Bank Opacity and Deposit Rates

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Abstract

We propose a novel mechanism explaining why bank portfolios are opaque: banks maintain opacity to boost profits at the expense of depositors. We show that banks' choice of opacity is guided by their desire to raise cheap long term funding, while managing the trade-off between insolvency and illiquidity. Opacity enables banks to raise cheaper deposits. However, because opacity forces depositors to rely on noisy signals about solvency and to behave more cautiously, it can also raise the probability of early withdrawals when interest rates are high. We find that, in high-interest-rate environments, banks strategically tilt their portfolios toward excessive opacity to lower deposit rates, at the cost of more frequent early failure.

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1 Introduction

Banks are opaque by design, and opacity remains a deliberate and enduring feature of modern banking. Investors and depositors often have limited insight into the composition and riskiness of a bank's assets—especially during times of stress. There are two opposing views on the role of opacity in the financial sector. One perspective views opacity as a source of fragility—by increasing insolvency risk which fuels runs, panic, and systemic crises, while another highlights its role in mediating illiquidity risk by enabling the creation of safe, information-insensitive debt (see Dang et al. (2017)). The tension over whether opacity is a flaw or a feature remains unresolved. Moreover, a key question arises: Given that in reality banks face both insolvency and illiquidity risk, under what conditions would a bank strategically choose to be opaque?

We approach the question by developing a model in which a bank must manage a trade-off between two distinct sources of failure: illiquidity and insolvency. The bank raises short-term funding from depositors to finance long-term risky projects. Because depositors have the option to withdraw early, the bank faces the possibility of both interim illiquidity and ex-post insolvency. When choosing her portfolio of projects, the bank fundamentally cares about two objectives: avoiding failure and minimizing the promised deposit rates to maximize returns conditional on not failing. Under a deposit contract with interim withdrawal, the risk of failure has two components: the risk of illiquidity, when depositors withdraw early, and the risk of insolvency at maturity, when the bank lacks sufficient resources as a result of high promised rates. If the bank can influence both the return distribution and the information available to depositors through her portfolio choices, a trade-off emerges. To raise cheaper deposits and reduce long-term funding costs, she may accept a higher risk of illiquidity. In this sense, opacity becomes a strategic tool—not to obscure information for its own sake, but to reduce the funding costs while optimally balancing short-term fragility against long-term gain.

Our model formalizes this trade-off. The bank chooses both how to allocate the funds she raises from depositors across different projects (portfolio allocation) and whether the composition of the portfolio is transparent, opaque, or fully unobservable to depositors. Depositors retain the option to withdraw early and receive a safe outside alternative. Under transparency, depositors observe the return of each project at an interim date; under opacity, they observe only the aggregate return of the portfolio; and under full unobservability, no information is revealed before maturity. The bank's choices jointly determine both the risk profile of her portfolio and the informativeness of the signals that depositors use to decide whether to withdraw.

To understand the underlying mechanism, it is useful to note that in both the transparent and unobservable benchmark cases, the bank faces no trade-off between illiquidity and insolvency because depositor information is fixed and independent of the bank's portfolio allocation. In the transparent case, depositors observe interim returns perfectly, eliminating uncertainty about the bank's solvency at the withdrawal date. The bank's objective is thus to minimize illiquidity risk and the associated return on deposits. She typically chooses a highly diversified portfolio that increases the likelihood of repayment and induces depositors to accept a lower return on deposits. However, when depositors outside option is very appealing, she shifts toward a more concentrated portfolio to capture upside gains—engaging in risk shifting. In the unobservable case, depositors receive no interim information, which makes their decision to withdraw perfectly predictable and eliminates illiquidity risk. As a result—much like in the transparent case—the bank is encouraged to diversify, since a more stable return distribution lowers insolvency risk and reduces the return on deposits. In both settings, portfolio allocation affects only the return distribution, not the informativeness of depositor signals. This stands in contrast to the opaque case, where the bank can strategically manage both the risk profile and the flow of information to depositors.

The trade-off between illiquidity and insolvency arises only when the bank's portfolio is opaque. Her ability to shape depositor beliefs becomes central to managing this tension. In the presence of opacity, the bank's portfolio allocation plays a dual role: it determines both the return distribution and the informativeness of depositor signals. This dual function generates conflicting incentives. Greater diversification improves solvency and lowers the long-term return on deposits, but it also blurs the information depositors use, increasing the likelihood of early withdrawals. Conversely, a more concentrated portfolio sharpens depositor signals and reduces illiquidity risk, but raises funding costs and insolvency risk. As a result, the bank chooses a portfolio allocation that balances—rather than minimizes—either risk.

The key result is that opacity gives the bank a lever to substitute between insolvency risk and illiquidity risk—enabling her to accept more frequent early withdrawals in exchange for a lower promised deposit rate and likelihood of default. This trade-off makes opacity an optimal and deliberate choice. In equilibrium, opacity emerges as both a source of fragility—fueling runs and early liquidation—and a mechanism for creating safer debt by reducing funding cost of the bank. This is consistent with the evidence offered by Chen et al. (2022a) who find that transparent US banks pay higher deposit rates. In particular, we find that in high interest rate environments, opaque banks pay a lower deposit rate compared to transparent ones. Put differently, our model suggests that in high interest rate environments, bank strategically manipulate their portfolio towards being excessively opaque in order or minimize the transmission of monetary policy.

The model has implications about how the level of government rates disciplines banks' information choices. In the context of government interest rates, the key insight of the model is that banks choose their degree of opacity to minimize the transmission of monetary policy to their depositors. When safe outside option is attractive—i.e., when government rates are high—depositors become less tolerant of opacity, forcing banks to be transparent and offer higher returns to retain funding. When government rates are low, banks can afford to be fully opaque, as depositors have little incentive to withdraw. It is only at intermediate government rates that the bank optimally adopts partial opacity, balancing illiquidity and insolvency risk through portfolio allocation. This implies that changes in the broader interest rate environment can directly shape the nature of financial fragility—not just through asset valuations, but by altering banks' incentives to be transparent or opaque. In this way, the model provides a micro-foundation for why fragility, opacity, and maturity mismatch tend to co-move with interest rate cycles.

Related Literature. This paper contributes to a growing literature on the interplay between information disclosure, liquidity risk, and financial stability.[1]

It is well acknowledged that the banking system is opaque (see, e.g., Morgan (2002); Flannery et al. (2013)). The financial crisis of 2007–2008 emphasized the opacity of the financial system and prompted a line of research focused on the effects of financial transparency on systemic risk and the role of disclosure policies. Bouvard et al. (2015), Alvarez and Barlevy (2015), Goldstein and Leitner (2018), and Orlov et al. (2017) provide models that study the costs and benefits of disclosing bank-specific information. They show that increasing transparency is generally beneficial during financial crises but has ambiguous effects in normal economic times. These papers generally treat information disclosure as exogenous or policy-imposed, whereas we model it as a bank's endogenous choice.

