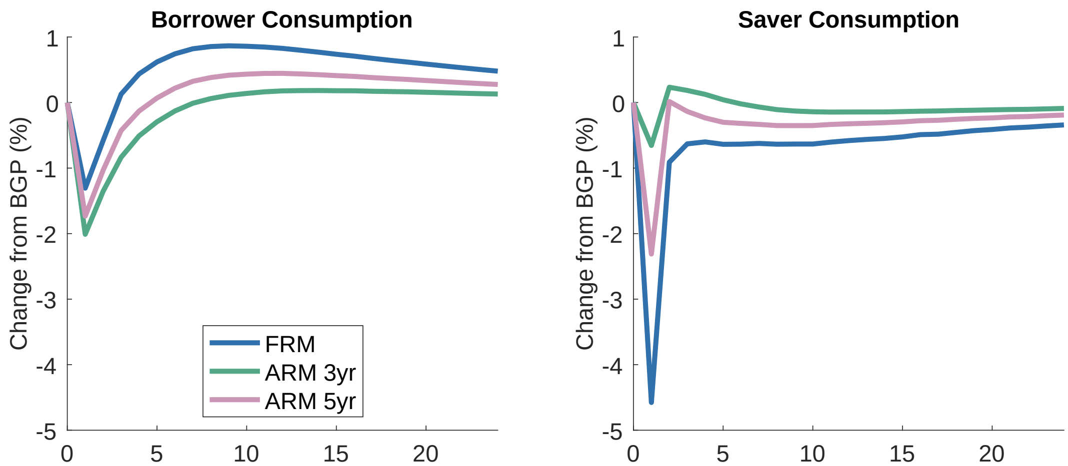
Figure 9 reports the results in deviations from the level in the ROE volatility-minimizing economy.[33]

Panel (b) shows that intermediate mortgage fixation lengths lead to the best attainable risk-sharing arrangements between borrowers and savers as $\mathcal{R}_{BS}$ is minimized at a fixation length

but since these effects turn out to be quantitatively negligible, we do not report these separately.

[33]At a fixation length of 3 years, $\mathcal{R}_{iB}$ is 0.17, and $\mathcal{R}_{BS}$ is 0.005. Since these measures are scale-free, the level of undiversifiable idiosyncratic risk faced by borrowers is considerably larger than aggregate risk shared between borrowers and savers, consistent with many macroeconomic models.

Figure 10: Consumption Responses To Positive Rate Shock



Notes: The figure shows impulse responses to a positive interesr rate shock. The left panel shows the response of aggregate borrower consumption C_t^B, while the right panel shows the response of aggregate saver consumption C_t^S. Benchmark FRM economy is in blue. Volatility-minimizing ARM 3yr economy is in green, and the $\mathcal{R}_{BS}$-minimizing ARM 5yr economy is in purple.

of 5 years. This is a slightly longer fixation length relative to the contract that minimizes the volatility of intermediary ROE, suggesting a small trade-off between financial stability and aggregate risk sharing.

To illustrate this trade-off directly, we plot the responses of borrower and saver consumption to a positive rate shock for the benchmark FRM economy, the volatility-minimizing ARM 3yr economy, and the risk-sharing optimizing ARM 5yr economy in Figure 10. In the FRM economy, savers – who own banks – have a much greater exposure to this shock than borrowers. When the fixation length is chosen to minimize bank volatility (ARM 3yr, green), savers become almost insulated from the shock, but it is now borrowers whose consumption suffers. From a risk-sharing perspective, lowering the fixation length to 3 years leads to an over-correction. A less aggressive choice of 5 years (purple) leads to similar consumption responses for both borrowers and savers, and thus minimizes $\mathcal{R}_{BS}$.

However, low exposure to aggregate risk leads borrowers to endogenously choose higher exposure to idiosyncratic risk (Panel (a) of Figure 9). At intermediate fixation lengths, they choose the largest mortgages, and hence the largest mortgage payments, should they choose to make them rather than defaulting. When payments constitute a larger fraction of liquid income, the effect of idiosyncratic income shocks on consumption is amplified. Moreover, higher mortgage balances lead to a higher probability of default. Since consumption levels in and out of default are different, a higher probability of default also leads to higher consumption volatility. This is reflected in the higher $\mathcal{R}_{iB}$ at intermediate fixation lengths.

Overall, mortgage structure most strongly affects the sharing of interest rate risk between borrowers and savers, with the best attainable outcome occurring at an intermediate fixation length of 5 years. The findings on idiosyncratic risk sharing between borrowers highlight a somewhat subtle downside: a more efficient (aggregate) risk-sharing arrangement leads borrowers to take on more idiosyncratic risk, which the mortgage structures under consideration cannot diversify away.

5.5 Role of Aggregate Income Shocks

The results above show how financial stability and risk sharing are affected by mortgage structure in an environment in which the only source of aggregate risk is shocks to interest rates. It is the source of risk whose allocation between borrowers and savers is most directly affected by mortgage fixation length.

In the data, households also face aggregate income shocks, and these shocks may be correlated with interest rates. For instance, times when interest rates rise may also be times when incomes rise, as would be the case in an economy dominated by aggregate demand shocks. Alternatively, interest rate increases may coincide with income declines if supply shocks predominate.[34] How do our results change if we allow for the possibility of correlated aggregate income and interest rate shocks?

To answer this question, we relax the restriction $Y_t = 1$ and calibrate a VAR(1) process to govern the joint dynamics of $(\log Y_t, r_t^p)$, where $\log Y_t$ is measured as the cyclical component of log GDP and r_t^p is as before. Over the baseline 1987-2024 sample period, we find a positive correlation between innovations to the two series, with a correlation coefficient of 0.313. Relative

[34] In a New Keynesian framework, a positive demand shock increases both output and inflation, to which central banks respond by raising nominal rates. With nominal rigidities, this leads to an increase real rates. In contrast, a negative supply shock increases inflation while reducing output. If the central bank's policy rule responds to inflation more strongly than to output, it would raise nominal rates, leading real rates to rise as well. See Woodford (2003) for a canonical treatment.

Table 5: Measures of Financial Stability

Income Correlation:	Calibrated ($\rho_{yr} = 0.313$)			Uncorrelated ($\rho_{yr} = 0$)		
Mortgage Structure:	ARM (1yr)	ARM (3yr)	FRM	ARM (1yr)	ARM (3yr)	FRM
Excess ROE (mean)	2.16	1.56	1.62	2.00	1.56	1.66
ROE (st. dev.)	18.04	4.51	8.04	16.37	4.82	8.65
Excess ROA (mean)	0.22	0.16	0.18	0.20	0.17	0.18
ROA (st. dev.)	1.93	0.73	1.03	1.74	0.76	1.08
Leverage	90.34	91.48	91.41	90.64	91.45	91.37
Fraction of constraint binding	32.08	55.38	52.02	37.94	53.71	47.95
Duration of bank net worth	-12.58	-2.57	1.53	-11.01	-1.75	2.21
PTI (OLS coef.)	1.44	0.33	-0.27	1.45	0.35	-0.23
LTV (OLS coef.)	1.67	-0.18	-1.23	1.63	-0.08	-1.03
Default Rate (mean)	2.12	2.34	2.34	2.15	2.34	2.33
Default Rate (std. dev.)	0.37	0.23	0.31	0.40	0.25	0.30
Default Rate (OLS coef.)	0.10	-0.01	-0.07	0.09	-0.00	-0.05
DTI (mean)	146.57	150.85	150.42	147.02	150.81	150.58
LTV (mean)	55.95	59.45	59.22	56.35	59.37	59.23
Deposits / Income (mean)	25.55	24.48	24.51	25.47	24.51	24.51

Notes: Unconditional moments from a long simulation of the model. Except for the duration of bank net worth, all quantities are reported in percent. Rows marked "OLS coef." report the coefficient of a regression of the variable on the policy rate r_t^p.

to the rate-only process in the baseline model, we also find a lower volatility of the innovations in rates – 0.013 vs. 0.014. Intuitively, in a VAR some of the variation in rates is now attributed to the contemporaneous and lagged effects of income innovations. Appendix V.1 contains details on the VAR estimation and the resulting impulse response functions. We then re-solve the model with the new process for each of the mortgage structures.[35]

The results are shown in Table 5. The left panel shows the results for the three main fixation lengths – ARM with a one-year fixation length, ARM with a three-year fixation length, and FRM – in the exogenous environment with both income and rate shocks, calibrated to the data. The right panel shows the results for the same three fixation lengths when the correlation of income and rate innovations is counterfactually set to zero. This allows us to separately consider the effect of introducing an extra source of aggregate risk into the model from the effect of it being correlated with interest rates.

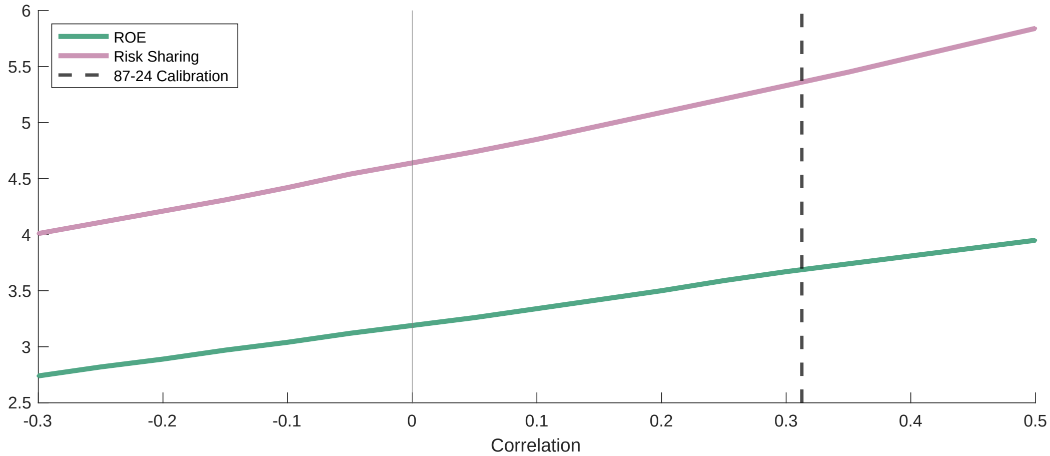
[35]As before, the FRM economy represents the data generating process. A change in the exogenous environment leads to different values for the moments governing our internal calibration. In principle, this could require re-calibrating the internally calibrated parameters. However, we find the fit of the model with income shocks to be comparable to the baseline model without. For parsimony, we do not re-calibrate the model.

In the presence of income shocks, changing mortgage fixation becomes somewhat less effective at reducing volatility than it was in the baseline, whether of intermediary returns on equity (top panel, second row) or default rates (bottom panel, fourth row). In the baseline economy, going from an economy with FRMs to an economy with a 3-year fixation length lowers intermediary ROE volatility from 9.11 to 1.41. With uncorrelated income shocks, the corresponding reduction is from 8.65 to only 4.82. This smaller effect occurs for two reasons. First, unlike rate shocks, income shocks affect both households in a similar way. A positive rate shock benefits borrowers at the expense of savers in the FRM economy, but a positive income shock benefits both. Shortening the fixation length improves the sharing of interest rate risk because that risk is allocated asymmetrically to begin with, but has little effect on the sharing of income risk. Second, the reduction in endogenous volatility achieved by intermediate fixation length (ARM (3y) column vs. FRM column) reduces incentives for precautionary savings. Both borrowers and savers take on more debt, with mortgage DTI and LTV slightly higher (bottom panel, rows 6-7) and with intermediary constraints binding more often (top panel, row 5). These riskier portfolios leave households more exposed to income shocks, partly offsetting the reduction in volatility due to better sharing of interest rate risk.

Next, consider what happens to financial stability in the FRM economy when income and rate shocks become positively correlated, as they are in the data. An increase in rates now lowers default rates not just because it lowers market-value LTVs (bottom panel, row 2), as in the baseline with only rate shocks, but also because of a concurrent increase in income. Default rates become more countercyclical in rates (bottom panel, row 5) leading to a stronger hedging force offsetting the market value losses on long-term mortgages stemming from an increase in rates. As a result, intermediary ROE volatility in the FRM economy is lower than in the uncorrelated case (top panel, row 2).

The opposite is true for ARMs. A rise in rates increases borrower payments but they can afford more of that increase because their incomes also rise. Intermediaries earn higher cash flows because promised mortgage payments are less offset by rising default rates, weakening the default hedging force that was present in the baseline model. In addition, defaults rise in the state of the world when net worth is low, namely when rates are low, due to lower incomes. In sum, positive correlation makes the FRM economy safer and the ARM economy riskier,

Figure 11: Optimal Fixation Lengths as a Function of Income Correlation



Notes: For each correlation (x-axis), blue line plots the fixation length that minimizes ROE volatility, and red line plots the fixation length that minimizes $\mathcal{R}_{BS}$ (lower values mean better risk sharing between borrowers and savers). The vertical dashed line shows the calibrated correlation of 0.313. To determine minima, we solve a grid of economies with different fixation lengths and different correlations between income and rate shocks and fit cubic splines to the ROE and risk sharing measures.

suggesting that a higher fixation length may be optimal.

To confirm this intuition, we solve a grid of economies with different fixation lengths and different correlations between income and rate shocks. For each correlation, we find (1) the fixation length that minimizes the volatility of intermediary ROE, and (2) the fixation length that optimizes risk sharing between borrowers and savers (minimizes $\mathcal{R}_{BS}$). The results are shown in Figure 11. Indeed, as we increase the correlation from -0.3 to 0.5, the ROE-minimizing fixation length rises from 2.7 to 3.9 years. The fixation length that optimizes risk sharing rises from 4 to 5.8 years, consistently remaining 1-2 years higher than the ROE-minimizing value as in the baseline. At the calibrated correlation of 0.313, the ROE volatility is minimized by a fixation length of 3.6 years while risk sharing is optimized by a length of 5.3 years.

The magnitude of these effects are not large enough to overturn the main findings of the paper. A mortgage with an intermediate fixation length of a few years does the best job of promoting financial stability and risk sharing in the presence of income shocks, whether the correlation is positive, as it has been in the recent sample, or zero, as it has been on average in a longer 1962-2024 period.[36]

6 Conclusion

This paper highlights the role of mortgage structure on financial stability and risk sharing between households and financial intermediaries. To evaluate these effects in equilibrium, we build a quantitative model with flexible mortgage contract structures, borrowers, and an intermediary sector. Borrowers endogenously default for liquidity and net worth-related reasons, and default is more sensitive to interest rates in the adjustable-rate mortgage regime. In addition, intermediary distance to capital constraints affects equilibrium mortgage pricing. As a result, our model captures complex interaction effects between interest rate and credit risk, and intermediary net worth.

Our findings reveal that mortgage structure is key to understanding differential financial stability risks in response to interest rate fluctuations. In an ARM economy, rising rates lead to increased household mortgage payments, higher default rates, and declining house prices. Despite higher credit losses, banks benefit from increased net interest margins and asset values, ultimately raising their net worth. Conversely, an FRM economy shields households from higher payments, thereby reducing defaults, but banks experience rising deposit costs and falling asset values, reducing their net worth and profitability.

We identify a “U-shaped” relationship between mortgage structure and financial stability risks. Pure ARM economies exhibit high net worth volatility due to strong interest rate sensitivity, whereas FRM economies partially hedge interest rate risk through sticky deposit rates. Yet ARM economies better hedge defaults by concentrating them in states when banks’ net worth is high. Intermediate fixation lengths, around 3 to 5 years, optimally balance these opposing forces, minimizing volatility and maximizing aggregate risk-sharing. Additionally, introducing correlated aggregate income and interest rate shocks suggests that a more positive correlation increases the optimal fixation length.

The paper does not study endogenous transitions from one mortgage structure to another. In the data, broad differences in mortgage fixation lengths across countries have remained relatively

[36] Appendix V.1 contains estimation details.

stable over the past decade (see Figure 2 which shows information from mortgages outstanding in 2013 as well as new origination flows in 2023), suggesting that these differences could arise from persistent policy differences and other factors such as mortgage funding structures across countries, which are outside the scope of this paper. It is worth emphasizing that the U.S. stands out as an outlier, and that most countries originate mortgages with intermediate fixed-rate length on average. Liu (2022) and Sanchez Sanchez (2023) show that the U.S. 30-year fixed-rate mortgage is an unlikely equilibrium outcome without the government-sponsored enterprises (GSEs) that subsidize credit risk-taking. Consistent with this, Rose (2018) argues that the shift towards longer-term contracts in the U.S. between 1930 and 1940 reflects "the advent of federal involvement in the residential mortgage market." Campbell (2012) further shows that the U.S. has the highest degree of government participation in housing finance compared to most countries except Singapore, but homeownership rates remain close to countries like the U.K., Ireland and Canada. The prevalence of intermediate mortgage fixation lengths globally could thus reflect equilibrium outcomes under lower degrees of government participation in housing finance, in line with the paper's finding that these types of contracts naturally balance sources of volatility from both pure ARM and FRM structures.