Several studies highlight the role of information sensitivity in triggering bank runs and the endogenous dynamics of bank fragility. Goldstein and Pauzner (2005) and Morris and Shin (2003) show that, with a fixed information structure, depositor signals critically affect the probability and nature of runs. He and Manela (2016) model rumor-based runs, showing how endogenous information acquisition by depositors can trigger liquidity crises even at solvent institutions. Dang et al. (2017) emphasize that opacity can preserve the liquidity of short-term debt by preventing information-sensitive withdrawals, while Dang et al. (2015) argue that financial stability often relies on maintaining investors' ignorance. Our contribution is to focus on how banks strategically shape the information environment through opacity

[1] Our analysis builds on the tradition initiated by Diamond and Dybvig (1983), where banks provide liquidity insurance but are vulnerable to runs. While Diamond and Dybvig (1983) model runs driven by random liquidity shocks and coordination failures, we focus on information-driven fragility, where depositors withdraw based on noisy signals about bank asset performance. In addition, rather than treating opacity as a passive feature, we model banks as strategic designers of their information environment, balancing liquidity and solvency considerations.

choices that balance interim liquidity risk and ex-post solvency.

Empirical evidence supports the importance of opacity in banking outcomes. Most directly related to our mechanism, Chen et al. (2022a) documents that transparent US banks pay higher deposit rates. In particular, we find that in high interest rate environments, opaque banks pay a lower deposit rate compared to transparent ones. Chen et al. (2022b) show that greater transparency increases the sensitivity of uninsured deposit flows to banks' performance signals, amplifying liquidity risk. This underscores the practical relevance of modeling opacity choices and their implications for financial stability.

Our work also relates to broader studies on information provision and disclosure incentives in financial markets. Frenkel et al. (2020) and Malenko and Malenko (2019) analyze how information supplied to market participants shapes voluntary disclosure and governance outcomes. While these papers focus on corporate settings, they similarly demonstrate how the design of the information environment shapes agent behavior and systemic dynamics.

Building on this literature, our main contribution is to model opacity as a strategic decision by banks. We link portfolio transparency choices to the trade-off between interim liquidity and ex-post solvency risks. We show that banks optimally choose intermediate levels of opacity, and that this choice varies systematically with market conditions, particularly with depositors' outside options. This approach offers a new perspective on the origins of financial fragility, emphasizing opacity as an equilibrium outcome of banks' optimization behavior.

The rest of the paper is organized as follows. In Section 2 we introduce the model set-up. In Section 3 we introduce the full information and no information benchmarks to highlight the role of opacity. Our main results are presented in Section 4. Section 5 discusses the implications of our model. Section 6 concludes.

2 The Model

The focus of our analysis is to understand how banks use opacity in their portfolios. To this end, we introduce a framework where a bank chooses her portfolio of risky loans not only to optimize her risk exposure but, crucially, to influence the information available to depositors. Naturally, information is particularly important for depositors when they have the bargaining power to use it. For this reason, we develop a model in which the return on deposits is set to maximize depositors expected payoffs, while the bank's bargaining power is reflected in her asset portfolio choice. This approach ensures that both the depositors and the bank make positive profits, leading to interesting trade-offs.

2.1 The Environment

Consider an economy with three dates, $t = 0, 1, 2$, and two types of agents. First, a bank that can finance long-term risky projects but has no funding. Second, households who own c units of endowment in aggregate but do not have direct access to risky projects. All agents are risk-neutral.

To invest in projects, the bank must raise funds from households in the form of *demand deposits*. In exchange for borrowing funds from the depositors, the bank issues a long-term deposit contract which matures at date 2 and pays face value D at maturity. Throughout the paper, we refer to D as the return on deposits.

Risky Projects There is a universe of long-term risky projects which the bank can invest in. All the project have a common scale c. Each project i has scale returns R_i at date 2. Project returns are independent draws from a uniform probability distribution $G(\cdot)$, with support $[0, 1]$.

Projects are of two types: *transparent* (T) and *unobservable* (U). The return, R_i, of a transparent project is perfectly revealed to depositors at date 1. In contrast, depositors receive no information about the return of an unobservable project. Let s_i denote the type of project i with $s_i \in \{R_i, \varnothing\}$, where $\varnothing$ represents that there is no information about the project.

Bank Portfolio Choice The bank raises funds from depositors and invests in projects at date 0. We assume that the bank can invest in at most two projects, $(1, 2)$, to keep the analysis tractable. The bank makes two decisions when choosing her portfolio.

First, she chooses a *portfolio allocation* $(\phi, 1 - \phi)$, representing the fraction of the bank portfolio in projects 1 and 2, respectively, with $\phi \in [0, 1]$. Bank's portfolio allocation governs the return on the bank portfolio at date 2.

Second, she chooses a *portfolio composition*, $\mathbf{s} = (s_1, s_2)$, which denotes the project types in her portfolio. Bank's portfolio composition governs the interim information structure of depositors at date 1. If $s = (R1_1, R_2)$ we call the bank portfolio *transparent*, if $s = (R_1, \varnothing)$ we call it *opaque*, and if $s = (\varnothing, \varnothing)$ we call it *unobservable*.

Portfolio Return The return on bank portfolio at date 2 is given by

$$V(\phi) = \phi R_1 + (1 - \phi) R_2.$$

As such, the cumulative distribution of return on bank's portfolio at date 2, $H(.; \phi)$, is determined solely by the portfolio allocation, ϕ, and is independent of bank's portfolio composition.[2] Figure 1 depicts the cumulative distribution of return for three different values of portfolio allocation.

Note that the mean of the return distribution is $\bar{R} = \frac{1}{2}$, independent of the portfolio allocation, ϕ. However, the riskiness of the return distribution, as measured by second-order stochastic dominance, does depend on the portfolio allocation. The distribution of returns is symmetric around $\phi = \frac{1}{2}$: allocations at $\phi = 0$ or $\phi = 1$ correspond to fully undiversified portfolios and yield the riskiest return distribution. In contrast, $\phi = \frac{1}{2}$ corresponds to a

[2] In Section A we provide the details of the return distribution $H(z; \phi)$.

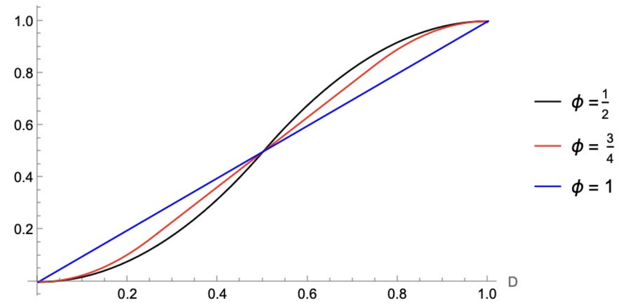

Figure 1: Cumulative Distribution of Bank's Portfolio Return $V(\phi)$

perfectly diversified portfolio, which minimizes return variance and generates the most concentrated distribution around the mean. Moving ϕ closer to $\frac{1}{2}$ makes the return distribution more concentrated and stochastically dominant in the second order. For this reason, we refer to portfolios with ϕ near $\frac{1}{2}$ as *well-diversified*.

Interim Information Structure of Depositors Depositors (might) receive information about the bank's portfolio return at date 1. In particular, the depositors observe a signal about any project in the bank portfolio that is transparent. For simplicity, assume that for each transparent project the interim information perfectly reveals the project return. This information is important as depositors face a withdrawal decision at date 1, which is governed by their information structure.[3]

Thus, bank's portfolio composition determines the informativeness of date 1 signal to bank depositors. When the portfolio is *transparent* (T), and $\mathbf{s} = (R_1, R_2)$, depositors receive perfect information about the bank's date-2 return already at date $t = 1$. When the portfolio is *unobservable* (U), and $\mathbf{s} = (\varnothing, \varnothing)$, depositors receive no information at date $t = 1$ about the bank's return at date 2. Finally, when the portfolio is opaque (O), depositors have partial information at date $t = 1$ about the bank's return at date 2. Without loss of generality, we let R_1 be the return of the transparent project, and R_2 the return of the unobservable project, so that $\mathbf{s} = (R_1, \varnothing)$.

[3]Note that as the bank does not make any decision at date 1, bank's interim information structure is irrelevant.

Bank's portfolio allocation ϕ is also relevant in determining the extent of depositors' interim information iff bank's portfolio composition is opaque.

Early Withdrawal Decision of Depositors After observing information about returns according to the portfolio composition $\mathbf{s}$, depositors decide whether to demand their deposits early at date 1, or wait to receive D at date 2. We represent a depositor's decision at date 1 through a function

$$\omega(\mathbf{s}) = \begin{cases} 1 & \text{if depositor withdraws,} \\ 0 & \text{if depositor continues.} \end{cases}$$

In our setup all depositors are identical and there are no coordination concerns. We focus on symmetric equilibria where if one depositor finds it optimal to withdraw early, then all depositors withdraw early.

If depositors choose to withdraw early, they receive a redemption value $r < E(R_i)$. We assume $c < r$, which is sufficient to ensure that the participation constraint of depositors in lending to the bank at date 0 is satisfied.

We provide two different interpretations for r. First, if the bank faces early withdrawals, she has to sell her portfolio of projects to second-best users at a fire sale prices r, which is transferred to depositors entirely.[4] A second interpretation is that if depositors withdraw early, the bank is forced to liquidate her portfolio of projects early at one-to-one return, and the liquidation value c is transferred to depositors. At this point, the depositors have access to an outside short-term riskless investment opportunity that returns $\frac{r}{c}$ per unit investment at date 2. Under this interpretation, r represents the interest rate paid on government bonds.

What is important is that, in both interpretations, the bank lacks sufficient resources to meet early withdrawals while continuing its investment, and thus she fails early. Put differently, depositors' willingness to accept r as a redemption value renders the bank *illiquid*

[4]Projects are worth less to second-best users because of the misallocation mechanism proposed by Shleifer and Vishny (1992) and Kiyotaki and Moore (1997), and adopted by Lorenzoni (2008). As such, they can be sold only at a discount.

at date 1.

Payoffs If depositors choose to continue, they receive D at date 2 provided the bank's portfolio returns enough, that is, if $V(\phi) \geq D$. Otherwise, if $V(\phi) < D$, the bank is *insolvent* and enters costly bankruptcy at date 2, having insufficient resources to repay depositors when the projects mature. For tractability, we make the stark assumption that bankruptcy absorbs all the projects' payoff. In other words, the fraction that the depositors receive from the bank's portfolio is 0.[5]

Considering the decision depositors make at date 1, the payoff they receive at date 2 can be expressed as

$$(1 - \omega(\mathbf{s})) \cdot \left(D \cdot \mathbf{1}_{\{V(\phi) \geq D\}}\right) + \omega(\mathbf{s}) \cdot r, \tag{1}$$

where $\mathbf{1}_{\{V(\phi) \geq D\}}$ is an indicator function such that it is equal to 1 if $V(\phi) \geq D$ and 0 otherwise.

The bank is the residual claimant on the return of her portfolio and receives at date 2

$$\max\{V(\phi) - D, 0\}. \tag{2}$$

Thus, the bank receives 0 both if she is illiquid and depositors withdraw at date 1 or if she is insolvent and default at date 2.

Timing At date 0, the bank chooses simultaneously the types of projects in her portfolio and the allocation for each of them, i.e. the opacity and diversification levels. Afterwards, at date 1, depositors may observe information about the projects, depending on the types of projects the bank chose. Subsequently, they decide whether to withdraw or continue with their investment with the bank. If depositors withdraw their money early at this point, they receive r and the bank gets 0. If depositors continue their investment, projects mature and

[5] The results are robust to the alternative assumption that the fraction that the depositors receive of the bank's portfolio is a small positive α in the event of bank default at date 2. The robustness results are available upon request.

returns are realized at date 2. If the bank is solvent at date 2, depositors are paid the return on deposits D and the bank gets $V(\phi)$. Otherwise, the depositors and the bank receive 0.

2.2 Equilibrium

Both bank and depositors are rational and internalize how their choices will affect others' actions and subsequent payoffs. Thus, the bank decides strategically the composition of project types, $\mathbf{s} = (s_1, s_2)$, in her portfolio and the portfolio allocation $(\phi, 1-\phi)$. In other words, the bank chooses $\mathbf{s}$ and ϕ to maximize her expected profit, as we describe in detail below.

Similarly, depositors decide strategically whether to withdraw early. For this decision to be meaningful and to interact with the bank's choice, we assume the bank offers depositors a competitive deposit contract. Specifically, the return on deposit, D, is set to maximize the depositors' expected surplus, given that they lend all their funds. This assumption ensures that both the bank and the depositors receive non-trivial payoffs.

In this set-up we use the following equilibrium concept.

Definition 1 (**Equilibrium).** *An equilibrium is a composition of project types* $\mathbf{s} = \{s_1, s_2\}$ *and a portfolio allocation* $(\phi^*, 1-\phi^*)$*, a return on deposits* D^**, and depositors' continuation decision* $\omega^*(s_1, s_2)$ *such that*

1. *the continuation decision maximizes depositors' expected payoff at date* 1

$$\max_{\omega} \{(1-\omega(\mathbf{s})) \cdot D \cdot \Pr(D \leq V(\phi) |\mathbf{s}) + \omega(\mathbf{s}) \cdot r\};$$

2. *the return on deposits maximizes the depositors' expected payoff at date* 0

$$\max_{D} \mathbb{E}_{\mathbf{s}} \{(1-\omega(\mathbf{s})) \cdot D \cdot \Pr(D \leq V(\phi) |\mathbf{s}) + \omega(\mathbf{s}) \cdot r\};$$

3. *the project type decision and portfolio allocation maximize the bank's expected payoff*

at date 0

$$\max_{\phi,\mathbf{s}} \mathbb{E}_{R_1,R_2}\{(1-\omega(\mathbf{s}))\cdot \max[(V(\phi)-D),0]\}.$$

Implicitly, the optimal withdrawal decision is a function of the return on deposits as well as the fraction of transparent and unobservable projects in the bank's portfolio, i.e. $\omega^*(s_1, s_2) = \omega^*(s_1, s_2; D, \phi)$. Similarly, the return on deposits is a function of the bank's projects type and portfolio allocation, i.e. $D^* = D^*(\phi, s_1, s_2)$. In the exposition below, we take these dependencies as implicit so not to burden excessively the notation.

2.3 Solution Concept

Characterizing the equilibrium involves two steps. The first step is to solve for the bank's portfolio allocation choice keeping portfolio composition $\mathbf{s}$ fixed. This implies finding ϕ^* for when the bank holds: i) only transparent project(s) with $\mathbf{s} = (R_1, R_2)$; ii) only unobservable project(s) with $\mathbf{s} = (\varnothing, \varnothing)$, iii) both an unobservable and a transparent project with $\mathbf{s} = (R_1, \varnothing)$.