Overall, our findings have implications for monetary policy and macroprudential regulation. Our model provides a framework for understanding how changes in policy rates affect financial stability differentially across mortgage structures, and suggests that macroprudential concerns following rate rises differ, for instance, for the Federal Reserve compared to the European Central Bank. Our results inform optimal mortgage design that aims to improve financial stability and risk-sharing between households and financial intermediaries.

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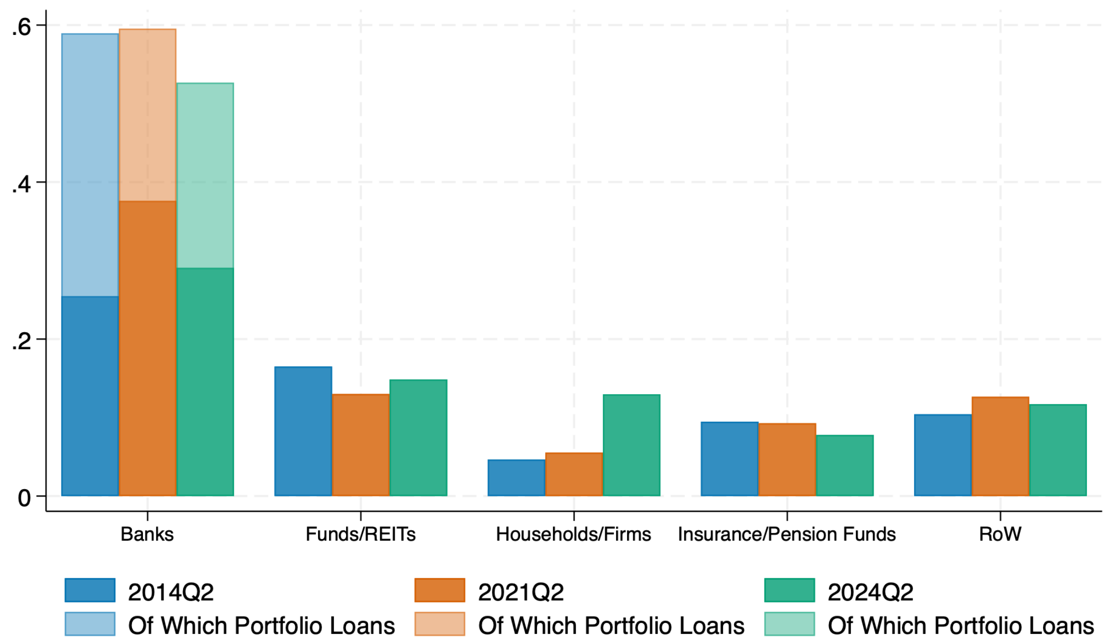
Internet Appendix

for "Mortgage Structure, Financial Stability, and Risk Sharing"

Vadim Elenev and Lu Liu

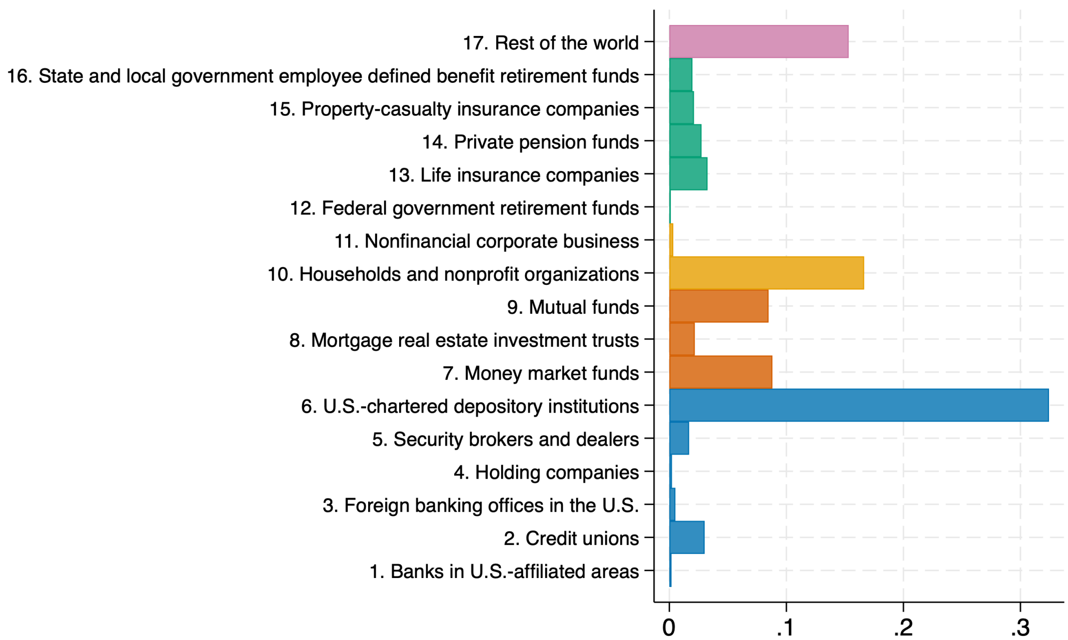
I Additional Figures and Tables

Figure IA.1: Non-Government Residential Mortgage Holdings by Sector (Portfolio & MBS)



Notes: This figure shows the composition of non-government residential mortgage holdings, including bank portfolio loans (I) and agency- and GSE-backed securities holdings (II), excluding direct government holdings, and holdings by the GSEs and the Federal Reserve. Data for (I) is based on the Urban Institute Housing Chartbook ("Unsecuritized First Liens (Bank Portfolio)"). Data for (II) comes from Table L211 from the US Financial Accounts (Flow of Funds) split into Banks, Funds/REITs, Households/Firms, Insurance/Pension Funds, and the Rest of World (RoW) in 2014Q2, 2021Q2, and 2024Q2. A detailed breakdown of constituent sector definitions for (II) is provided in Table IA.I. The data is retrieved from the Federal Reserve. Since (II) is reported at quarterly frequency, we obtain (I) from the Urban Institute Housing Chartbook from August 2014, and September 2021 and 2024, which reflect data as of 2014, 2021, and 2024 for the second quarter of the year, respectively.

Figure IA.2: Non-Government MBS Holdings (Detailed Sector Breakdown)



Notes: This figure shows the composition of non-government agency- and GSE-backed securities holdings from Table L211 from the US Financial Accounts (Flow of Funds), with a breakdown into underlying sectors that form the groups of Banks, Funds/REITs, Households/Firms, Insurance/Pension Funds, and the Rest of World (RoW) in Figure IA.1, in 2024Q2.

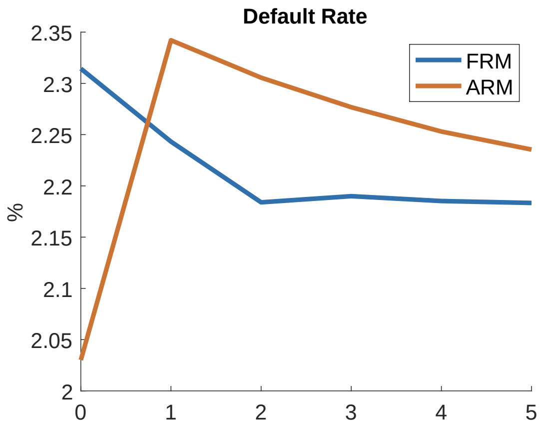
Table IA.I: Overview of Sector Definitions for Non-Government MBS Holdings

Sector	**Constituent Groups**
Banks	U.S.-chartered depository institutions
	Foreign banking offices in the U.S.
	Banks in U.S.-affiliated areas
	Credit unions
	Security brokers and dealers
	Holding companies
Funds/REITs	Mutual funds
	Mortgage real estate investment trusts
	Money market funds
Households/Firms	Households and nonprofit organizations
	Nonfinancial corporate business
Insurance/Pension Funds	Property-casualty insurance companies, including those held by U.S. residual market reinsurers
	Life insurance companies
	Private pension funds
	Federal government retirement funds
	State and local government employee defined benefit retirement funds
Rest of World (RoW)	Rest of the world

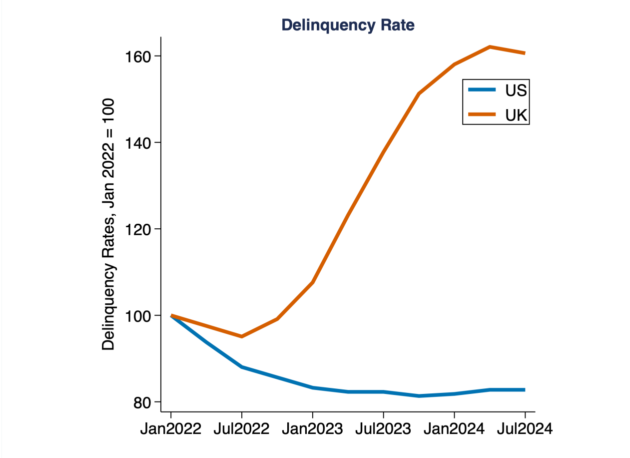
Notes: Constituent groups from Table L211 of the US Financial Accounts (Flow of Funds).

Figure IA.3: Model Predictions & Evidence: Delinquencies

(a) IRF for Defaults (FRM vs. ARM)



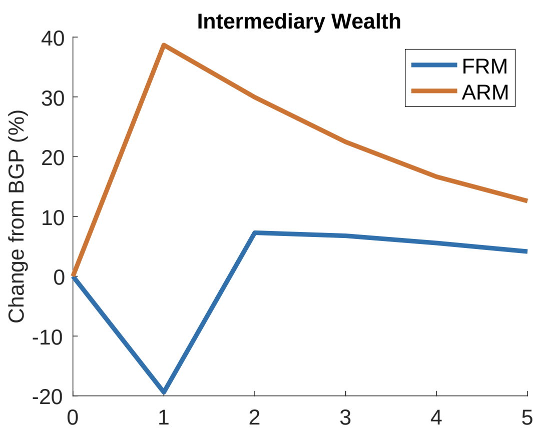
(b) Delinquencies since 2022 (US vs. UK)



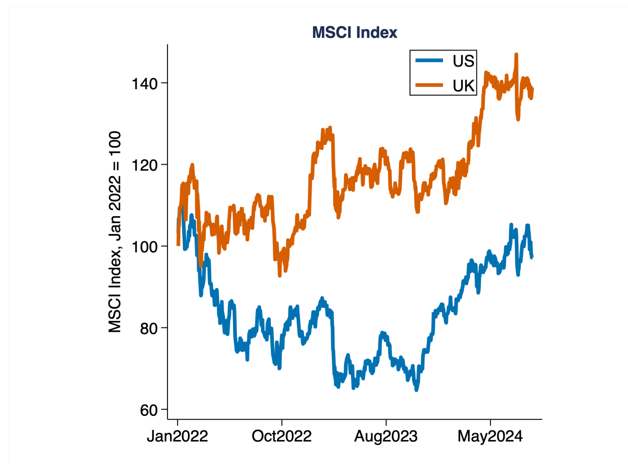
Notes: Panel (a) shows the impulse response function for default rates in response to an exogenous interest rate shock as shown in section 5.1. Panel (b) shows delinquency measures in the US and UK, indexed to 2022 Q1. US delinquencies are measured on single-family residential mortgages from FRED, reflecting loans past due 30 days or more and still accruing interest as well as those in nonaccrual status. UK delinquencies are arrears balances as percent of total outstanding balances reported by the FCA, reflecting loans where the amount of actual arrears is 1.5% or more of the borrower's current loan balance.

Figure IA.4: Model Predictions & Evidence: Bank Equity Prices

(a) IRF for Intermediary Net Wealth (FRM vs. ARM)



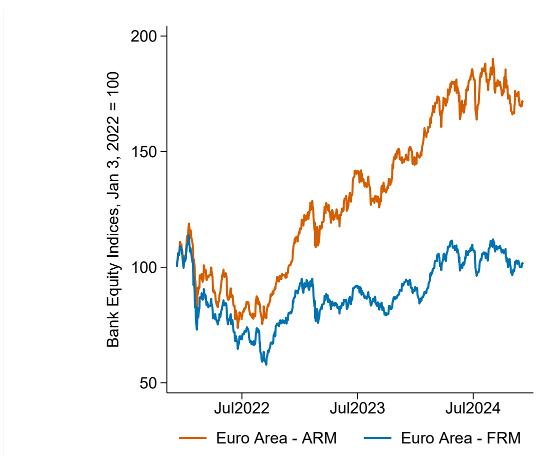
(b) Bank Equities since 2022 (US vs. UK)



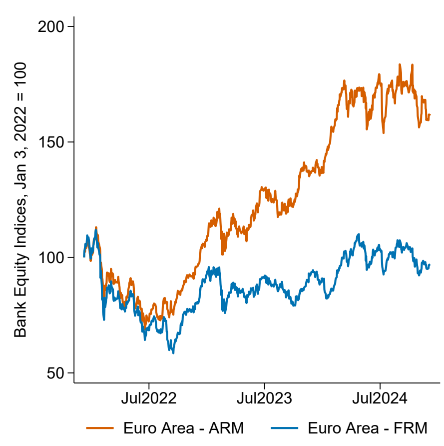
Notes: Panel (a) shows the impulse response function for intermediary net wealth in response to an exogenous interest rate shock as shown in section 5.1. Panel (b) shows MSCI bank equity indices in the US and UK indexed to January 1, 2022.

Figure IA.5: ARM and FRM Bank Equity Indices (Euro Area)

(a) Equal Weights Across Countries



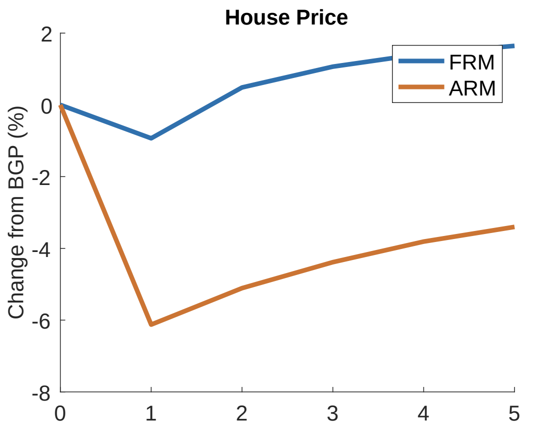
(b) Market Cap Weights Across Countries



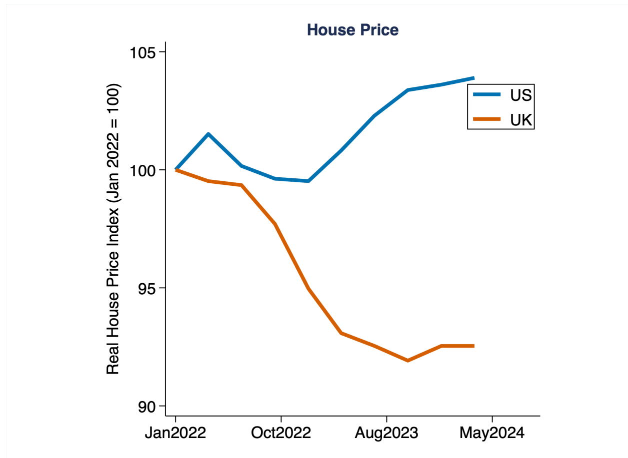
Notes: This figure shows Euro Area ARM and FRM bank equity indices constructed from country-level bank equity indices using equal weights (Panel (a)) and market-capitalization weights (Panel (b)) across Euro Area countries, between January 3, 2022 and December 31, 2024. Country-level bank equity indices are constructed from publicly listed bank equities data from Bloomberg using market capitalization weights. Euro Area - ARM contains the countries Spain (6), Finland (3), Greece (6), Italy (35), Poland (11), Ireland (4), and Portugal (1), while Euro Area - FRM contains the countries Belgium (2), Germany (5), Netherlands (2), and France (16), with the number of individual banks indicated in parentheses. Country-level indices are normalized to have a January 3, 2022 value of 100.