In each of these three cases, we solve the model backwards. We characterize the depositors' optimal withdrawal decision, $\omega^*(\mathbf{s})$, and the optimal return on deposit, D^*, that satisfy conditions (1) and (2) in Definition 1, given a bank's portfolio allocation $(\phi, 1-\phi)$. Then, we solve for the optimal portfolio allocation, $(\phi^*, 1-\phi^*)$, taking into account that the bank understands that the depositors behave optimally.

In the second step, we solve for the bank's optimal choice of project types, given that she understands the optimal portfolio allocation associated with each portfolio composition, $\mathbf{s}$. This second step delivers the equilibrium outcomes when the bank is able to choose both her portfolio allocation *and* the type(s) of projects she invests in. Characterizing the interaction of bank's optimal portfolio composition and optimal portfolio allocation and their impact of depositor withdrawal decision and the ensuing bank fragility is our main interest.[6]

[6] Details on the solution of the second step are in Section D. There we show that the solution involves comparing the bank's expected payoff across the possible opacity levels, given that the bank understands

Solving the model delivers four variables that characterize equilibrium: bank's optimal portfolio composition $\mathbf{s}^*$ and optimal portfolio allocation $(\phi^*, (1-\phi^*))$; the optimal return on deposit, D^*; and the depositors early withdrawal decision ω^*. While the first three variables are *ex-ante* equilibrium outcomes, the early withdrawal decision represents an *interim* outcome.

In addition, it is useful to include in the equilibrium analysis the probability of early withdrawal or *probability of illiquidity* of the bank at date 0, defined as

$$\begin{aligned}\Omega(\phi, D) &= \Pr(\text{bank fails at date } t = 1)\\ &= \Pr(\omega\left(\mathbf{s}\right) = 1),\end{aligned} \tag{3}$$

as well as the *probability of insolvency* of the bank at date 1, defined as

$$\begin{aligned}\Psi(\phi, D; \mathbf{s}) &= \Pr(\text{bank fails at date } t = 2|\text{reaching date } t = 2)\\ &= \Pr(V(\phi) < D|\omega\left(\mathbf{s}\right) = 0).\end{aligned} \tag{4}$$

From a high level perspective, the bank wants to maximize its profit. High profitability has three components: 1) staying liquid in the interim date 1, i.e. low Ω; 2) staying solvent ex-post at date 2, i.e. low Ψ; 3) paying a low rate of return on deposits at date 2, i.e. low D. The important observation is that the failure probability of the bank has two components. We will show that perhaps counterintuitively, minimizing the two components is not necessarily aligned, in which case the bank faces a trade-off in her portfolio choice.

In order to illustrate the trade-offs faced by the bank, we first discuss the bank's optimal choice of portfolio allocation ϕ, taking the portfolio composition $\mathbf{s}$ as given, in Propositions 1 and 2 and Example 1. We then characterize bank's optimal portfolio composition and allocation, $(\mathbf{s}, \phi)$, when the bank jointly chooses them.

the equilibrium diversification associated with each of them.

3 Benchmark Cases

As a preamble to the full equilibrium characterization, we consider two special cases: a fully transparent portfolio and a fully unobservable portfolio. In both settings, the bank chooses a portfolio allocation $\phi \in [0, 1]$ across two risky projects and offers depositors receive a contract characterized by a repayment D at date 2. Bank's decision only affects the distribution of her portfolio returns, and not depositors information structure. These polar cases clarify how the bank's incentives interact with the information structure, particularly how they depend on whether depositors can observe the return realization at the interim date.

Throughout the analysis of these benchmark cases, we adopt the convention that the bank's optimal portfolio allocation ϕ lies in the interval $[0, \frac{1}{2}]$, exploiting the symmetry of the return distribution. Accordingly, the boundary cases $\phi = 0$ and $\phi = 1$ are treated as equivalent and represent fully undiversified portfolios.

All proofs for the environment with a transparent portfolio and with an unobservable portfolio are provided in Appendix B and Appendix C.

3.1 Transparent Portfolio

We start by analyzing the bank's portfolio allocation choice when both projects are transparent. When the bank holds a transparent portfolio, with $\mathbf{s} = (R_1, R_2)$, depositors have perfect information about the return on bank portfolio when they make their decision whether to withdraw their funds at date 1. Thus, they continue to fund the bank if they are certain that the bank does not default at date 2. Conversely, they withdraw their deposit early when they learn that the bank's portfolio returns will be insufficient to cover their return on deposit. In other words, the depositors withdrawal decision at date 1 is as follows

$$\omega(\mathbf{s}) = \begin{cases} 1 & \text{if } V(\phi) < D \\ 0 & \text{if } V(\phi) \geq D \end{cases} .$$

The bank understands that if depositors choose not to withdraw at date 1, it is certain that she will remain solvent at date 2 and will therefore receive the residual payoff $(V(\phi)-D)$. In other words, the probability of insolvency conditional on depositors not withdrawing, is $\Psi = 0$. This implies that the date-0 probability that the bank is illiquid and that depositors withdraw early is $\Omega = \Pr(V(\phi) < D)$.

Since insolvency is no longer a concern once the bank is continued, the only risk she is facing is illiquidity. Thus, for a given return on deposits D, the bank's expected payoff at date 0 is given by

$$\mathbb{E}_{R_1,R_2}\left[(V(\phi)-D)\middle|V(\phi)\geq D\right]\cdot \Pr\left[V(\phi)\geq D\right]. \tag{5}$$

For a fixed return on deposit, the bank benefits from a lower probability of early withdrawals, which translates one-for-one into a lower risk of illiquidity. At the same time, as the residual claimant, she captures the excess return on the portfolio above the promised payment to depositors. The following lemma formalizes this trade-off.

Lemma 1. *For any fixed portfolio allocation ϕ, the bank's expected payoff is decreasing in the return on deposits, D.*

Lemma 1 simply states that conditional on the return distribution of the bank's portfolio, bank expected payoff in Equation (5) is decreasing in the return on deposits. This is intuitive as the return on deposits impacts bank's expected profit through two channels that go in the same direction. First, when bank holds a transparent portfolio, the depositors receive perfect information about bank's future return at date 1. This implies that their withdrawal decision is perfectly aligned with the bank's (in)solvecy: They will withdraw early if and only if the (to-be-realized) bank's future return is below the return on deposits. As such, a higher return on deposits leads to a higher probability of early withdrawal and thus lower bank expected profit. Second, a higher return on deposits implies that whenever the depositors do not withdraw early, they receive a higher share of the portfolio return at date 2, which in turn means that bank's profit will be lower.

In our environment, however, the return on deposits D is not exogenous, and must solve the following maximization problem

$$\max_{D} \quad D \cdot \Pr\left(V(\phi) \geq D\right) + r \cdot \Pr\left(V(\phi) < D\right). \tag{6}$$

The optimal return on deposits depends on the depositors' outside option, r, as well as the portfolio allocation chosen by bank, ϕ, which is in itself a function of r. Intuitively, the return on deposits is increasing in depositors' outside option r. At the same time, the return on deposits can be lowered when the bank chooses a well-diversified portfolio, i.e. a portfolio allocation closer to $\frac{1}{2}$.

In order to determine the equilibrium outcome, it remains to characterize bank's optimal portfolio allocation ϕ. The following proposition takes the above observations that the endogenous return on deposits and investor withdrawal decision are both impacted by bank's portfolio allocation ϕ into account and characterizes the equilibrium for each value of exogenous depositor outside option, r.