Figure IA.6: Model Predictions & Evidence: House Prices

(a) IRF for House Prices (FRM vs. ARM)



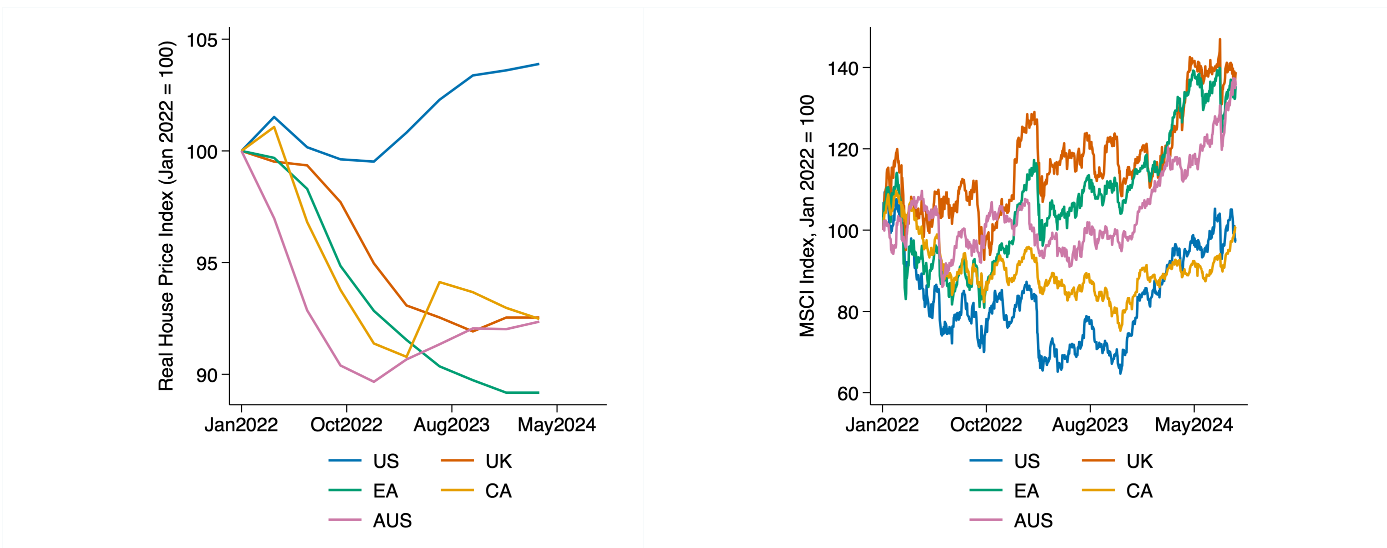
(b) Real House Prices since 2022 (US vs. UK)



Notes: Panel (a) shows the impulse response function for house prices in response to an exogenous interest rate shock as shown in section 5.1. Panel (b) shows real house prices in the US and UK indexed to 2022 Q1.

Figure IA.7: House and Bank Equity Prices (Cross-Country)

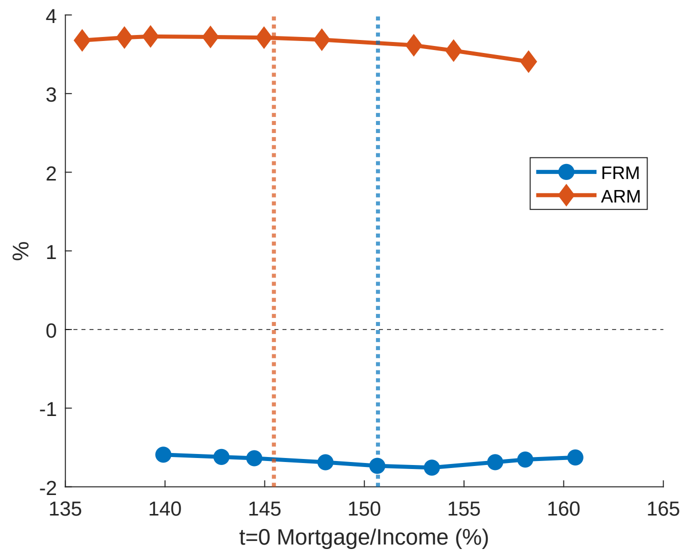
(a) Real House Prices since 2022 (Cross Country) (b) Bank Equities since 2022 (Cross Country)



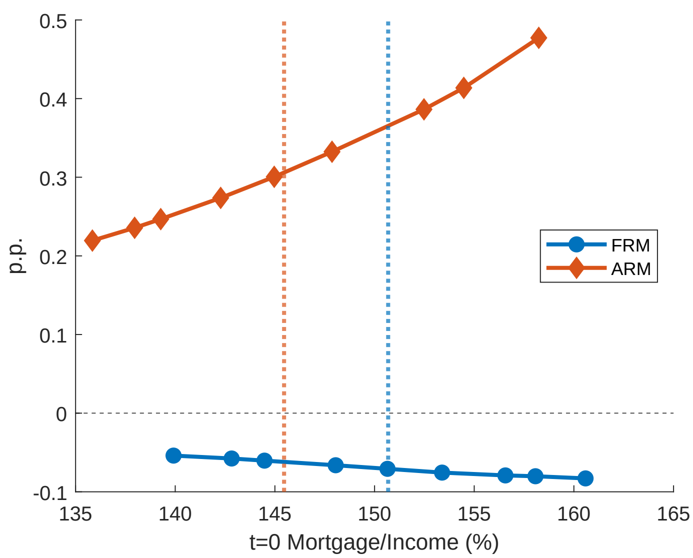
Notes: Panel (a) shows real house prices in the US, UK, Canada, Australia, and Euro Area indexed to 2022 Q1. Panel (b) shows MSCI bank equity indices in the US, UK, Canada, Australia, and Euro Area indexed to January 1, 2022.

Figure IA.8: State-Dependent Impulse Responses: Net Worth and Default by Mortgage-to-Income

(a) Deviation in Intermediary Net Worth / GDP (%)

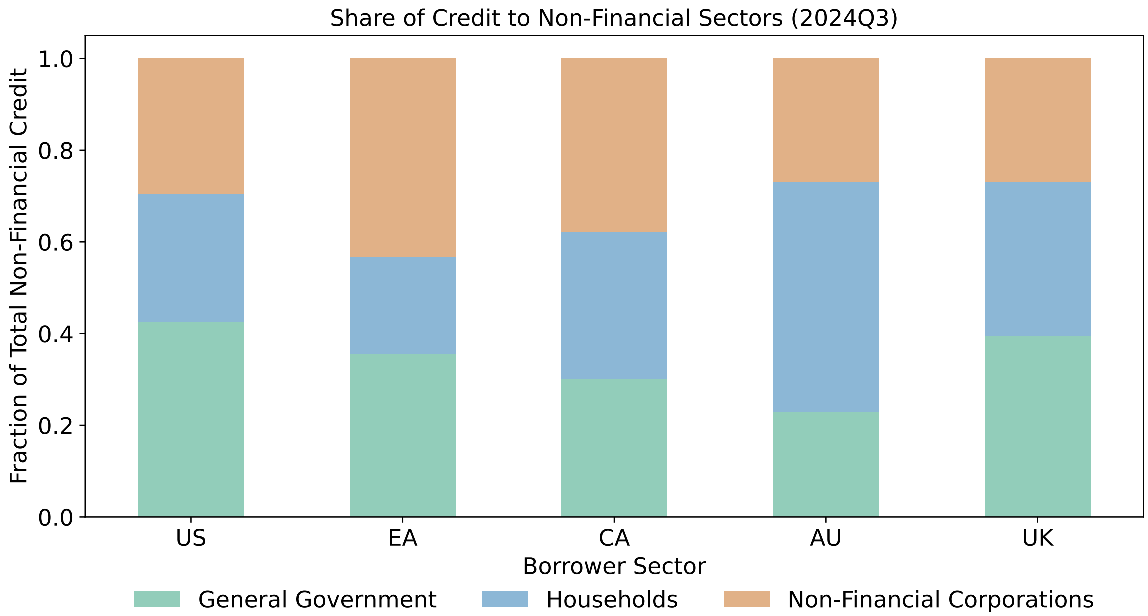


(b) Deviation in Default Rate (p.p.)



Notes: This figure shows $t = 1$ impulse responses to a rate rise, conditional on different borrower mortgage-to-income ratios. For default rates, the figure shows the percentage point difference between the shocked path and the unconditional path. For intermediary net worth, the figure shows the percentage deviation of the shocked path relative to the unconditional path. Points in the state space were chosen as percentiles $p \in \{1, 5, 10, 30, 50, 70, 90, 95, 99\}$ of the borrower wealth distribution conditional on the average ("medium") level of interest rates. Since this distribution is correlated with intermediary wealth, we also condition on the mean of intermediary net worth, conditional on average interest rates and borrower wealth being in the neighborhood of the given borrower wealth, i.e. $WI_j = E[WI | r_t^p = \bar{r}, WB_j \in \mathcal{B}(WB_j)]$ where $\mathcal{B}$ defines the neighborhood, set to 5 percentiles. Dotted vertical lines indicate baseline impulse responses for the FRM and ARM economies.

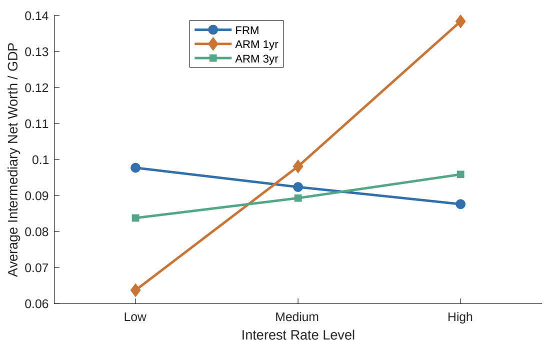
Figure IA.9: Credit to the Non-Financial Sector Across Countries & Country Groups



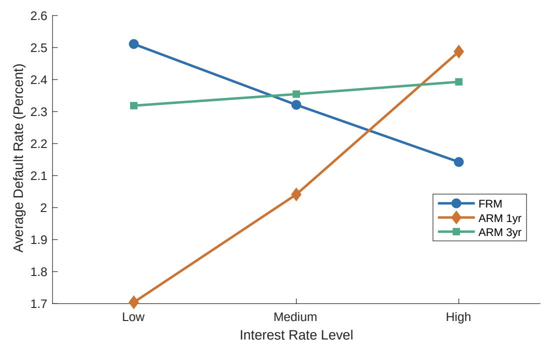
Notes: This figure shows credit shares to the non-financial sector, split by general government, households and non-profit institutions, and non-financial corporations as measured by the BIS, across different countries and country groups (U.S., Euro Area, Canada, Australia, U.K.), as of 2024 Q3.

Figure IA.10: Net Worth and Default by Interest Rate Level (Intermediate ARM)

(a) Intermediary Net Worth / GDP

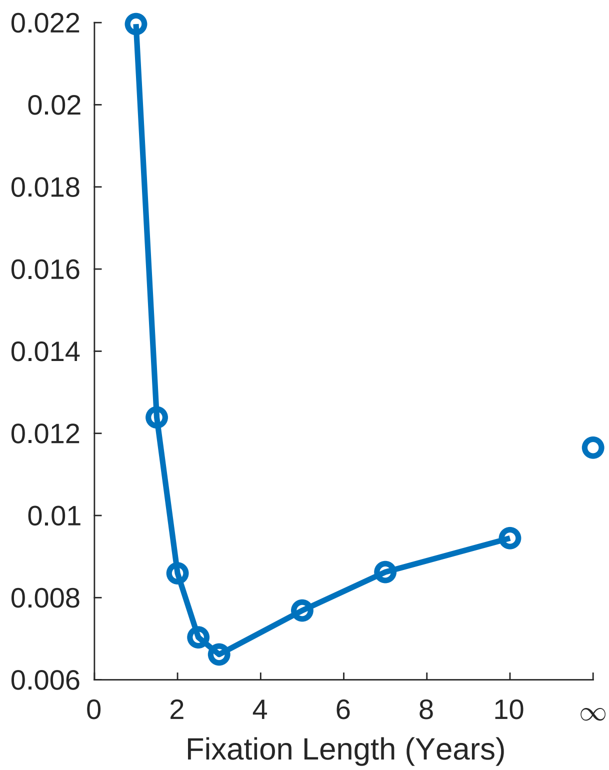


(b) Default Rate



Notes: This figure shows simulation-based average rates of default and levels of intermediary net worth across different levels of interest rates, for the full-ARM (1-year fixation length), intermediate ARM (3-year fixation length), and full-FRM (infinite fixation length) economies.

Figure IA.11: Mortgage Return Volatility



Notes: This figure shows the volatility of mortgage returns, measured as the standard deviation of net income over total assets, i.e. reflecting volatility of return on assets (ROA). The x-axis reflects an annual rate reset probability of $\pi_\tau \in \{1, 2/3, 0.5, 0.4, 1/3, 0.2, 1/7, 0.1, 0\}$, which corresponds to fixed-rate lengths of 1, 1.5, 2, 2.5, 3, 5, 7, 10 years and ∞, respectively.

II Analytical Results: Net Worth Hedging, Interest Rate vs. Credit Risk Channel, and Robustness

In this section, we characterize the sensitivity of mortgage portfolio values to interest rates in closed form. We decompose the response into an *interest rate channel*, capturing the effect of higher rates on mortgage payments and prices, and a *credit channel*, reflecting the effect of higher rates on losses in value caused by higher rates of default and changes in recovery rates. We show the conditions under which a net-worth hedging effect exists, i.e., the credit channel has the opposite sign to the interest rate channel, and, in the case of ARMs, a weaker magnitude than the interest rate channel.

Specifically, we find that for ARMs, the default rate sensitivity to rate rises would have to be at least *14 to 15 times greater* than what is generated in our baseline calibration, for the positive cash flow effect from an increase in mortgage income on performing loans to be offset by an increase in credit losses. We show that ARMs thus have negative duration under a wide range of plausible processes for interest rates and default responses.

Interest Rate and Credit Channels We show that the effect of a rise in rates on the value of the mortgage portfolio can be decomposed into two channels: 1) an interest rate channel, and 2) a credit channel.

We start by decomposing the mortgage portfolio response to shocks to the policy rate r_t^f. Recall that the per-bond mortgage payoff $\mathcal{X}_t$ is:

$$\mathcal{X}_t = (1 - F_t^\eta)(x_t + [1 - \delta_m]q_t^m) + F_t^\eta(1 - LGD_t),$$

i.e. the mortgage payment plus the ex-payment value of the mortgage at equilibrium prices q_t^m, times the probability of survival, plus recoveries (one minus the losses-given-default) times the probability of default.

We can write the response of this payoff to interest rate increases as:

$$\frac{\partial \mathcal{X}_t}{\partial r_t^p} = (1 - F_t^\eta)\left(\frac{\partial x_t}{\partial r_t^p} + [1 - \delta_m]\frac{\partial q_t^m}{\partial r_t^p}\right) - \frac{\partial F_t^\eta}{\partial r_t^p}(x_t + [1 - \delta_m]q_t^m - [1 - LGD_t]) - F_t^\eta \frac{\partial LGD_t}{\partial r_t^p}$$

In steady state, $q_t^m \approx 1$ (exact in deterministic steady state), and $x_t \approx \iota_f + \delta_m$ (exact in deterministic steady state) in either the FRM or ARM economy. So the steady-state sensitivity can be written as:

$$\left.\frac{\partial \mathcal{X}_t}{\partial r_t^p}\right|_{r_t^p=\bar{r}} = \underbrace{(1-F^\eta)\left(\frac{\partial x_t}{\partial r_t^p} + [1-\delta_m]\frac{\partial q_t^m}{\partial r_t^p}\right)}_{\text{Interest rate channel}} \quad \underbrace{-\frac{\partial F_t^\eta}{\partial r_t^p}(\iota_f + \overline{LGD}) - F^\eta \frac{\partial LGD_t}{\partial r_t^p}}_{\text{Credit channel}}$$

Mortgage portfolio values change (1) because payments and prices of surviving mortgages change, and (2) because default rates and recovery rates change. We term (1) the interest rate channel, and (2) the credit channel, and expand on each partial derivative in the following.

Interest Rate Channel By aggregation, we can interpret the payoff $\mathcal{X}_t$ as a payoff on a portfolio of mortgages. For an economy with a given reset probability π_τ, a fraction $1-\pi_\tau$ of mortgages will be in the fixed stage and a fraction π_τ will be in the floating stage. Likewise we can interpret q_t^m as the price of the mortgage portfolio, i.e. the price of the claim to all future mortgage payments x_{t+s} for $s = 1, 2, \ldots$ Define the mortgage yield-to-maturity (ytm) as the discount rate that prices future promised mortgage cash flows:[37]

$$q_t^m = \sum_{s=1}^{\infty} \frac{(1-\delta_m)^{s-1}}{(1+ytm_t)^s} E_t[x_{t+s}] \tag{IA.1}$$

The total portfolio cash flows are $\iota_f + \delta_m$ for loans in the fixed stage, and $r_{t+s}^f + \iota_a + \delta_m$ for loans in the floating stage. At any period, the fraction of loans in the floating stage is given by π_τ, so we can write:

$$x_{t+s} = \delta_m + \iota_f + \pi_\tau(r_{t+s}^f - \bar{r}),$$

where we used $r_{t+s}^f + \iota_a = r_{t+s}^f - \bar{r} + \bar{r} + \iota_a$ and $\bar{r} + \iota_a = \iota_f$. We can see from here (and from

[37]Note that this formulation incorporates future default risk into ytm. In that sense, it is consistent with the definition of credit risky yields used by practitioners.

the aggregation result) that

$$\frac{\partial x_t}{\partial r_t^f} \;=\; \pi_\tau. \tag{IA.2}$$

Given the AR(1) process for rates, $E_t[r_{t+s}^f - \bar{r}] = \rho^s(r_t^p - \bar{r})$. Plugging in, we get: $q_t^m = \sum_{s=1}^{\infty} \frac{(1-\delta_m)^{s-1}}{(1+ytm_t)^s}\left(\delta_m + \iota_f + \pi_\tau \rho^s(r_t^p - \bar{r})\right)$. Using geometric series formulas, we obtain:

$$q_t^m = \underbrace{\frac{\iota_f + \delta_m}{ytm_t + \delta_m}}_{\text{Fixed}} + \pi_\tau \underbrace{\frac{\rho(r_t^p - \bar{r})}{1 + ytm_t - \rho(1-\delta_m)}}_{\text{Fixed-for-floating swap}} \tag{IA.3}$$

which yields a key insight: the first term reflects the present value of a claim to the fixed cash flows (i.e. a fixed bond), while the second term reflects π_τ claims to a fixed-for-floating swap which pays discounted future spreads $r_t^f - \bar{r}$.[38] In steady state, $ytm = \iota_f$ and $r_t^p = \bar{r}$ so the first term is one and the second term is zero. Out of steady state, the value of the swap moves against the value of the fixed component, and so hedges rate risk, consistent with the intuition that floating-rate bonds are less sensitive to rates than fixed-rate ones.

Differentiating with respect to r_t^p at steady state we get:

$$\left.\frac{\partial q_t^m}{\partial r_t^p}\right|_{r_t^p = \bar{r}} = \underbrace{-\frac{ytm'(\bar{r})}{\iota_f + \delta_m}}_{\text{Discount rate effect}} + \pi_\tau \underbrace{\frac{\rho}{1 + \iota_f - \rho(1-\delta_m)}}_{\text{Future cash flow effect}} \tag{IA.4}$$

where the first term represents the change in value coming from discount rates, including changing credit spreads, and the second term represents changes in future cash flows, which go to zero if the entire portfolio is FRMs ($\pi_\tau = 0$), or if interest rate shocks are not persistent ($\rho = 0$).

Note: Textbook Macaulay Duration is defined as $-\frac{1}{\bar{q}^m} \left.\frac{\partial q_t^m}{\partial r_t^p}\right|_{r_t^p = \bar{r}}$. Assuming a *parallel and permanent* rate shift ($\rho = 1$) and *full pass-through* ($ytm'(\bar{r}) = 1$), and because the swap's

[38] The present value of spreads is $\pi_\tau \sum_{s=1}^{\infty} \frac{(1-\delta_m)^{s-1}}{(1+ytm_t)^s} \rho^s \left(r_t^f - \bar{r}\right) = \pi_\tau \frac{\rho\,(r_t^f - \bar{r})}{1+ytm_t - \rho\,(1-\delta_m)}$.

present value is zero in steady state, using equation IA.4 we get:

$$\delta_{\text{Macaulay}} = \frac{1-\pi_\tau}{\iota_f + \delta_m} \qquad \begin{cases} \pi_\tau = 0 \text{ (pure FRM)}: & \delta_{\text{Macaulay}} = \dfrac{1}{\iota_f + \delta_m}, \\ \pi_\tau = 1 \text{ (pure ARM)}: & \delta_{\text{Macaulay}} = 0. \end{cases}$$

Credit Channel: Upper Bound Because steady state default rates are low (our calibration target is 2.45%), the recovery term ($F^\eta \frac{\partial LGD_t}{\partial r_t^p}$) is quantitatively small. To develop intuition, we consider the extreme case where $LGD_t = 1$, i.e., there are no recoveries. This makes defaults much costlier than in the calibrated model, in which $LGD \approx 0.2$, and as a result puts an upper bound on how much credit effects can offset/"hedge" interest rate effects.[39]

Then, leaving the evaluation at $r_t^p = \bar{r}$ implicit, the total sensitivity simplifies to:

$$\frac{\partial \mathcal{X}_t}{\partial r_t^p} = \underbrace{(1-F^\eta)\left(\frac{\partial x_t}{\partial r_t^p} + [1-\delta_m]\frac{\partial q_t^m}{\partial r_t^p}\right)}_{\text{Interest rate channel}} \underbrace{-\frac{\partial F_t^\eta}{\partial r_t^p}(1+\iota_f)}_{\text{Credit channel}} \tag{IA.5}$$

What governs the strength of the credit channel is thus the sensitivity of default with respect to interest rates ($\frac{\partial F_t^\eta}{\partial r_t^p}$).

Overview of Quantitative Results Our quantitative analysis shows that for *any* mortgage structure, the interest rate and credit channel have *opposite* signs:

- With FRMs, $\frac{\partial x_t}{\partial r_t^p} = 0$ since payments are fixed. The effect on performing loans is the mark-to-market loss on remaining principal: $\frac{\partial q_t^m}{\partial r_t^p} < 0$, i.e. a valuation effect. As a result, the first term is negative. But this same valuation effect decreases the household's net-worth incentive to default, so $\frac{\partial F_t^\eta}{\partial r_t^p} < 0$. Therefore, the second term is positive.
- For an adjustable-rate mortgage (ARM), the signs for both terms flip. Both $\frac{\partial x_t}{\partial r_t^p} = 1 > 0$ (interest income effect) and $\frac{\partial q_t^m}{\partial r_t^p} > 0$ (since future coupons will also be higher), making the first term positive. Higher payments strengthen the borrowers' liquidity incentive

[39] Formally, an assumption of zero recoveries places an upper bound on the magnitude of the credit channel as long as $\frac{\partial F_t^\eta}{\partial r_t^p}\frac{1-\overline{LGD}}{F^\eta} > \frac{\partial LGD_t}{\partial r_t^p}$. Since in our calibration $\frac{1-\overline{LGD}}{F^\eta} = \frac{1-0.2}{0.0245} = 32.6$, it would have to be $\frac{\partial LGD_t}{\partial r_t^p} < 32.6 \times \frac{\partial F_t^\eta}{\partial r_t^p}$, for this condition to be violated. Thus the upper bound assumption holds for virtually all plausible levels of default and LGD sensitivities.

to default, while lower home values strengthen the net worth-driven default incentive. Therefore, $\frac{\partial F_t^\eta}{\partial r_t^p} > 0$, and the second term is negative.

Table IA.II: Directional Effects of a Rate Rise on FRMs vs. ARMs

Mortgage type	$\frac{\partial x_t}{\partial r_t^f}$	$\frac{\partial q_t^m}{\partial r_t^f}$	$\frac{\partial F_t^\eta}{\partial r_t^f}$	Interest Rate Channel	Credit Channel
FRM	0	< 0	< 0	$-$	$+$
ARM	> 0	> 0	> 0	$+$	$-$

As a result, credit risk hedges interest rate risk for both ARM and FRMs.

Next, we develop conditions under which the credit channel could offset the positive mortgage cash flow gains of the interest rate channel, focusing on the case of pure ARMs.

Conditions for Credit Channel to Dominate Interest Rate Channel Combining equations IA.5, IA.2, IA.4 and using steady state substitutions $q_t^m = 1$ and $x_t = \iota_f + \delta_m$, the change in the value of the mortgage portfolio in response to a rate shock is:

$$\frac{\partial \mathcal{X}_t}{\partial r_t^p} = \underbrace{(1 - F^\eta)\left(-\frac{1-\delta_m}{\iota_f + \delta_m} ytm'(r) + \pi_\tau \frac{1+\iota_f}{1+\iota_f - \rho(1-\delta_m)}\right)}_{\text{Interest rate channel}} \underbrace{-\frac{\partial F_t^\eta}{\partial r_t^p}(1+\iota_f)}_{\text{Credit channel}}$$

Consider a sufficiently short fixation period (sufficiently high π_τ) such that the interest rate effect is positive, i.e., $\pi_\tau \frac{1+\iota_f}{1+\iota_f-\rho(1-\delta_m)} > \frac{1-\delta_m}{\iota_f+\delta_m} ytm'(r)$. Then, for the total effect to be negative, i.e., for the credit channel to dominate the positive mortgage cash flow effect of the interest rate channel, it must be that:

$$\pi_\tau \frac{1+\iota_f}{1+\iota_f - \rho(1-\delta_m)} < \frac{\partial F_t^\eta / \partial r_t^p}{1 - F^\eta}(1+\iota_f) + \frac{1-\delta_m}{\iota_f + \delta_m} ytm'(\bar{r})$$

In other words:

1. Default rates must be increasing enough in rates (high $\partial F_t^\eta / \partial r_t^p$)

2. Default rates must be high enough to begin with (low survival probability $1 - F^\eta$)

3. Discount rates need to be not too decreasing in rates (high $ytm'(\bar{r})$).

For the case of a pure ARM ($\pi_\tau = 1$), this condition becomes:

$$\underbrace{\frac{1}{1+\iota_f-\rho(1-\delta_m)}}_{\text{Interest channel: future cash flow effect}} < \underbrace{\frac{\partial F^\eta_t/\partial r^p_t}{1-F^\eta}}_{\text{Credit channel}} + \underbrace{\frac{1-\delta_m}{(1+\iota_f)(\iota_f+\delta_m)} ytm'(\bar{r})}_{\text{Interest channel: discount rate effect}} \tag{IA.6}$$

As a result, the interest rate channel can be decomposed into a future cash flow effect (first term of inequality IA.6), and a discount rate effect (third term of inequality IA.6).

- The future cash flow effect is always positive. When the policy rate rises, each adjustable-rate loan's coupon rises one for one, scaled by the loan's effective duration (in the denominator).
- The credit channel reflects how much default rates rise in response to higher rates, scaled by the survival probability.
- A higher ytm lowers the present value of the remaining principal on every surviving loan (discount rate effect).

As a result, the right-hand-side of the inequality measures the per-unit value loss coming from 1) more loans defaulting, and 2) lower values of the fixed portion of future cash flows. For the credit channel to dominate the interest rate channel (and more precisely, to offset the positive effect of future cash flows), the relative size of all three components matters.

The sign and size of the interest rate channel depends on $ytm'(\bar{r})$, i.e. how much the mortgage yield-to-maturity changes in response to the policy rate ("YTM pass-through"). Once we know this number, we can show how big the default sensitivity must be to offset the positive cash flow effects of higher future mortgage payments. We start by evaluating a possible upper bound, the YTM pass-through under the expectations hypothesis, before providing an alternative estimate given our calibration of sticky deposits, as well as a model-implied comprehensive estimate.

YTM Pass-through Benchmark: Expectations Hypothesis A 1 p.p. increase in the policy rate expressed as deviation from steady state is $r^p_t - \bar{r} = 0.01$. Under the expectations hypothesis, with zero term premia and an AR(1) policy rate process $\mathrm{E}_t[r^f_{t+1} - \bar{r}] = \rho(r^f_t - \bar{r})$, the expected deviation s periods ahead is

$$E_t\left[r^f_{t+s} - \bar{r}\right] \;=\; \rho^s\,(0.01).$$

Let $y_t^{(n)}$ denote the n-period zero coupon yield. Under the expectations hypothesis, the steady-state yield curve is flat $\bar{y}^{(n)} = \bar{r}$. Away from the steady state, n-period zero-coupon yield, in logs, is the sum of expected future short rates, so its deviation from steady state is given by:

$$y_t^{(n)} - \bar{y}^{(n)} \equiv \sum_{s=0}^{n} \mathrm{E}_t[r_{t+s} - \bar{r}] = \frac{\rho(1-\rho^n)}{n(1-\rho)}(r_t^f - \bar{r}).$$

Suppose that the credit spread, equal to $\overline{ytm} - \bar{r}$ in steady state, is unchanged by the rate shock.[40] Then, with $y\bar{t}m = \iota_f$, the constant discount rate for an expected mortgage payment s periods ahead $\mathrm{E}_t[x_{t+s}]$ therefore becomes $\iota_f + \frac{1}{s}\frac{1-\rho^s}{1-\rho}(r_t^f - \bar{r})$.

Discounting every cash flow of the mortgage pool with that rate and summing yields an infinite sum expression for q_t^m. Differentiating q_t^m with respect to the policy rate,[41] then evaluating at steady state, gives

$$\frac{\partial q_t^m}{\partial r_t^f} = -\frac{1-\pi_\tau\rho}{1+\iota_f-\rho(1-\delta_m)},$$

the difference between a fixed-leg present value loss and a floating-leg present value gain.

Equating this to the derivative of the price to the rate expressed in terms of a single yield-to-maturity $ytm(\bar{r})$ (IA.4) $-\frac{ytm'(\bar{r})}{\iota_f+\delta_m} + \frac{\pi_\tau\rho}{1+\iota_f-\rho(1-\delta_m)}$ and solving for $ytm'(\bar{r})$ gives the *full pass-through* under the Expectations Hypothesis:

$$ytm'_{\mathrm{EH}}(\bar{r}) = \frac{\iota_f+\delta_m}{1+\iota_f-\rho(1-\delta_m)},$$

which equals 1 only if the rate increase is permanent ($\rho = 1$) and hence the entire yield curve shifts up in parallel.

[40] This implicitly assume that neither expected future credit losses, nor the risk premium associated with them, changes - this assumption will be relaxed when we use the model-implied comprehensive estimate.

[41] The mortgage price is given by the sum $q_t^m = \sum_{s=1}^{\infty} \frac{(1-\delta_m)^{s-1}}{(1+\iota_f+\frac{1}{n}\frac{1-\rho^s}{1-\rho}(r_t^p-\bar{r}))^s}\left(\delta_m+\iota_f+\pi_\tau\rho^s(r_t^p-\bar{r})\right)$, and so its derivative is given by the sum $\frac{\partial q_t^m}{\partial r_t^p} = \sum_{s=1}^{\infty}\left[-\frac{(1-\delta_m)^{s-1}}{(1+\iota_f)^{1+s}}\frac{1-\rho^s}{1-\rho}(\iota_f+\delta_m)s + \pi_\tau\frac{(1-\delta_m)^{s-1}}{(1+\iota_f)^s}\rho^s\right]$.

YTM Pass-through with Sticky Deposits How do things change with imperfect pass-through of indexation rates r_t^p to the rates off of which the mortgage is priced, deposit funding rate $r_t^d = r_t^p - \alpha_d + \beta_d(r_t^p - \bar{r})$? The future path of deposit rate deviations is given by $r_{t+s}^d - \bar{r}^d = \beta_d \rho^s (r_t^p - \bar{r})$. Then long rates are $y_t^{(n)} = \frac{1-\rho^n}{1-\rho}\beta_d(r_t^p - \bar{r})$. As a result, the entire zero-coupon curve simply scales by β_d. Hence

$$ytm'_{\beta_d}(\bar{r}) = \beta_d ytm'_{\mathrm{EH}}(\bar{r}).$$

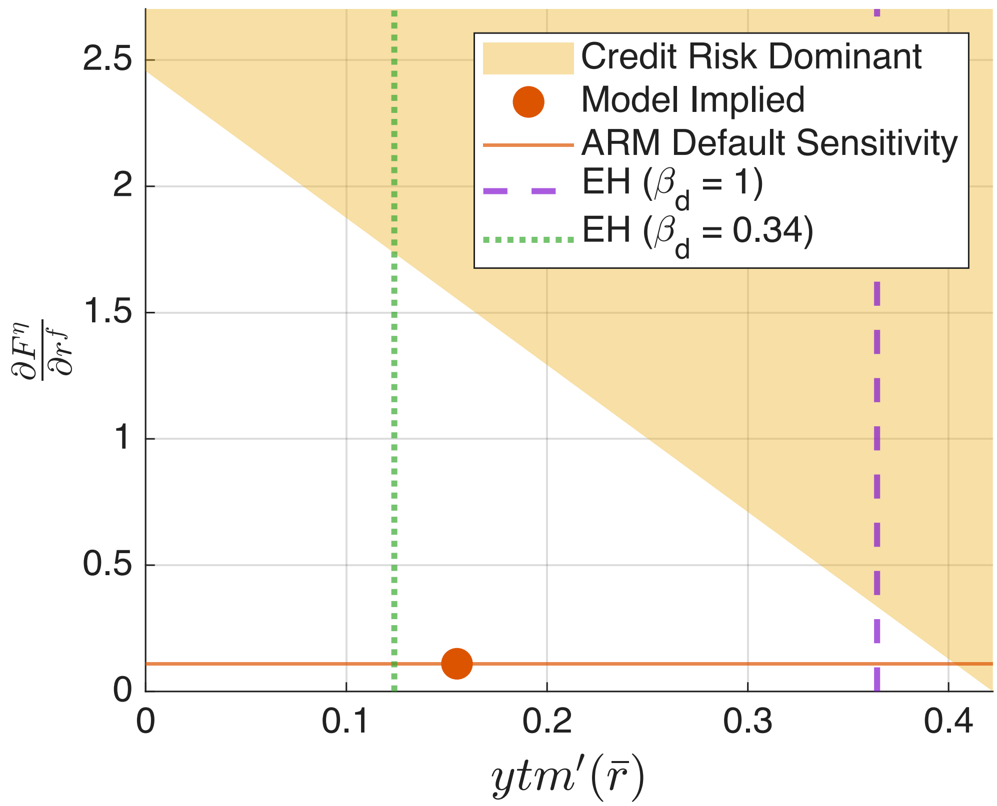
With $\rho = 0.724$, $\iota_f = 0.059$, $\delta_m = 0.085$ and $\beta_d = 1$, we get an EH pass-through of 0.364, and for $\beta_d = 0.34$ (matching our calibration of deposit rate stickiness), we get a pass-through of 0.124.