Proposition 1 (**Equilibrium Transparent Portfolio**). *When bank's portfolio composition is transparent, there exists a threshold $\bar{r}_T$ such that*

1. *If $r < \bar{r}_T$, the bank chooses a well-diversified portfolio allocation that minimizes the return on deposits at date* 2*.*

2. *If $r \geq \bar{r}_T$, the bank chooses a fully undiversified portfolio allocation.*

Observe that when bank's portfolio composition is transparent, depositors have full information when they decide whether to withdraw their deposits early. This observation has two consequences. First, the probability of illiquidity Ω reflects that the realization of the signal reveals perfectly the future realization of the return of both projects in the bank portfolio, above and beyond its dependence on r. Second, the withdrawal decision is perfect. Put differently, whenever the bank is liquid she is solvent as well, $\Psi = 0$, and it is not possible to increase the return on deposits D without making the bank insolvent in at least one state of the world at date 2.

In other words, there is no information friction at the time of depositors' withdrawal decision. This implies that there is no trade-off between minimizing the probability of illiquidiy and insolvency, and the bank's target is to minimize a single-dimensional failure probability and the rate she pays. As such, in order to build some intuition about the bank portfolio allocation, it is useful to think about a single-period debt contract and a distribution of return.

With a simple debt contract, once the return is realized, the bank is either solvent or insolvent, depending on the rate of return on debt. If she is solvent, then she receives the residual claim $R - D$. When D, the rate of return on debt is endogenous, then it is the inverse of solvency: minimizing the probability of insolvency minimizes what the bank has to pay the debt holders as well. The argument underlies the general intuition behind the bank's preference for holding *diversified* portfolios. Higher diversification, $\phi \sim \frac{1}{2}$, makes the return distribution second order stochastically dominant and a reasonably high return is more certain. As lenders are more likely to be paid back, they are willing to accept a lower rate of return.

Next, consider the case in which the return on deposits is exogenously fixed under a simple single-period debt contract. In this setting, the bank cares only about upside outcomes. Diversification compresses the tails of the return distribution, but since the return on deposits does not adjust, this only reduces the likelihood of very high returns—precisely the states that are most profitable for the bank. As a result, when the promised return D is high, the bank avoids diversification and instead engages in *risk shifting*, concentrating its portfolio to maximize the chance of extreme positive outcomes.

In our framework, although the rate of return on deposits is determined endogenously, the same risk shifting behavior arises when r is high. That is so because r is the outside option of depositors for withdrawing and so it acts as an exogenous lower bound on the required rate of return on debt. Thus, when r is high, the only feasible rates of return on deposits are high as well. As such, the bank risk shifts and avoids diversification.

This argument shows two important points. First, the risk shifting behavior of the bank is independent of the demandable nature of deposit contract. Second, that when portfolio composition is transparent, bank's incentives to minimize the probability of illiquidity, insolvency, and rate of return on deposits are mainly well aligned, except when r is too high when the bank engages in risk shifting.

3.2 Unobservable Portfolio

Next we analyze the bank's portfolio allocation choice when both projects are unobservable. When the bank holds an unobservable portfolio, with $\mathbf{s} = (\varnothing, \varnothing)$, depositors have no information about the return on bank portfolio when they make their decision whether to withdraw their funds at date 1.

It follows that for any portfolio allocation ϕ and return on deposits D, both determined at date 0, depositors' withdrawal decision at date 1 is already determined at date 0. Depositors will keep their deposits in the bank at date 1 if and only if the amount the expected repayment at date 2, $D \cdot \Pr(V(\phi) \geq D)$, exceeds the reservation value, r. Otherwise, they withdraw their deposit and receive r. Thus, the depositors withdrawal decision at date 1 is known at date 0, when bank chooses its portfolio allocation ϕ, to be:

$$\omega(\mathbf{s}) = \begin{cases} 1 & \text{if } D \cdot \Pr(V(\phi) \geq D) < r \\ 0 & \text{if } D \cdot \Pr(V(\phi) \geq D) \geq r \end{cases} .$$

This implies that at date 0 that bank faces two possible outcomes. If the bank portfolio allocation ϕ and the pursuant return on deposits $D(\phi)$ are such that

$$D(\phi) \cdot \Pr\left(V(\phi) \geq D(\phi)\right) \geq r, \tag{7}$$

then the depositors do not withdraw early and the probability of illiquidity is date 0. The only risk remaining for the bank is the risk of insolvency, $(1 - \Pr\left(V\left(\phi\right) \geq D\right))$. In this case, bank's expected payoff at date 0 is given by

$$\mathbb{E}_{R_1,R_2}\left[V\left(\phi\right) - D)|V\left(\phi\right) \geq D\right] \cdot \Pr\left[V\left(\phi\right) \geq D\right]. \tag{8}$$

Alternatively, if condition (7) is violated, the bank gets 0 at date 2.

Note that for any fixed return on deposits D, the bank's expected payoff in Equation (8) is identical to that in the transparent portfolio case, as given in Equation (5). This equivalence arises because the bank faces only one source of risk in both settings. Consequently, Lemma 1 applies equally in this setting, provided the participation constraint is satisfied, i.e., $D \cdot \Pr\left(V(\phi) \geq D\right) \geq r$.

However, when the bank holds an unobservable portfolio, there is not always be a return on deposits that persuades depositors to continue with the bank, regardless of the portfolio allocation the bank chooses. In particular, the return on deposits must solve the following problem

$$\max_{D} D \cdot \Pr\left(V(\phi) \geq D\right). \tag{9}$$

This is because, at date 0, it is certain that depositors will continue with the bank at date 1 if the solution to Problem (9) satisfies $D \cdot \Pr\left(V(\phi) \geq D\right) \geq r$; otherwise they will choose to liquidate. Without loss of generality, we assume that if the depositors are anticipated to withdraw with probability one, they do not lend to the bank in the first place.

Indeed, when the depositors' outside option is sufficiently attractive, it is straightforward to verify that the solution to Problem (9) does not satisfy the threshold for which depositors continue with the bank for any ϕ that the bank chooses. This implies that in the unobservable case, depositors effectively commit to their decision at date 0. Without information at date 1, they lack the flexibility to revise their withdrawal decision made at date 0. The return on deposits reflects the compensation provided to depositors for this lack of flexibility, which, in some cases, may exceed what the bank can afford to pay.

For any return on deposits that satisfies that satisfies Problem (9) and depositors continue the bank, the following proposition characterizes bank's portfolio allocation choice.

Proposition 2 (**Equilibrium Unobservable Portfolio**). *When bank's portfolio composition is unobservable, there exists a threshold $\bar{r}_U$ such that*

1. *If $r < \bar{r}_U$, the bank chooses a well-diversified portfolio allocation that minimizes the return on deposits at date 2.*

2. *If $r \geq \bar{r}_U$, the bank cannot implement any portfolio allocation that prevents depositors withdrawing early.*

Consider that r is sufficiently small. In this case, depositors will not withdraw in the interim date. As such, bank's probability of illiquidity is trivially zero, $\Omega = 0$. However, the bank might still be insolvent at date 2 as the realization of project returns can be arbitrarily low, $\Psi \geq 0$. It follows that in setting portfolio allocation ϕ when portfolio composition is unobservable, the bank's target is to minimize a single-dimensional failure probability and the rate she pays, similar to the case of transparent portfolio in Subsection 3.1. As such, the trade-off that the bank faces reduces to simple one-period trade-off which results in a high degree of diversification: A more diversified portfolio decreases both the probability of insolvency and the rate of return on deposits paid to depositors.