Quantifying Credit vs. Interest Rate Channel We find that the default sensitivity with respect to rates is $\partial F_t^\eta / \partial r_t^p = 0.1096$ (from the ARM IRFs). At the calibration target $F^\eta = 0.0245$, this is far too small to overturn the rate effect even assuming full pass-through of rates to YTM under the Expectations Hypothesis. At the deposit-adjusted passthrough of 0.124, a 1 p.p. increase in rates would need to lead to a 1.7 p.p. increase in defaults, 15x larger than the effect we find.

Figure IA.12 plots how high the default sensitivity with respect to rates would have to be to offset the interest rate channel, for a given level of the YTM pass-through. The EH YTM pass-through under $\beta_d = 1$ is marked as a purple dashed line, and the EH YTM pass-through under $\beta_d = 0.34$ is marked as a green dotted line. It also plots the calibrated default sensitivity of 0.1096 as a horizontal orange line. Lastly, recall that this closed-form measure does not capture: the persistence of default increases, which would increase the passthrough; and falling risk premia, which would decrease the pass-through. To quantify the full YTM response that takes into account all of these effects, we compute the model-implied value from an IRF of mortgage YTM to a rate shock, shown in Figure Figure IA.12 as the orange dot, at 0.1551.

To conclude, default sensitivities with respect to rate rises would have to be 5x (full pass-through assumption), 14x (model-implied comprehensive effect) to 15x (deposit-adjusted expectations hypothesis) greater, to offset the positive cash flow effect of future payment increases. Moreover, taking into account some foreclosure recovery ($LGD < 1$) would further amplify the required magnitude of credit losses in default. As a result, we conclude that the interest rate channel is greater than the credit channel under the vast majority of plausible paths for interest rates and default rates.

Figure IA.12: Illustration of Conditions for Credit Channel to Dominate Interest Rate Channel



Notes: This figure shows how high the default sensitivity with respect to rates ($\partial F_t^\eta / \partial r_t^p$) would have to be to offset the interest rate channel as described in Equation IA.6, for a given level of the pass-through of rate changes to mortgage yields ($ytm'(\bar{r})$). The area where the credit channel outweighs the interest rate channel is shaded in yellow. YTM pass-through under the Expectations Hypothesis and $\beta_d = 1$ is marked as a purple dashed line, and that under $\beta_d = 0.34$ is marked as a green dotted line. The model-implied YTM pass-through is shown as an orange dot. The calibrated $\partial F_t^\eta / \partial r_t^p = 0.1096$ is shown as a horizontal orange line.

III Model Derivations

III.1 Borrowers

The complete borrower's problem is given by:

$$V(w_t^i, \mathcal{Z}_t) = \max_{d_t^i, h_t^i, s_t^i, m_t^i} \beta \mathrm{E}_t \left[\max \left\{ \max_{a_t^i \geq 0} u(c_{t+1}^{i,nd}, h_t^i) + V(w_{t+1}^{i,nd}, \mathcal{Z}_t), \eta_t^i \left(u(c_{t+1}^{i,d}, h_t^i) + V(w_{t+1}^{i,d}, \mathcal{Z}_t) \right) \right\} \right] \tag{IA.7}$$

where $\mathcal{Z}_t = \{Y_t, r_t^p, W_t^B, W_t^I\}$ and

$$u(c_t^i, h_{t-1}^i) = \frac{\left[(c_t^i)^{1-\theta}(h_{t-1}^i)^{\theta}\right]^{1-\gamma}}{1-\gamma}$$

such that

$$w_t^i + \mathcal{R}_t^i = \frac{d_t^i}{1+r_t^d} + q_t^m m_t^i + p_t^h h_t^i + p_t^s s_t^i + \Phi \left(\frac{q_t^m m_t^i}{p_t^h h_t^i} - \overline{LTV} \right) \tag{IA.8}$$

$$c_t^{i,nd} + x_t^i m_{t-1}^i + \delta_h h_{t-1}^i + a_t^i = s_{t-1}(Y_t + \epsilon_t^i) + d_{t-1}^i \tag{IA.9}$$

$$c_t^{i,d} = s_{t-1}(Y_t + \epsilon_t^i) + d_{t-1}^i \tag{IA.10}$$

$$w_t^{i,nd} = a_t^i - (1-\delta_m) m_{t-1}^i q_t^m + p_t^h h_{t-1}^i + p_t^s s_{t-1}^i \tag{IA.11}$$

$$w_t^{i,d} = (1-\lambda) p_t^s s_{t-1}^i \tag{IA.12}$$

$$a_t^i \geq 0 \tag{IA.13}$$

where $\mathcal{R}_t^i$ is a rebate of the LTV adjustment cost Φ proportional to wealth w_t^i. With this parametrization, the adjustment cost does not have income effects.

Notice that $u(c,h)$ is homogeneous of degree $1-\gamma$ in c and h and that all constraints are linear in wealth w_t^i in the sense that if a given allocation is feasible for a wealth of 1, then w_t^i times that allocation is feasible for a wealth of w_t^i. By Proposition 1 of Diamond and Landvoigt (2022), these two properties imply that the borrower's value function can be decomposed into $\frac{(w_t^i)^{1-\gamma}}{1-\gamma}$ and a term $v(\mathcal{Z})$ that only depends on state variables exogenous to the borrower.

For a given choice g_t^i, define $\hat{g}_t^i = \frac{g_t^i}{w_t^i}$. Then, the value function can be rewritten as:

$$v(\mathcal{Z}_t)\frac{(w_t^i)^{1-\gamma}}{1-\gamma} = \max_{\hat{d}_t^i,\hat{h}_t^i,\hat{s}_t^i,\hat{m}_t^i} \beta \mathrm{E}_t \left[\max \left\{ \max_{\hat{a}_t^i \geq 0} (w_t^i)^{1-\gamma} u(\hat{c}_{t+1}^{i,nd}, \hat{h}_t^i) + v(\mathcal{Z}_{t+1}) \frac{(w_t^i \hat{w}_{t+1}^{i,nd})^{1-\gamma}}{1-\gamma}, \right. \right.$$

$$\left. \left. \eta_t^i \left(u(\hat{c}_{t+1}^{i,d}, \hat{h}_t^i) + v(\mathcal{Z}_{t+1}) \frac{(w_t^i \hat{w}_{t+1}^{i,nd})^{1-\gamma}}{1-\gamma} \right) \right\} \right]$$

Divide both sides by $(w_t^i)^{1-\gamma}$ and drop i subscripts on hatted trading stage choice variables following the proposition cited above, getting the following recursion:

$$v(\mathcal{Z}_t) = (1-\gamma) \max_{\hat{d}_t,\hat{h}_t,\hat{s}_t,\hat{m}_t} \beta \mathrm{E}_t \left[\max \left\{ \max_{\hat{a}_t \geq 0} u(\hat{c}_{t+1}^{i,nd}, \hat{h}_t) + v(\mathcal{Z}_{t+1}) \frac{(\hat{w}_{t+1}^{i,nd})^{1-\gamma}}{1-\gamma}, \right. \right.$$

$$\left. \left. \eta_t^i \left(u(\hat{c}_{t+1}^{i,d}, \hat{h}_t) + v(\mathcal{Z}_{t+1}) \frac{(\hat{w}_{t+1}^{i,d})^{1-\gamma}}{1-\gamma} \right) \right\} \right]$$

such that

$$1 = \frac{\hat{d}_t}{1+r_t^d} + q_t^m \hat{m}_t + p_t^h \hat{h}_t + p_t^s \hat{s}_t + \Phi\left(\frac{q_t^m \hat{m}_t^i}{p_t^h \hat{h}_t^i} - \overline{LTV} \right) - \hat{\mathcal{R}}_t \tag{IA.14}$$

$$\hat{c}_t^{i,nd} + x_t^i \hat{m}_{t-1} + \delta_h \hat{h}_{t-1} + a_t^i = \hat{s}_{t-1}(Y_t + \epsilon_t^i) + \hat{d}_{t-1} \tag{IA.15}$$

$$\hat{c}_t^{i,d} = \hat{s}_{t-1}(Y_t + \epsilon_t^i) + \hat{d}_{t-1} \tag{IA.16}$$

$$\hat{w}_t^{i,nd} = \hat{a}_t^i - (1-\delta_m)\hat{m}_{t-1} q_t^m + p_t^h \hat{h}_{t-1} + p_t^s \hat{s}_{t-1} \tag{IA.17}$$

$$\hat{w}_t^{i,d} = (1-\lambda) p_t^s \hat{s}_{t-1} \tag{IA.18}$$

$$\hat{a}_t^i \geq 0 \tag{IA.19}$$

$$\tag{IA.20}$$

The remaining dependence on i is in consumption stage shock realizations and choices, which enter the value function through the continuation values inside the expectations operator. Therefore, if we can write the consumption stage problem as a function of state variables exogenous to the borrower and i.i.d. idiosyncratic shocks, we will have confirmed the validity of our aggregation.

No Default Branch Consumption Decision If the borrower chooses not to default, they choose $\hat{c}_t^{i,nd}$ and $\hat{a}_t^i$ to maximize $u(\hat{c}_{t+1}^{nd}, \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd})^{1-\gamma}}{1-\gamma}$ subject to the budget constraint (IA.15), wealth evolution (IA.17), and the non-negative intraperiod savings constraint (IA.19). The first order condition for $\hat{a}_t^i$ is:

$$u_c(\hat{c}_{t+1}^{i,nd}, \hat{h}_t) = v(\mathcal{Z}_{t+1})\left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} + \kappa_{t+1}^{i,nd}$$

where $\kappa_{t+1}^{i,nd}$ is the Lagrange multiplier on the nonnegativity constraint (IA.19). We will use the functions $\hat{c}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i)$ and $\hat{w}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i)$ to explicitly denote the dependence of the consumption decision on the idiosyncratic realizations borrower's income and the mortgage regime.

Default Decision Given the consumption decision above, a household decides to default iff

$$\underbrace{u(\hat{c}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i), \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd}(y_t^i, \mathbb{1}_\tau^i))^{1-\gamma}}{1-\gamma}}_{v^{nd}(d_t^i,h_t^i,s_t^i,m_t^i,\epsilon_t^i,\mathbb{1}_\tau^i)} < \eta_t^i \underbrace{\left[u(\hat{y}_t^i + \hat{d}_{t-1}, \hat{h}_t) + v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{d}(y_t^i))^{1-\gamma}}{1-\gamma}\right]}_{v^{d}(d_t^i,h_t^i,s_t^i,m_t^i,\epsilon_t^i)}$$

This expression implies that there exist a default threshold $\eta^*(\epsilon_t^i, \mathbb{1}_\tau^i)$ at which the household is indifferent between defaulting and not defaulting. Which side of the threshold leads to a default vs. no-default decision depends on the sign of the value function, which depends on whether or not $\gamma > 1$. For the rest of these derivations, assume that $\gamma > 1$, the more common case, in which case value functions are negative, and so the default region is given by $[0, \eta^*(y_t^i, \mathbb{1}_\tau^i)]$.

Using the Law of Iterated Expectations, we can separate the conditional expectation E_t in the definition of the value function into an expectation over the realization of aggregate shocks $\mathrm{E}_t^{\mathcal{Z}}[\cdot]$, the expectation over the realizations of i.i.d. idiosyncratic shocks to income ϵ_t^i and reset probability $\mathbb{1}_\tau^i$ denoted by $\mathrm{E}_i[\cdot]$, and the expectation over i.i.d. default utility shocks η^i denoted by $\mathrm{E}_\eta[\cdot]$. Let F_η denote the c.d.f. of the η^i distribution. Then the expectation in the value function can be written as:

$$\mathrm{E}_t^{\mathcal{Z}}\left[\mathrm{E}_i\left[F_\eta(\eta^*(\epsilon,\tau))\mathrm{E}_\eta\left[\eta_t^i\left(u(\hat{s}_{t-1}(Y_t+\epsilon_t^i)+\hat{d}_{t-1},\hat{h}_t)+v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^d)^{1-\gamma}}{1-\gamma}\right)|\eta_t^i>\eta^*(\epsilon_t^i,\mathbb{1}_\tau^i)\right]\right.\right.$$
$$\left.\left.+\left(1-F_\eta(\eta^*(\epsilon_t^i,\mathbb{1}_\tau^i))\right)\left(u(\hat{c}_{t+1}^{nd}(\epsilon_t^i,\mathbb{1}_\tau^i),\hat{h}_t)+v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd}(\epsilon_t^i,\mathbb{1}_\tau^i))^{1-\gamma}}{1-\gamma}\right)\right]\right]$$

Since idiosyncratic shocks are i.i.d., they affect the household problem only through the laws of motion for wealth, admitting aggregation.

We model shocks to ϵ_t^i as discrete. Shocks to the ARM stage $\mathbb{1}_\tau^i$ are Bernoulli. In this case, the expectation $\mathrm{E}_i[\cdot]$ above can be written as:

$$
\begin{aligned}
&\sum_{\tau\in\{0,1\}}\sum_{\epsilon\in\mathcal{E}}\mathcal{P}_\epsilon(\epsilon_t^i=\epsilon)\mathcal{P}_\tau(\tau_t^i=\tau)\times\\
&\mathrm{E}_i\Bigg[F_\eta(\eta^*(\epsilon,\tau))\mathrm{E}_\eta\left[\eta_t^i|\eta_t^i>\eta^*(\epsilon,\tau)\right]\left(u(\hat{s}_{t-1}(Y_t+\epsilon)+\hat{d}_{t-1},\hat{h}_t)+v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^d)^{1-\gamma}}{1-\gamma}\right)\\
&+(1-F_\eta(\eta^*(\epsilon,\tau)))\left(u(\hat{c}_{t+1}^{nd}(\epsilon,\tau),\hat{h}_t)+v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^{nd}(\epsilon,\tau))^{1-\gamma}}{1-\gamma}\right)\Bigg]
\end{aligned}
$$

Note that conditional on default, the borrower's value function does not depend on the specific realization of the utility penalty, meaning that $u(\hat{s}_{t-1}(Y_t+\epsilon)+\hat{d}_{t-1},\hat{h}_t)+v(\mathcal{Z}_{t+1})\frac{(\hat{w}_{t+1}^d)^{1-\gamma}}{1-\gamma}$ can be brought outside the $\mathrm{E}_\eta[\cdot]$ expectation.

Distribution of η Shocks Let $\log\eta_t^i\sim\mathcal{N}\left(-\frac{\sigma_\eta^2}{2},\sigma_\eta^2\right)$. This implies that the average penalty for default is purely pecuniary and governed by λ, while the dispersion of η shocks given by σ_η governs the sensitivity of default rates to economic conditions.

The log-normal distribution admits a simple expression for the partial expectation of the default penalty:

$$
\begin{aligned}
F_\eta^-(\epsilon,\tau)\equiv F_\eta\left(\eta^*(\epsilon,\tau)\right)\mathrm{E}_\eta\left[\eta_t^i|\eta_t^i\le\eta^*(\epsilon,\tau)\right] &= \int_0^{\eta^*(\epsilon,\tau)}\frac{\eta}{\sigma_\eta\sqrt{2\pi}}\exp\left(-\frac{(\log\eta^*(\epsilon,\tau)+\sigma_\eta^2/2)^2}{2\sigma_\eta^2}\right)d\eta\\
&=\Phi\left(\frac{\log\eta^*(\epsilon,\tau)-\sigma_\eta^2/2}{\sigma_\eta}\right)
\end{aligned}
$$

As well as for the survival probability:

$$
\tilde{F}_\eta(\epsilon,\tau)\equiv 1-F_\eta\left(\eta^*(\epsilon,\tau)\right)=1-\Phi\left(\frac{\log\eta^*(\epsilon,\tau)+\sigma_\eta^2/2}{\sigma_\eta}\right)=\Phi\left(\frac{-\log\eta^*(\epsilon,\tau)-\sigma_\eta^2/2}{\sigma_\eta}\right)
$$

Therefore, for a given ϵ and τ, the continuation value of the borrower's problem can be written as:

$$F_\eta^-(\epsilon,\tau)v_t^d(d_t^i,h_t^i,s_t^i,m_t^i,\epsilon)+\tilde{F}_\eta(\epsilon,\tau)v_t^{nd}(d_t^i,h_t^i,s_t^i,m_t^i,\epsilon,\tau)$$

where

$$\eta^*(\epsilon,\tau)=\frac{v_t^d(d_t^i,h_t^i,s_t^i,m_t^i,\epsilon)}{v_t^{nd}(d_t^i,h_t^i,s_t^i,m_t^i,\epsilon,\tau)}$$

III.1.1 First Order Conditions

Preliminaries For a generic choice variable g, write the continuation value of the borrower's problem as:

$$\mathrm{E}_t\left[\underbrace{\left(\int_0^{\eta^*(g)}\eta dF_\eta(\eta)\right)}_{F_\eta^-(g)}v_{t+1}^d(g)+\underbrace{\left[1-F_\eta(\eta^*(g))\right]}_{\tilde{F}_\eta(g)}v_{t+1}^{nd}(g)\right]$$

Differentiating with respect to g yields and collecting terms:

$$\mathrm{E}_t\left[\frac{\partial v_{t+1}^d(g)}{\partial g}F_\eta^-(g)+\frac{\partial v_{t+1}^{nd}(g)}{\partial g}\tilde{F}_\eta(g)+f_\eta(\eta^*(g))\frac{\partial\eta^*(g)}{\partial g}\left(-\eta^*(g)v_{t+1}^d(g)+v_{t+1}^{nd}(g)\right)\right]$$

Plugging in the default condition $v_{t+1}^{nd}(g)=\eta^*(g)v_{t+1}^d(g)$ leads the last term to become zero:

$$\mathrm{E}_t\left[\frac{\partial v_{t+1}^d(g)}{\partial g}F_\eta^-(g)+\frac{\partial v_{t+1}^{nd}(g)}{\partial g}\tilde{F}_\eta(g)\right]$$

Which is the expression we will use to calculate the first order conditions below.

Define the LTV adjustment cost Φ to be $\Phi(x) = \frac{\phi}{2}x^2$.

Denote by μ_t the Lagrange multiplier on the time t budget constraint (IA.14).

Deposits Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of (inter-period) deposits $\hat{d}_t$ in the default and no-default states, respectively, are given by:

$$\frac{\partial V^d_{t+1}}{\partial \hat{d}_t} = u_c(\hat{c}^d_{t+1}(\epsilon), \hat{h}_t) + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,d}_{t+1}\right)^{-\gamma} \frac{\partial \hat{w}^{i,d}_{t+1}}{\partial \hat{d}^i_t} = u_c(\hat{c}^d_{t+1}(\epsilon), \hat{h}_t)$$
$$\frac{\partial V^{nd}_{t+1}}{\partial \hat{d}_t} = u_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t) + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,nd}_{t+1}\right)^{-\gamma} \frac{\partial \hat{w}^{i,nd}_{t+1}}{\partial \hat{d}^i_t} = u_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t)$$

The FOC for (inter-period) deposits $\hat{d}^i_t$ is then given by:

$$\frac{\mu_t}{1 + r^d_t} = \beta \mathrm{E}_t \left[F^-_\eta(\epsilon, \tau) u_c(\hat{c}^d_{t+1}(\epsilon), \hat{h}_t) + \tilde{F}_\eta(\epsilon, \tau) u_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t) \right]$$

Lucas Tree Shares Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of Lucas tree shares $\hat{s}_t$ in the default and no-default states, respectively, are given by:

$$\frac{\partial V^d_{t+1}}{\partial \hat{s}_t} = u_c(\hat{c}^d_{t+1}(\epsilon), \hat{h}_t)(Y_t + \epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,d}_{t+1}\right)^{-\gamma} (1 - \lambda) p^s_{t+1}$$
$$\frac{\partial V^{nd}_{t+1}}{\partial \hat{s}_t} = u_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t)(Y_t + \epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,nd}_{t+1}\right)^{-\gamma} p^s_{t+1}$$

The FOC for shares $\hat{s}^i_t$ is then given by:

$$\mu_t p^s_t = \beta \mathrm{E}_t \Bigg[F^-_\eta(\epsilon, \tau) \left(u_c(\hat{c}^d_{t+1}(\epsilon), \hat{h}_t)(Y_t + \epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,d}_{t+1}\right)^{-\gamma} (1 - \lambda) p^s_{t+1} \right)$$
$$+ \tilde{F}_\eta(\epsilon, \tau) \left(u_c(\hat{c}^{nd}_{t+1}(\epsilon, \tau), \hat{h}_t)(Y_t + \epsilon) + v(\mathcal{Z}_{t+1}) \left(\hat{w}^{i,nd}_{t+1}\right)^{-\gamma} p^s_{t+1} \right) \Bigg]$$

Houses Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of houses $\hat{h}_t$ in the default and no-default states, respectively, are given by:

$$
\begin{aligned}
\frac{\partial V_{t+1}^{d}}{\partial \hat{h}_t} &= u_h^B(\hat{c}_{t+1}^{d}(\epsilon), \hat{h}_t) \\
\frac{\partial V_{t+1}^{nd}}{\partial \hat{h}_t} &= u_h^B(\hat{c}_{t+1}^{nd}(\epsilon), \hat{h}_t) - u_c(\hat{c}_{t+1}^{nd}(\epsilon, \tau), \hat{h}_t)\delta_h + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} p_{t+1}^{h}
\end{aligned}
$$

The FOC for houses $\hat{h}_t^i$ is then given by:

$$
\begin{aligned}
\mu_t p_t^h = \Phi_h \frac{q_t^m \hat{m}_t^i}{(\hat{h}_t^i)^2} &+ \beta \mathrm{E}_t \Big[F_\eta^-(\epsilon, \tau) u_h^B(\hat{c}_{t+1}^{d}(\epsilon), \hat{h}_t) \\
&+ \tilde{F}_\eta(\epsilon, \tau) \left(u_h^B(\hat{c}_{t+1}^{nd}(\epsilon), \hat{h}_t) - u_c(\hat{c}_{t+1}^{nd}(\epsilon, \tau), \hat{h}_t)\delta_h + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} p_{t+1}^{h} \right) \Big]
\end{aligned}
$$

Mortgages Given the realizations of idiosyncratic shocks (ϵ, τ), the marginal values of houses $\hat{m}_t$ in the default and no-default states, respectively, are given by:

$$
\begin{aligned}
\frac{\partial V_{t+1}^{d}}{\partial \hat{m}_t} &= 0 \\
\frac{\partial V_{t+1}^{nd}}{\partial \hat{m}_t} &= u_c(\hat{c}_{t+1}^{nd}(\epsilon, \tau), \hat{h}_t)x_t^i + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} (1 - \delta_m) q_{t+1}^{m}
\end{aligned}
$$

The FOC for shares $\hat{s}_t^i$ is then given by:

$$
\mu_t q_t^m \left(1 - \Phi_m \frac{q_t^m}{p_t^m \hat{h}_t^i}\right) = \beta \mathrm{E}_t \left[\tilde{F}_\eta(\epsilon, \tau) \left(u_c(\hat{c}_{t+1}^{nd}(\epsilon, \tau), \hat{h}_t)x_t^i + v(\mathcal{Z}_{t+1}) \left(\hat{w}_{t+1}^{i,nd}\right)^{-\gamma} (1 - \delta_m) q_{t+1}^{m} \right) \right]
$$

III.1.2 Market-Clearing Conditions and Aggregation

To calculate intermediary wealth and market clearing, we must integrate over the distribution of borrower shocks. First, note that identical choices by borrowers in per-wealth units mean that for any quantity g_t^i that is a function of borrower choices, we can express it is a product of the common per-wealth choice $\hat{g}_t$ and aggregate borrower wealth w_t^B:

$$\int_0^\ell g_t^i di = \hat{g}_t \int_0^\ell w_t^i di = \hat{g}_t w_t^B$$

Aggregate share of defaulting mortgages F_t^η is given by:

$$F_t^\eta = \int_0^\ell \mathbb{1}_d^i di = \sum_{\tau \in \{0,1\}} \sum_{\epsilon \in \mathcal{E}} \mathcal{P}_\epsilon(\epsilon_t^i = \epsilon) \mathcal{P}_\tau(\tau_t^i = \tau) F_\eta(\eta^*(\epsilon, \tau))$$

Aggregate per-unit mortgage payment x_t is given by:

$$x_t = \mathrm{E}_i[x_t^i | \eta^i \leq \eta^{*,i}(\epsilon_t^i, \mathbb{1}_\tau^i)]$$

For other quantities,

- Mortgages: $\int_\ell^1 m_t^I di = \int_0^\ell m_t^i di$ implies $M_t^I = \hat{m}_t W_t^B$
- Borrower Tree Shares: $\alpha = \hat{s}_t W_t^B$
- Houses: $\bar{H} = \hat{h}_t W_t^B$

Finally, the law of motion for aggregate borrower wealth is:

$$\begin{aligned} W_{t+1}^B &= \int_0^\ell w_{t+1}^i di \\ &= W_t^B \mathrm{E}_i \left[\tilde{F}_\eta((\epsilon_t^i, \mathbb{1}_\tau^i)) \hat{w}_{t+1}^{i,d}(\epsilon_t^i) + F_\eta((\epsilon_t^i, \mathbb{1}_\tau^i)) \hat{w}_{t+1}^{i,nd}(\epsilon_t^i, \mathbb{1}_\tau^i) \right] \end{aligned}$$

III.2 Banks

III.2.1 Problem

Banks are not subject to idiosyncratic shocks and are ex-ante identical. As a result, we can solve the problem for the representative *aggregate* bank. Denote aggregate quantities with capital letters. The bank's complete problem is given by:

$$V^I(W_t^I, \mathcal{Z}_t) = \max_{\mathrm{Div}_t^I, D_t^I, M_t^I} \mathrm{Div}_t^I + \mathrm{E}_t \left[\mathcal{M}_{t+1}^S V^I(W_{t+1}^I, \mathcal{Z}_{t+1}) \right] \tag{IA.21}$$

subject to

$$W_t^I = \frac{D_t^I}{1 + r_t^d} + q_t^m M_t^I + \mathrm{Div}_t^I \tag{IA.22}$$

$$W_{t+1}^I = (1 - \nu)\mathcal{X}_{t+1} M_t^I + D_t^I \tag{IA.23}$$

$$D_t \leq \xi \left(\kappa \bar{q}^m + (1 - \kappa) q_t^m \right) M_t^I \tag{IA.24}$$

where $\mathcal{X}_t$ is the aggregate mortgage payment per unit of mortgage debt given borrowers' choices:

$$\mathcal{X}_t = \tilde{F}_t^\eta (x_t + (1 - \delta_m) q_t^m) + E_i \left[F_\eta(\epsilon_t^i, \mathbb{1}_t^i) \frac{h_{t-1}^i}{M_{t-1}^I} p_t((1 - \zeta) - \delta_h) \right]$$

Since default decisions do not depend on wealth levels and since housing choices $h_t^i = \hat{h}_t w_t^i$ are proportional to borrower wealth for all borrowers,

$$E_i \left[F_\eta(\epsilon_t^i, \mathbb{1}_t^i) h_{t-1}^i \right] = E_i \left[F_\eta(\epsilon_t^i, \mathbb{1}_t^i) \right] E_i \left[h_{t-1}^i \right] = F_t^\eta H_{t-1}^B = F_t^\eta \alpha_h$$

As a result, the mortgage payoff can be written:

$$\mathcal{X}_t = \tilde{F}_t^\eta (x_t + (1 - \delta_m) q_t^m) + F_t^\eta \frac{\alpha_h}{M_t^I} p_t((1 - \zeta) - \delta_h)$$

III.2.2 First Order Conditions

Solve the budget constraint (IA.22) for Div_t^I and plug into the intermediary problem (IA.21). Then, differentiating with respect to the remaining control variables M_t^I and D_t^I yields the following FOCs:

Mortgages The FOC for mortgages M_t^I is given by:

$$q_t^m = \mu_t^L \xi \left(\kappa \bar{q}^m + (1-\kappa) q_t^m\right) + E_t\left[\mathcal{M}_{t+1}^S \mathcal{X}_{t+1}\right]$$

where μ_t^L is the Lagrange multiplier on the leverage constraint (IA.24).

Deposits The FOC for deposits D_t^I is given by:

$$\frac{1}{1+r_t^d} = \mu_t^L + \mathrm{E}_t\left[\mathcal{M}_{t+1}^S\right]$$

III.3 Savers

Likewise, we write and solve the representative saver's problem using aggregate quantities. For symmetry, we define saver wealth inclusive of their Lucas Tree shares and housing, even though neither is tradeable by them.

$$V^S(W_t^S, \mathcal{Z}_t) = \max_{C_t^S, E_t} u(C_t^S, H_t^S) + \beta E_t[V^S(W_{t+1}^S, \mathcal{Z}_{t+1})]$$

subject to

$$W_t^S = p_t^s S_t^S + p_t^h H_t^S + E_t p_t^e + C_t^S \tag{IA.25}$$

$$W_{t+1}^S = S_t^S(p_{t+1}^s + Y_t) + H_t^S(p_{t+1}^h - \delta_h) + E_t(p_{t+1}^e + \mathrm{Div}_{t+1}^I) + \mathcal{R}_{t+1}^S \tag{IA.26}$$

where $\mathcal{R}_{t+1}$ are (1) borrower costs of default, parametrized by λ, (2) banks' foreclosure costs, parametrized by ζ, and (3) banks' intermediation costs, parametrized by ν, rebated lump-sum:

$$\mathcal{R}_t^S = F_t^\eta\left(\lambda p_t^s \alpha + \zeta p_t^h \alpha_h\right) + \nu \mathcal{X}_t M_t^I$$

The first order condition for bank equity E_t is

$$p_t^e = \mathrm{E}_t \left[\beta \left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\gamma} (\mathrm{Div}_{t+1} + p_{t+1}^e) \right]$$

which implies the saver's stochastic discount factor $\mathcal{M}_{t+1}^S = \beta \left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\gamma}$.

Normalize the supply of bank shares E_t to 1. Then, iterating on both the bank's value function and the saver's FOC for bank equity, we get that $V_t^I = \mathrm{Div}_t + p_t^e$.

III.4 Resource Constraint

In this section, we verify that aggregate consumption and housing investment are financed by the aggregate output of Lucas trees and by changes in the net deposit position of the economy.