What is common between the unobservable and transparent portfolio compositions is that the bank has no ability to finetune the extent of depositors' interim information through its choice of portfolio allocation. Depositors will have full information if the portfolio composition is transparent and no information if it is unobservable, independent from bank's choice of portfolio allocation ϕ. The portfolio allocation solely determines the distribution of future return, which in turn governs the probability of solvency and bank profit. Thus, the bank often chooses a highly diversified portfolio, or engage in risk shifting—only when portfolio composition is transparent and r is prohibitively high.

Alternatively, with an opaque portfolio composition, the portfolio allocation ϕ has a dual role. The bank uses it to both control the ex-post return distribution, which governs the probability of solvency and bank profits, and finetune depositors' interim information, which governs probability of illiquidity. As such, the bank might face a trade-off between staying liquid and being solvent with cheap funding.

4 Main Result: Emergence of Opaque Banks

In this section, we characterize the optimal bank decision to jointly choose the portfolio composition and portfolio allocation. We begin with an example that anticipates the novel trade-off between being liquid versus being solvent while raising cheap deposits, that is specific to opaque portfolio composition——one that does not arise under transparent and unobservable portfolios.

Example 1 (**Bank Portfolio Allocation Trade-off with an Opaque Portfolio**). *If the portfolio composition is opaque, there is a range $r \in [r_l, r_h]$ for which the optimal portfolio allocation ϕ^* does not minimize neither the probability of illiquidty nor the return on deposits.*

This example illustrates that, unlike in the transparent or unobservable portfolio cases, a bank holding an opaque portfolio does not choose a portfolio allocation that minimizes the return on deposits. While she benefits when the return on deposits is low—since she is the residual claimant on the portfolio—she understands that this may increase the likelihood of early withdrawals. Conversely, reducing illiquidity risk requires a return on deposits that is too high from her perspective, thereby lowering her expected payoff. As the example shows, she avoids both extremes, choosing instead a portfolio allocation that balances the trade-off rather than minimizing either risk in isolation.

This example highlights a misalignment between two core determinants of the bank's payoff and spells out the central trade-off that emerges between the probability of illiquidity Ω on one side, and the probability of insolvency Ψ and the corresponding bank residual profit on the otehr side.

To see this, recall that the bank's profit depends on three components: the likelihood of early liquidation, Ω, as defined in Equation (3), the probability of default at maturity, Ψ, as defined in Equation (4), and the promised return on deposits, D. The bank's ex-ante expected payoff can be written as:

$$\big(1 - \Omega(\phi, D)\big) \times \mathbb{E}_{R_1}\left\{\big(1 - \Psi(\phi, D; R_1)\big) \times \mathbb{E}_{R_2}\Big[(V(\phi) - D)|V(\phi) > D\Big]\right\}.$$

In contrast to the transparent and unobservable cases, where the bank faces either illiquidity or insolvency risk but not both, an opaque portfolio exposes her to both. Crucially, decreasing Ω and Ψ concurrently can be conflicting. This is because the portfolio allocation ϕ simultaneously determines (i) the return distribution $V(\phi) \sim \phi G(\cdot) + (1-\phi)G(\cdot)$ and (ii) the informativeness of the signal available to depositors. When these two components affect the bank's payoff in opposing ways, a trade-off arises. Opacity then becomes a strategic lever the bank uses to substitute between insolvency and illiquidity risk in a way that maximizes expected profits.

We now explain these two impacts separately. First, like the unobservable and transparent portfolio composition, the portfolio allocation ϕ determines the *return distribution* of bank portfolio at date 2, which in turn determines bank solvency, Ψ, and the long-term rate of return required by the depositors D, through that. Second, unlike the unobservable and transparent portfolio compositions, the portfolio allocation also governs the *depositor information* with an opaque portfolio. How much information depositors have determines their early withdrawal decision, which in turn determines illiquidity risk, Ω.

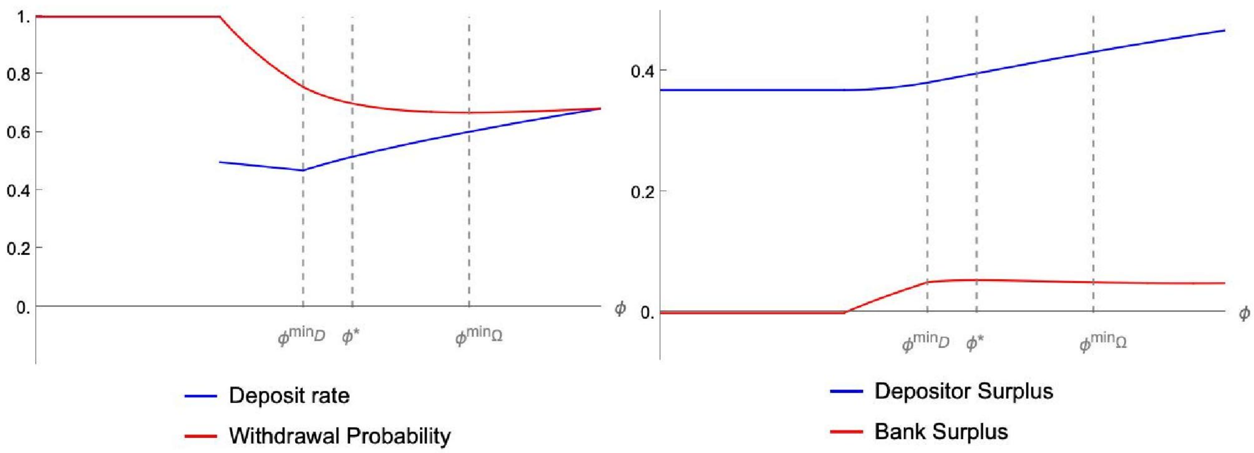

Figure 2: Equilibrium outcomes for opaque portfolio composition when $r = 0.37$. The left plot depicts the return on deposits bank default probability as a function of different choices of portfolio allocationϕ. The right plot depicts the depositor and bank surplus as a function of ϕ. Both plots also indicate the optimal portfolio allocation chosen by the bank, as well as the portfolio allocations that minimize the return on deposits and default probability.

It follows that when the portfolio composition is opaque, the choice of portfolio allocation involves a trade-off. On the one hand, there is a diversification channel which is independent of the portfolio composition, as it is present in the unobservable and transparent portfolios as well. As $\phi \to \frac{1}{2}$ the ex-post return distribution becomes more diversified, which implies a lower D, and thus a lower Ψ, on average. However, a lower ϕ reduces the precision of depositors' information and makes them withdraw more to avoid the risk of bank becoming insolvent and not being paid, increasing the Ω. Thus, there is a misalignment between Ω and Ψ if $\phi \to \frac{1}{2}$ to increase the chance of bank solvency.

On the other hand, a high ϕ improves depositor information and allows them to withdraw more effectively, when they are less likely to be paid if the bank continues to operate until date 2. This in turn reduces the probability of early withdrawal and lowers Ω. On the other hand, it makes the ex-post return less diversified, increasing the required long-term return on debt D and the probability of bank insolvency Ψ, on average.