Define aggregate borrower consumption in terms of conditional expectations of individual consumption:

$$\begin{aligned} C_t^B &= W_{t-1}^B \mathrm{E}_i \left[F_\eta(\eta^{*,i}) \hat{c}_t^{i,nd} + \tilde{F}_\eta(\eta^{*,i}) \hat{c}_t^{i,d} \right] \\ &= W_t^B \left(F_t^\eta \mathrm{E}_i \left[\hat{c}_t^{i,d} | \eta^i \leq \eta^{*,i} \right] + \tilde{F}_t^\eta \mathrm{E}_i \left[\hat{c}_t^{i,nd} | \eta^i > \eta^{*,i} \right] \right) \end{aligned}$$

From the consumption stage budget constraints:

$$\begin{aligned} \mathrm{E}_i \left[\hat{c}_t^{i,d} | \eta^i \leq \eta^{*,i} \right] &= \hat{s}_{t-1} \mathrm{E}_i \left[Y_t + \epsilon_t^i | \eta^i \leq \eta^{*,i} \right] + \hat{d}_{t-1} \\ \mathrm{E}_i \left[\hat{c}_t^{i,nd} | \eta^i > \eta^{*,i} \right] &= \hat{s}_{t-1} \mathrm{E}_i \left[Y_t + \epsilon_t^i | \eta^i > \eta^{*,i} \right] + \hat{d}_{t-1} - \hat{m}_{t-1} x_t - \delta_h \hat{h}_{t-1} - \mathrm{E}_i \left[\hat{a}_t^i | \eta^i > \eta^{*,i} \right] \end{aligned}$$

From the no-default branch wealth evolution equation, we get that intra-period savings $\hat{a}_t^i = \hat{w}_t^{i,nd} - p_t^h \hat{h}_{t-1} - p_t^s \hat{s}_{t-1} + (1 - \delta_m) q_t^m \hat{m}_{t-1}$. Furthermore, observe that

$$\tilde{F}_t^\eta \mathrm{E}_i \left[Y_t + \epsilon_t^i | \eta^i > \eta^{*,i} \right] + F_t^\eta \mathrm{E}_i \left[Y_t + \epsilon_t^i | \eta^i \leq \eta^{*,i} \right] = Y_t + \mathrm{E}_i[\epsilon_t^i] = Y_t$$

Define aggregate borrower deposits $D_t^B = W_t^B \hat{d}_t$. Use market-clearing in Lucas trees and housing to write $W_t^B \hat{s}_t = \alpha$ and $W_t^B \hat{h}_t = \alpha_h$. Use market-clearing in mortgages to write

$W_t^B \hat{m}_t = M_t^I$. Assembling,

$$
\begin{aligned}
C^B &= \alpha Y_t + D_{t-1}^B \\
&\quad + \tilde{F}_t^\eta \left[\alpha_h (p_h - \delta_h) + \alpha p_t^s - M_{t-1}^I \left(x_t + (1 - \delta_m) q_t^m \right) - W_{t-1}^B \mathrm{E}_\tau \left[\hat{w}_t^{i,nd} | \eta > \eta^{*,i} \right] \right]
\end{aligned}
$$

Recall that $W_t^B = W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^i \right]$. We can break up the expectation as follows:

$$
\mathrm{E}_i \left[\hat{w}_t^i \right] = \tilde{F}_t^\eta \mathrm{E}_i \left[\hat{w}_t^{i,nd} | \eta > \eta^{*,i} \right] + F_t^\eta \mathrm{E}_i \left[\hat{w}_t^{i,d} | \eta \leq \eta^{*,i} \right]
$$

Solving for the aggregate wealth of non-defaulters $\tilde{F}_t^\eta W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,nd} | > \leq \eta^{*,i} \right]$,

$$
\tilde{F}_t^\eta W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,nd} | \eta > \eta^{*,i} \right] = W_t^B - W_{t-1}^B F_t^\eta \mathrm{E}_i \left[\hat{w}_t^{i,d} | \eta \leq \eta^{*,i} \right]
$$

Use the default-branch wealth evolution equation and market clearing in Lucas trees to substitute

$$
W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,d} | \eta \leq \eta^{*,i} \right] = (1 - \lambda) p_t^s \alpha
$$

Multiply the trading stage budget constraint by W_t^B and plug in market-clearing conditions to get

$$
W_t^B = \frac{D_t^B}{1 + r_t^d} - q_t^m M_t^I + p_t^h \alpha_h + p_t^s \alpha
$$

Combining,

$$
\tilde{F}_t^\eta W_{t-1}^B \mathrm{E}_i \left[\hat{w}_t^{i,nd} | \eta > \eta^{*,i} \right] = \frac{D_t^B}{1 + r_t^d} + q_t^m M_t^I + p_t^h \alpha_h + p_t^s \alpha - F_t^\eta (1 - \lambda) p_t^s \alpha
$$

Plugging back into the expression for C^B,

$$
\begin{aligned}
C^B &= \alpha Y_t + D^B_{t-1} - \frac{D^B_t}{1+r^d_t} + q^m_t M^I_t - p^h_t \alpha_h - p^s_t \alpha + F^\eta_t (1-\lambda) p^s_t \alpha \\
&\quad + \tilde{F}^\eta_t \left[\alpha_h (p_h - \delta_h) + \alpha p^s_t - M^I_{t-1} \left(x_t + (1-\delta_m) q^m_t \right) \right] \\
&= \alpha Y_t + D^B_{t-1} - \frac{D^B_t}{1+r^d_t} + q^m_t M^I_t - F^\eta_t (p_h \alpha_h + \lambda p^s_t \alpha) \\
&\quad - \tilde{F}^\eta_t \left[\delta_h \alpha_h + M^I_{t-1} \left(x_t + (1-\delta) q^m_t \right) \right]
\end{aligned}
$$

This expression admits an economic interpretation. Borrowers earn income from their Lucas trees αY_t and deposits D^B_{t-1}. Those repaying their mortgages – a fraction $\tilde{F}^\eta_t$ – expend resources on housing maintenance $\delta_h \alpha_h$ and mortgage payments $M^I_{t-1} \left(\mathrm{E}_\tau \left[x^i_t | \eta > \eta^{*,i} \right] + (1-\delta_m) q^m_t \right)$. Those who default – a fraction $\tilde{F}^\eta_t$ – lose the value of their houses $p_h \alpha_h$ and a fraction λ of the value of their Lucas trees $p^s_t \alpha$. In the trading stage, they take out new mortgages $q^m_t M^I_t$ and make new deposits $\frac{D^B_t}{1+r^d_t}$.

Next, consider saver consumption. From the budget constraint and wealth evolution equation of savers,

$$
C^S_t = S^S_{t-1}(p^s_t + Y_t) + H^S_{t-1}(p^h_t - \delta_h) + E_{t-1}(p^e_t + \mathrm{Div}^I_t) + \mathcal{R}^S_t - p^s_t S^S_t - p^h_t H^S_t - E_t p^e_t
$$

Plug in market clearing conditions $E_t = 1$, $S^S_t = 1 - \alpha$, $H^S_t = 1 - \alpha_h$, to get

$$
C^S_t = (1-\alpha) Y_t - (1-\alpha_h)\delta_h + \mathrm{Div}^I_t + \mathcal{R}^S_t
$$

From the budget constraint for banks,

$$
\mathrm{Div}^I_t = (1-\nu) \mathcal{X}_t M^I_{t-1} + D^I_{t-1} - \frac{D^I_t}{1+r^d_t} - q^m_t M^I_t
$$

Plugging for Div^I_t and $\mathcal{R}_t$ and collecting terms,

$$
C^S_t = (1-\alpha) Y_t - (1-\alpha_h)\delta_h + \mathcal{X}_t M^I_{t-1} + D^I_{t-1} - \frac{D^I_t}{1+r^p_t} - q^m_t M^I_t + F^\eta_t \left(\lambda p^s_t \alpha + \zeta p^h_t \alpha_h \right)
$$

Next, subtitute the definition of $\mathcal{X}_t$:

$$C_t^S = (1-\alpha)Y_t + D_{t-1}^I - \frac{D_t^I}{1+r_t^d} - q_t^m M_t^I + F_t^\eta \lambda p_t^s \alpha$$
$$- (1-\alpha_h)\delta_h + \tilde{F}_t^\eta (x_t + (1-\delta_m)q_t^m)M_{t-1}^I + F_t^\eta p_t \alpha_h (1-\delta_h)$$

Define aggregate deposits as $D_t = D_t^B + D_t^I$. Then, adding C_t^B and C_t^S and collecting terms, we get the resource constraint:

$$\underbrace{C_t^B + C_t^S}_{\text{Aggregate Consumption}} + \underbrace{\delta}_{\text{Housing Investment}} = \underbrace{Y_t}_{\text{Output}} + \underbrace{D_{t-1} - \frac{D_t}{1+r_t^d}}_{\Delta\text{Net Foreign Assets}}$$

III.5 Risk Sharing Measures

III.5.1 Complete Markets Benchmark

An unconstrained social planner chooses allocations for each agent that are proportional to the weight that the planner puts on the utility of that agent.

Define the social welfare problem:

$$\max_{\{\{(c_t^i, h_{t-1}^i)\}_{i=0}^1\}_{t=0}^\infty} E_0 \left[\int_{i=0}^1 \lambda_i \sum_{t=1}^\infty \beta^t \frac{\left((c_t^i)^{1-\theta}(h_t^i)^\theta\right)^{1-\gamma} - 1}{1-\gamma} di \right]$$

such that the resource constraints for each good, in each period and each state of the world are satisfied:

$$\int_0^1 c_t^i = Y_t \quad \forall t, s^t$$
$$\int_0^1 h_{t-1}^i = \bar{H} \quad \forall t, s^t$$

where every variable x_t is implicitly a function of the random variable s^t, denoting the history of the economy up to time t.

Assign μ_t and ν_t as Lagrange multipliers to each of the constraints at time t, history s^t, respectively. Use $\pi(s^t)$ to denote the density of the unconditional history distribution at a given s^t. The first order condition for consumption for agent i at time t are:

$$\lambda_i \beta^t \pi(s^t)\left((c_t^i)^{1-\theta}(h_{t-1}^i)^{\theta}\right)^{-\gamma}(1-\theta)(c_t^i)^{-\theta}(h_{t-1}^i)^{\theta} = \mu_t$$

The first order condition for housing for agent i at time $t-1$ are:

$$\lambda_i \beta^t \pi(s^t)\left((c_t^i)^{1-\theta}(h_{t-1}^i)^{\theta}\right)^{-\gamma}\theta(c_t^i)^{1-\theta}(h_{t-1}^i)^{\theta-1} = \nu_{t-1}$$

Dividing them by each other, we get the optimal MRS between consumption and housing for a given state of the world, which is the same for all households:

$$\frac{c_t^i}{h_{t-1}^i} = \frac{1-\theta}{\theta}\frac{\mu_t}{\nu_{t-1}}$$

Substitute for housing in the consumption FOC:

$$h_{t-1}^i = \frac{\theta}{1-\theta}\frac{\mu_t}{\nu_{t-1}}c_t^i$$

$$\lambda_i \beta^t \pi^(s^t)\left((c_t^i)\left[\frac{\theta}{1-\theta}\frac{\mu_t}{\nu_{t-1}}\right]^{\theta}\right)^{-\gamma}(1-\theta)^{1-\theta}\theta^{\theta} = \mu_t^{1-\theta}\nu_{t-1}^{\theta}$$

Dividing the consumption FOCs for agents i and j at time t by each other, we get:

$$\frac{\lambda_i}{\lambda_j}\left(\frac{c_t^i}{c_t^j}\right)^{-\gamma} = 1$$

which means that the ratio of consumptions is constant over time and states of the world at:

$$\frac{c_t^i}{c_t^j} = \left(\frac{\lambda_i}{\lambda_j}\right)^{-1/\gamma}$$

Rewrite as:

$$c_t^i = \left(\frac{\lambda_i}{\lambda_j}\right)^{-1/\gamma} c_t^j$$

Integrate both sides with respect to i to get aggregate time t consumption:

$$C_t \equiv \int_0^1 c_t^i di = \lambda_j^{\frac{1}{\gamma}} c_t^j \int_0^1 \lambda_i^{-\frac{1}{\gamma}} di$$

Which implies that a given household's consumption c_t^j is a constant fraction of aggregate consumption C_t:

$$c_t^j = \frac{\lambda_j^{-\frac{1}{\gamma}}}{\int_0^1 \lambda_i^{-\frac{1}{\gamma}} di} C_t$$

The same argument applies to housing, i.e. it can be shown that the planner's optimal allocation of housing to agent j

$$h_{t-1}^j = \frac{\lambda_j^{-\frac{1}{\gamma}}}{\int_0^1 \lambda_i^{-\frac{1}{\gamma}} di} \bar{H}$$

is constant over time.

Since complete markets implement the planner allocation, this means that in a frictionless economy the volatility of the ratio of consumptions is zero. This likewise implies that each agent's consumption grows at the same rate. Formally, take the log:

$$\log c_t^i - \log c_t^j = -\frac{1}{\lambda} \left(\log \lambda_i - \log \lambda_j \right)$$

Let $\Delta \log c_t^i$ is defined as $\log c_t^i - \log c_{t-1}^i$. Then the log of the ratio of consumption growth rates is:

$$\begin{aligned}
\Delta \log c_t^i - \Delta \log c_t^j &\equiv (\log c_t^i - \log c_{t-1}^i) - (\log c_t^j - \log c_{t-1}^j) \\
&= (\log c_t^i - \log c_t^j) - (\log c_{t-1}^i - \log c_{t-1}^j)
\end{aligned}$$

Then in complete markets, it must be true that

$$\mathcal{R}_{ij} = \mathrm{Var}_0 \left[\Delta \log c_t^i - \Delta \log c_t^j \right] = 0$$

We refer to $\mathcal{R}_{ij}$ as a measure of "internal" risk sharing. In an incomplete markets economy, $\mathcal{R}_{ij} \geq 0$ and $\mathcal{R}_{ij}$ serves as a measure of risk sharing between households, with lower values denoting better risk sharing.

III.5.2 Complete Markets Open Economy

The open economy version of the complete markets model is similar to the closed economy version, except that the planner can now trade a risk-free bond with the rest of the world. The planner's problem is:

$$\max_{\{\{(c_t^i, h_{t-1}^i)\}_{i=0}^1, b_t\}_{t=0}^\infty} \mathrm{E}_0 \left[\int_{i=0}^1 \lambda_i \sum_{t=1}^\infty \beta^t \frac{\left((c_t^i)^{1-\theta}(h_{t-1}^i)^\theta\right)^{1-\gamma} - 1}{1-\gamma} di \right]$$

such that

$$\int_0^1 c_t^i + \frac{b_t}{1+r_t^d} = Y_t + b_{t-1} \quad \forall t, s^t$$
$$\int_0^1 h_{t-1}^i = \bar{H} \quad \forall t, s^t$$

The derivations above still hold. But now there is an additional choice variable of the planner. Bonds $b_t(s^t)$ show up in the resource constraint for t, s^t and in the resource constraints for all t, s^{t+1} that are reachable from s^t. Denote this set of possible states as $s_{t+1}|s^t$ and the . Then the additional first order condition for the bond is:

$$\frac{\mu_t}{1+r_t^d}\pi(s^t) = \int_{s_{t+1}|s^t} \pi(s^{t+1})\mu_{t+1}$$

Rearranging,

$$1 = (1+r_t^d) \int_{s_{t+1}|s^t} \pi(s_{t+1}|s^t)\frac{\mu_{t+1}}{\mu_t} = (1+r_t^d)\mathrm{E}_t\left[\frac{\mu_{t+1}}{\mu_t}\right]$$

where $\pi(s_{t+1}|s^t)$ denotes the conditional density of s_{t+1} given s^t, and where the second equality stems from the definition of a conditional expectation with $E_t[\cdot]$ denoting $\mathrm{E}[\cdot|s^t]$.

Plug in the FOC for consumption for the multipliers:

$$1 = (1 + r_t^d) E_t \left[\left(\frac{c_{t+1}^i}{c_t^i} \right)^{-\gamma(1-\theta)-\theta} \left(\frac{h_t^i}{h_{t-1}^i} \right)^{\theta(1-\gamma)} \right]$$

Recall that for any agent, the optimal housing allocation is constant and the growth rate of consumption is equal to the aggregate consumption growth rate. Then the above equation simplifies to:

$$1 = (1 + r_t^d) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma(1-\theta)-\theta} \right]$$

The problem admits aggregation, i.e. the planner's optimal choice of bonds is independent of the resource allocation problem.

Take logs

$$0 = \log(1 + r_t^d) + \log E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma(1-\theta)-\theta} \right]$$

and define

$$\mathcal{R}_{agg} = \mathrm{Var}_0 \left[\log(1 + r_t^d) + \log E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma(1-\theta)-\theta} \right] \right] \geq 0$$

as the "external" risk sharing measure. In complete markets, $\mathcal{R}_{agg} = 0$, while in incomplete markets larger values of $\mathcal{R}_{agg}$ indicate worse risk sharing between households in the economy and the rest of the world.

III.5.3 Internal Risk Sharing in our Model

In our model, there are two kinds of households: borrowers with consumption denoted by c_t^i and savers, with consumption denoted by c_t^S and identical across all savers. Let $C_t^B = \int_0^\ell c_t^i$ denote aggregate borrower consumption and $C_t^S = (1 - \ell) c_t^s$ denote aggregate saver consumption.

Borrowers are unconditionally identical, meaning internal risk sharing is summarized fully by two risk-sharing measures $\mathcal{R}_{iB}$ and $\mathcal{R}_{BS}$, where $\mathcal{R}_{iB}$ is the variance of the ratio of consumption growth rates between borrower i and the aggregate borrower, and $\mathcal{R}_{BS}$ is the variance of the ratio of aggregate consumption growth rates between borrowers and savers.

Recall, we can write borrower i's consumption at time t, c_t^i, as the product of borrower consumption per unit of wealth $\hat{c}_t^i$ and borrower wealth at time $t-1$, w_{t-1}^i. Consumption per unit of wealth only depends on the identity of the borrower i through the realizations of iid shocks to $\mathcal{S}_t^i = (\epsilon_t^i, \tau_t^i, \eta_t^i)$.

Write the log growth rate of borrower i's consumption as:

$$\Delta \log c_t^i = \log \hat{c}_t(\mathcal{S}_t^i) - \log \hat{c}_{t-1}(\mathcal{S}_{t-1}^i) + \log \hat{w}_{t-1}(\mathcal{S}_{t-1}^i)$$

where $\hat{w}_{t-1}(\mathcal{S}_{t-1}^i)$ represents the growth rate in wealth $\Delta \log w_{t-1}^i$, which also depends on the identity of the borrower i only through the realiations of iid shocks.

The definition of $\mathcal{R}_{iB}$ is $\mathrm{Var}_0[\Delta \log c_t^i - \Delta \log C_t^B]$. Using the law of total variance,

$$\mathcal{R}_{iB} = \mathrm{Var}_0 \left[\mathrm{E}_t \left[\Delta \log c_t^i - \Delta \log C_t^B \right] \right] + \mathrm{E}_0 \left[\mathrm{Var}_t \left[\Delta \log c_t^i - \Delta \log C_t^B \right] \right]$$

where the conditional moments Var_t and E_t are taken cross-sectionally with respect to realizations of idiosyncratic shocks. Simplifying,

$$\mathcal{R}_{iB} = \mathrm{Var}_0 \left[\mathrm{E}_t \left[\Delta \log c_t^i \right] - \Delta \log C_t^B \right] + \mathrm{E}_0 \left[\mathrm{Var}_t \left[\Delta \log c_t^i \right] \right]$$

Finally, $\mathcal{R}_{BS}$ is defined as $\mathrm{Var}_0[\Delta \log C_t^B - \Delta \log C_t^S]$.

III.6 Managing Bank Equity Duration Through Leverage

In our model, banks invest only in mortgages, so the duration of bank assets is determined exogenously by the structure of mortgage contracts. However, the duration of bank equity is endogenous and can be managed by banks through their choice of leverage.

Here, we illustrate the intuition behind this in a simple partial equilibrium setting. Let A denote assets, L denote liabilities (deposits), and E denote equity. Let D_X for $X \in \{A, L, E\}$ denote duration: $D_X \equiv -\frac{\partial \log X}{\partial r}$ and let $D_X^{\$}$ denote "dollar duration" $D_X^{\$} \equiv -\frac{\partial X}{\partial r}$, which means that $D^\$_X = XD_X$.[42]

From the standard accounting identity $E = A - L$, we get that $ED_E = D^\$_E = AD_A - LD_L$

Define leverage as liabilities over asserts $\lambda = L/A$. Then $L = \lambda A$ and $E = (1-\lambda)A$. We can express the equity dollar duration as $(1-\lambda)AD_E = AD_A - \lambda AD_L$. Capital requirements ensure that $\lambda \in [0, 1)$, so equity duration becomes:

$$D_E = \frac{D_A - \lambda D_L}{1 - \lambda} \tag{IA.27}$$

Given durations of assets and liabilities, leverage affects the duration of equity in two ways: (1) it governs how much of the asset duration is hedged/immunized by liabilities duration, and (2) it translates dollar gains/losses into percent gains/losses, with higher leverage meaning more amplification.

Consider a bank whose assets consist entirely of mortgages $A = M$, and whose deposit liabilities are sticky, so D_L is large. If $D_M \leq D_L$, then it is easy to see that a bank can achieve any target $D^*_E \leq D_M$ by choosing an appropiate $\lambda^* \in [0, 1)$.

Consider an exogenous change to mortgage structure that lowers $D'_M < D_M$, e.g., a shortening of the fixation length, as in the main text. This mechanically lowers the duration of assets, and, all else equal, lowers equity duration D_E. To keep $D_E = D^*_E$, banks can either (1) rebalance their asset portfolio towards a longer-duration asset to restore original asset duration, or (2) change their leverage to restore original D^*_E even at a lower level of $D'_M = D'_A$.

As long as $D^*_E \leq D_M \leq D_L$ with at least one of the inequalities being strict, banks can achieve their target equity duration by adjusting leverage:[43]

$$\lambda^*(D_A) = \frac{D^*_E - D_A}{D^*_E - D_L}$$

With a fixation length short enough, mortgage duration D_M becomes negative. At that point, banks can no longer exactly achieve their target D^*_E by adjusting leverage alone. (IA.27) shows

[42] $D_X = -\frac{\partial \log X}{\partial r} = -\frac{\partial \log X}{\partial X}\frac{\partial X}{\partial r} = \frac{1}{X} D^\$_X$

[43] Alternatively, $D^*_E \geq D_A \geq D_L$ would also work, but this case is less relevant, since here banks would use leverage to increase their exposure to interest rate risk rather than mitigate it.

that when $D_A < D_L$, $D_E < 0$ for any admissable λ.

But banks can raise their equity duration back towards target by de-levering because:

$$\frac{\partial D_E}{\partial \lambda} = \frac{D_A - D_L}{(1-\lambda)^2} < 0 \text{ if } D_A < D_L$$

In sum, in the case of moderate positive mortgage duration (FRM), banks can use leverage to exactly achieve target equity duration in lieu of buying other, longer-duration, assets. In other cases, they can't use leverage as effectively as asset portfolio choice, but they can still actively manage equity duration through leverage.

IV Zero Duration Benchmark

Our starting point for measuring financial stability is the volatility of intermediary net worth: $\mathbb{V}[\log W_t]$. In a world where $\log W_t$ only depends linearly on current interest rates r_t, we have:

$$\log W_t = \alpha - \delta r_t$$

As a result, we can interpret δ as the duration of intermediary net worth: $-\frac{\mathrm{d}\log W_t}{\mathrm{d}r_t} = \delta$. δ measures the percent decline in net worth for a 1 percentage point increase in rates. Minimizing $\mathbb{V}[\log W]$ is achieved when $\delta^* = 0$, i.e. in a "zero duration" financial system, the volatility-minimizing mortgage fixation length would match the duration of deposits.[44]

However, $\log W_t$ may further depend on other state variables represented by x_t, which yields:

$$\log W_t = \alpha - \delta r_t + \gamma x_t$$

The variance of $\log W_t$ is:

$$\mathbb{V}[\log W_t] = \delta^2 \mathbb{V}[r_t] + \gamma^2 \mathbb{V}[x_t] + \delta\gamma \mathrm{Cov}[r_t, x_t] \tag{IA.28}$$

[44]This duration-matching strategy to minimize the effect interest rate changes on portfolio values is also referred to as "immunization" by practitioners.

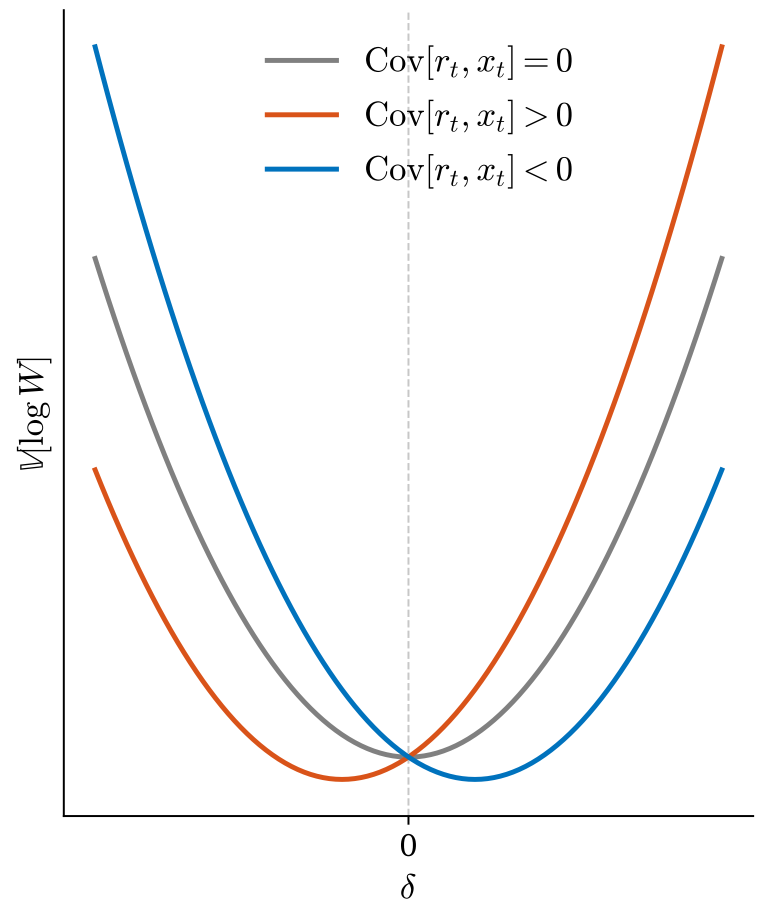
To find the new volatility-minimizing duration δ^{**}, we can take the first-order condition with respect to δ to obtain:

$$\delta^{**} = -\frac{\gamma}{2} \frac{\text{Cov}\left[r_t, x_t\right]}{\mathbb{V}\left[r_t\right]}$$

As a result, the volatility-minimizing duration is not zero, but instead also depends on $\text{Cov}\left[r_t, x_t\right]$. For $\text{Cov}\left[r_t, x_t\right] > 0$, the volatility-minimizing duration is smaller than zero, and for $\text{Cov}\left[r_t, x_t\right] < 0$, it is greater than zero. Equation IA.28 further shows that net worth variance is quadratic in duration, meaning duration is increasing in the absolute distance to the volatility-minimizing duration δ^{**}. Figure IA.13 illustrates this intuition for different values of $\text{Cov}\left[r_t, x_t\right]$.

"State variables" that may affect intermediary net worth beyond interest rates but that may be correlated with rates are those capturing endogenous default behavior by households as well as equilibrium pricing of mortgage rates, both of which may differ across mortgage structures.

Figure IA.13: Illustration of Net Worth Volatility and Duration



Notes: This figure plots the relationship between $\mathrm{V}[\log W_t]$ and duration δ from Equation IA.28, for fixed values of $\mathbb{V}[r_t]$, γ, and $\mathbb{V}[x_t]$, for $\mathrm{Cov}[r_t, x_t] = 0$, $\mathrm{Cov}[r_t, x_t] > 0$, and $\mathrm{Cov}[r_t, x_t] < 0$.

V Calibration Details

V.1 Estimation of Income and Interest Rate Process

We estimate the following VAR(1) for $y_t = \log Y_t$ and $\hat{r}_t$, where y_t is the cyclical component of log Real GDP and $\hat{r}_t$ is the demeaned real interest rate, using annual data from 1987 to 2024. Cyclical component of GDP is extracted using the one-sided Hodrick-Prescott filter. Real rates are 1-year real rates from the Federal Reserve Bank of Cleveland.

$$\begin{bmatrix} y_t \\ \hat{r}_t \end{bmatrix} = \begin{bmatrix} \phi_{yy} & \phi_{yr} \\ \phi_{ry} & \phi_{rr} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{y,t} \\ \epsilon_{r,t} \end{bmatrix},$$

where

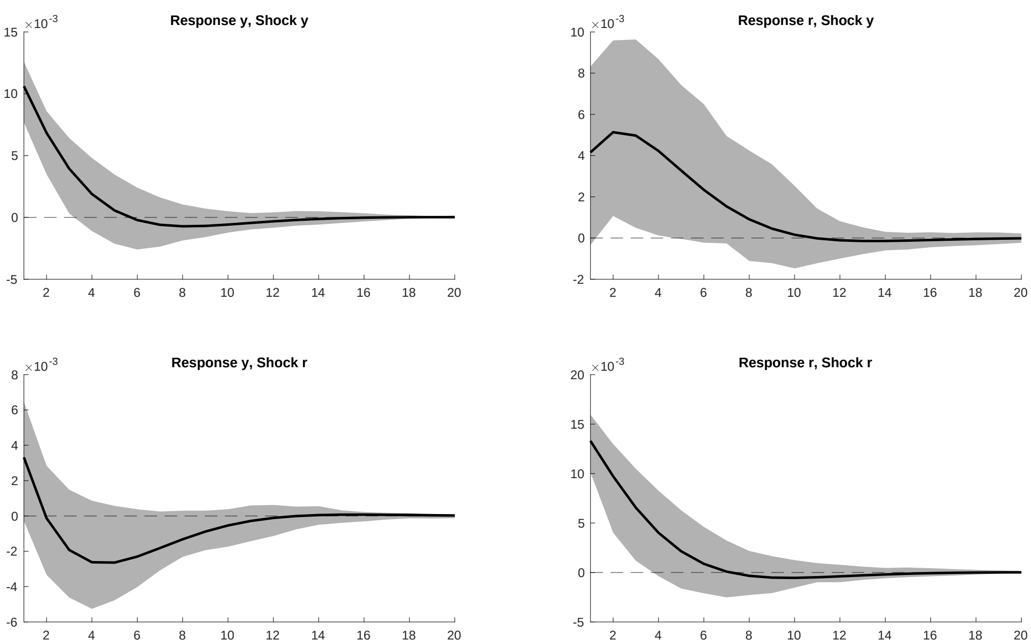
$$\begin{bmatrix} \epsilon_{y,t} \\ \epsilon_{r,t} \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}_2, \begin{bmatrix} \sigma_y^2 & \rho_{yr}\sigma_y\sigma_r \\ \rho_{yr}\sigma_y\sigma_r & \sigma_r^2 \end{bmatrix}\right)$$

The estimated values are $\phi_{yy} = 0.718$, $\phi_{yr} = -0.189$, $\phi_{ry} = 0.219$, $\phi_{rr} = 0.677$, $\sigma_y = 0.011$, $\sigma_r = 0.013$, and $\rho_{yr} = 0.313$.

These estimates yield generalized (not orthogonalized) impulse responses shown in Figure IA.14. Positive correlation yields positive comovement of the two series on impact, with the effect being more persistent in rates for innovations to income rather than vice versa.

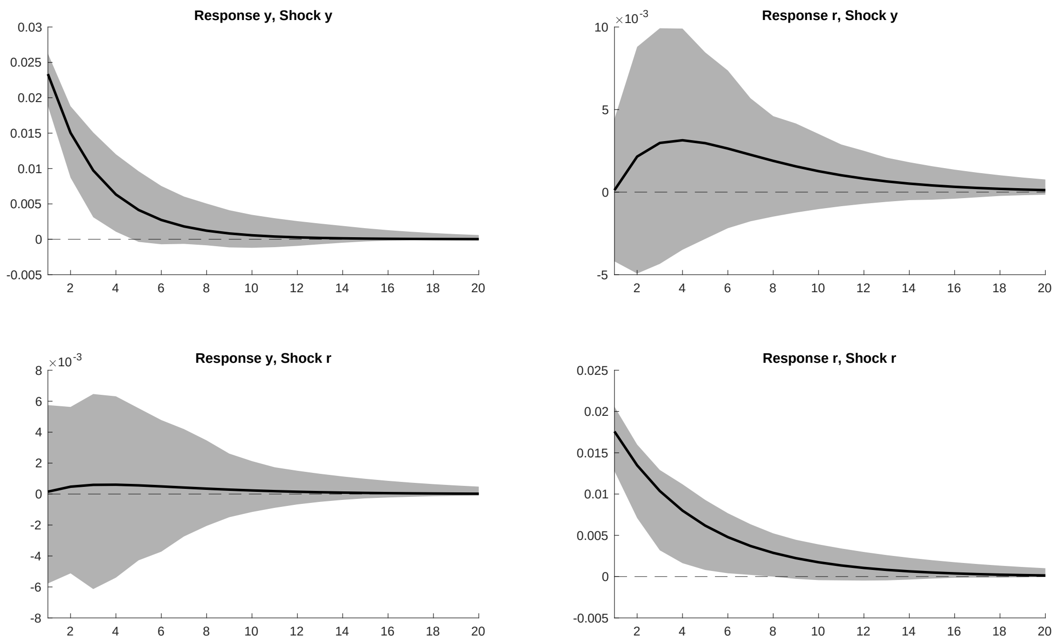
We also estimate the VAR(1) process for a longer sample 1962-2024. Because real rates from the Cleveland Fed are not available prior to 1982, we instead construct real rates as nominal constant maturity 1-year rates less realized inflation over the year. In the longer sample, the innovations are uncorrelated ($\rho_{yr} = 0.0066$) and impulse responses of one innovation on the other variable are not significant (see Figure IA.15).

Figure IA.14: Impulse Responses of Income and Interest Rates: 1987-2024



Notes: Impulse responses of income and interest rates to innovations in each series, given the estimated VAR(1) process. Shaded regions represent bootstrapped 95% confidence intervals. Innovations are not orthogonalized.

Figure IA.15: Impulse Responses of Income and Interest Rates: 1962-2024



Notes: Impulse responses of income and interest rates to innovations in each series, given the estimated VAR(1) process. Shaded regions represent bootstrapped 95% confidence intervals. Innovations are not orthogonalized.