Put differently, when $\phi \to \frac{1}{2}$, despite the favorable low probability of insolvency and low return on deposit, depositors' early withdrawal decision is done with mediocre information. It is quite imprecise and destroys sizeable ex-post value. The imprecision of early withdrawal decision necessitates a high Ω, something the bank dislikes. Alternatively, when ϕ is large, depositors are better informed about likelihood of ex-post default given the predetermined D. As such, they withdraw their deposits efficiently, exactly when $\Psi(R_1)$ is high–or equivalently $(1-\Psi(R_1))$ is low. Effective withdrawal enables them to withdraw less frequently, decreasing Ω, something the bank likes.

Example 1 shows that in equilibrium, the bank resolves this trade-off in a way that exposes it to defaulting early too frequently. In other words, bank chooses a "degree of opacity," ϕ, which does not minimize the probability of early withdrawal, i.e. bank illiquidity. One can think of this as a novel form of risk-shifting behavior: the bank loads on short-run risk of failure to decrease its long-run risk of failure.

With the insights about the optimal portfolio allocation with transparent and unobservable portfolio compositions in Section 3 and opaque portfolio composition in Example 1, the remaining question is the portfolio composition optimally chosen by the bank. In particular, whether the bank ever holds an opaque portfolio despite facing the extra trade-off between liquidity and solvency, which she can avoid by simply sticking to a transparent or an unobservable portfolio. Proposition 3, which is the he main result of the paper, addresses this question.

Proposition 3 (Jointly Optimal Portfolio Composition and Allocation). *When the bank chooses both the portfolio composition and the portfolio allocation simultaneously, there exist values $\bar{r}_U < \bar{r}_O$ such that:*

1. *when $r \leq \bar{r}_U$, the bank holds a well-diversified unobservable portfolio,*

2. *when $\bar{r}_U < r \leq \bar{r}_O$, the bank holds an opaque portfolio with imperfect diversification,*

3. *when $r > \bar{r}_O$, the bank holds a fully concentrated transparent portfolio.*

Proposition 3 shows that in equilibrium, the bank does choose an opaque portfolio for a wide range of intermediate values of r, the redemption rate of deposits. This result seems

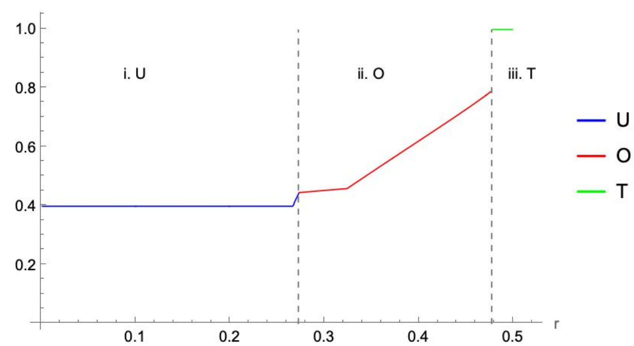

Figure 3: Bank's Portfolio Holding in Equilibrium

The three regions correspond to the bank's portfolio composition in equilibrium. In region i the bank holds an unobservable portfolio (U); in region ii an opaque portfolio (O); and in region iii a transparent portfolio (T). In the case of O, ϕ^* represents the fraction invested in the transparent project. The line is color-coded based on the regions to facilitate visual interpretation.

counter-intuitive as an opaque portfolio introduces an extra trade-off in bank's choice of portfolio allocation. Unlike transparent and unobservable portfolios, when the bank portfolio is opaque, minimizing the probability of illiquidity and insolvency concurrently becomes impossible. Thus, by choosing an opaque portfolio the bank would need to expose itself to early failure due to illiquidity too frequently.

Figure 3 depicts the three regimes that emerge in equilibrium as a function of the redemption value of deposits. In region i, the leftmost region, the bank holds a well-diversified unobservable portfolio. By choosing an unobservable portfolio composition, the bank withholds any information from depositors. This implies that if depositors choose to finance the bank, they never withdraw early.

As such, this region corresponds to the common intuition argued in the previous literature that keeping depositors uninformed prevents banking panics (Dang et al., 2017). The important point illustrated by Proposition 3 is that this mechanism is relevant only for low deposit redemption value, that is, when the depositor's outside option is low.

Alternatively, in region iii, the rightmost region, the bank chooses a transparent portfolio. Furthermore, she engages in risk shifting and foregoes all diversification. When depositors have a very high outside option, the only way to convince them to keep their deposits in the bank is to assure them that if they do not demand their deposits early, they will receive a high rate of return at date 2. The nest way for the bank to achieve this outcome is by holding a transparent portfolio. Even with a transparent portfolio, the high outside option of depositors implies that they withdraw their deposits rather frequently and require a high long-term rate of return as well. It follows from Proposition 1 that a response to this high required rate of return, the bank engages in risk-shifting and holds a fully concentrated portfolio in region *iii*.

The last region is region *ii*, the intermediate range of the redemption rate of deposits, $\bar{r}_U < r < \bar{r}_O$. What is important in this region is that r, the redemption value of deposits, is sufficiently high that if kept in the dark, depositors will panic and always withdraw their deposits early. Thus, an unobservable portfolio is not feasible. On the other hand, it is not high enough to justify providing full information to depositors, as with a transparent portfolio they would require an excessively high rate of return on their deposits. It follows that the bank's optimal choice is to hold an opaque portfolio.

When the bank portfolio is opaque, the portfolio allocation ϕ governs two equilibrium features concurrently: the degree of diversification, which determines the bank funding cost and in turn the bank profitability, and the degree of information provision to depositors, which determines the probability of illiquiidty. For an intermediate range of r, minimizing these two sources of bank failure requires different portfolio allocations. On the one hand, minimizing the average probability of insolvency and the rate of return on deposits requires a more diversified portfolio, $\phi \to \frac{1}{2}$. On the other hand, minimizing probability of illiquidity requires giving more information to the depositors, $\phi \to 1$.

Example 1 describes this trade-off in detail. In equilibrium, bank chooses an intermediate level of portfolio allocation ϕ^* that maximizes bank's profit but does not minimize the probability of illiquidity or the promised rate of return on deposits (equivalently, the average probability of insolvency), as illustrated in Figure 2. In this region, the bank uses portfolio opacity as a lever to substitute insolvency risk with illiquidity risk to maximize her profit at the cost of depositors.

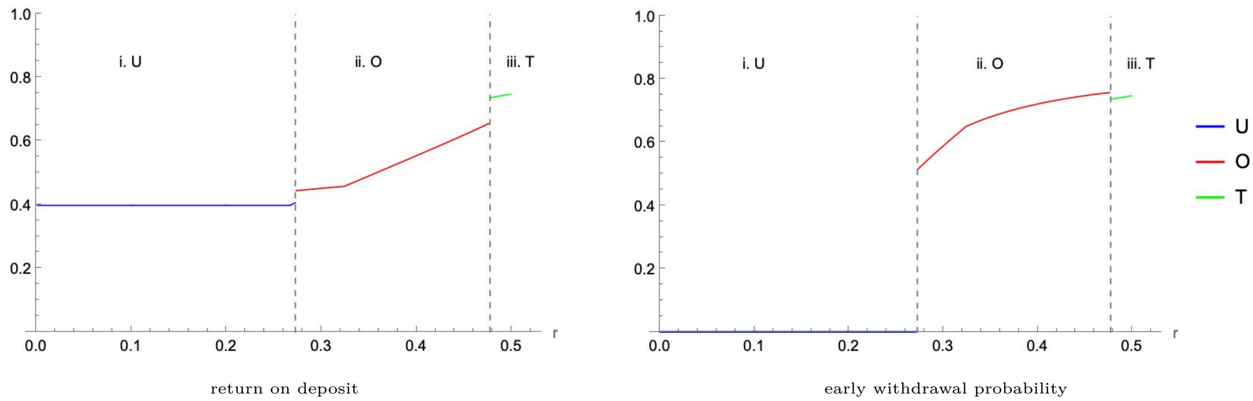

Figure 4: Equilibrium outcomes, return on deposits and investor early withdrawal probability

As such, Proposition 3 leads to a counter-intuitive observation. When the redemption rate of deposits is in an intermediate range, the bank optimally chooses to expose itself to excessive withdrawals. As such, one would observe too many banking panics in this region. Banks are willing to be exposed to this risk in order to reduce their funding costs.

This observation is most clear in the boundary between regions *ii* and *iii* in Figure 4. Recall that r corresponds to what the depositors receive if they withdraw early. As such, we would expect an increase in r to lead to an increase in the early withdrawal probability—the probability of illiquidity. Figure 4 shows that this intuition is correct for every value of r, except at the boundary between the two regions *ii* and *iii*, when the bank changes its portfolio composition from opaque to transparent.

In region *ii*, to the left of the threshold, the bank chooses an opaque portfolio and withholds some information from depositors. This leads to frequent early failures for the bank. However, the bank pays a low rate of return to depositors if solvent. As r crosses the threshold, it becomes too costly for the bank to withhold any information from depositors and thus, the optimal portfolio composition becomes transparent. Despite an increase in r, probability of illiquidity *declines* at this threshold. The reason is that when bank switches to a transparent portfolio, depositors can charge the bank a high return at date 2, which in turn motivates them to leave their deposits in the bank and do not withdraw early.

With an opaque portfolio, an alternative interpretation of the bank's optimal choice of portfolio allocation ϕ is as a *compound lottery.* The *outer lottery* corresponds to bank illiquidity—whether the bank fails early at date 1 or not. The *inner lottery* corresponds to bank solvency—whether the bank stays solvent at date 2 and if so, how much she pays the depositors. Ordinarily, one would assume that less frequent interim failures—a less risky outer lottery, is coupled with less frequent ex-post failures and a lower long-term return on deposits—a less risky inner lottery.

The model shows that this coupling breaks down when the depositors can realize a high value for withdrawing their deposits early from the bank. In this case, to prevent early withdrawal and stay liquid—a *less* risky outer lottery, the bank has to commit to a very high long-term return on deposits—a *more* risky inner lottery. However, the bank has a strong preference for a less risky inner lottery with low rate of return on deposits, as that is the only time that she is paid.

This preference leads the novel insight of the model, which is short-run risk taking behavior by the bank by holding an *opaque* portfolio. An opaque portfolio with the correct choice of portfolio allocation ϕ allows the bank to achieve a more risky outer lottery and a less risky inner one, with lower long-term return on debt. Put differently, an opaque portfolio enables the bank to shift its risk from insolvency to illiquidity, that is, from the inner lottery to the outer lottery, in a way that maximizes bank profit. In this case, the optimal portfolio allocation of the bank leads to a low long-term rate of return on debt and a high probability of solvency conditional on staying liquid, albeit at the cost of frequent illiquidity. In this case, we say that the bank exposes itself to *excessive fragility.*

Finally, in both regions *ii* and *iii* in the right panel of Figure 5, greater transparency increases the sensitivity of deposit flows to the bank's performance signals and increases the risk of illiquidity, as documented in Chen et al. (2022b).

5 Bank Fragility and Government Interest Rate

Since the global financial crisis of 2008, a large body of research has explored different causes of bank fragility and the consequences for the financial sector. Importantly, almost all of this research has been developed during a prolonged period of near-zero rates. However, after the pandemic, rates rose sharply to combat inflation, which in turn raised new questions about bank fragility. In this section, we use our framework to investigate the implications of bank opacity for bank fragility in different interest rate environments.

Recall that r, i.e. the short-term return on deposits, represents the outside option of depositors if they decide to withdraw their deposits before maturity. One possible interpretation is that depositors can withdraw their deposits from the bank at date 1 (at par) and invest in the government interest rate, r_f. The bank would need to liquidate the project early one-for-one to pay back each unit of deposits. With this interpretation, r will be an affine transformation of the government interest rate.[7] To avoid introducing new notation, we refer to r as the government interest rate in this section.[8]

This convention allows us to demonstrate our proposed mechanism that underlies banks' optimal choice of opacity, relate some of our results to the empirical evidence from the literature and distinguish the novel findings of the model. To make this comparison clear, it is insightful to compare the equilibrium outcome of the model vis-a-vis a *transparent portfolio with optimal allocation*, at different levels of interest rates. Figure 5 illustrated this comparison.

The most important observation is that independent of the prevailing monetary policy rate, a transparent bank uniformly pays a higher deposit rate compared to an opaque bank, consistent with findings of Chen et al. (2022a). However, the impact of opacity on bank fragility is non-uniform. In low interest rate environments, opacity makes bank portfolio

[7] Specifically, $r = c(1 + r_f)$, where c is the exogenous scale parameter.

[8] Furthermore, we normalize the interest rate on government bonds at date 0 to zero. As we assume that agents do not discount the future, this assumption is without loss of generality. Note that in this framework, discounting is equivalent to reducing bank payoff R at date 2 compared to r. Given that our results are for a general value of R, the no discounting assumption is without loss of generality as well.

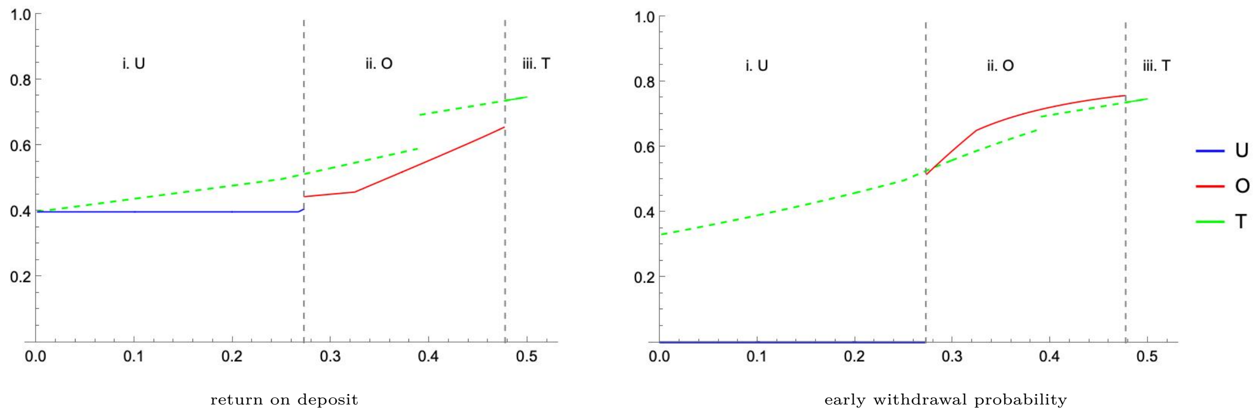

Figure 5: Equilibrium return on deposits and probability of early withdrawal: Comparison with the optimal transparent portfolio

information-insensitive and decreases the probability of illiquidity. However, this pattern reverses in high interest rate environments: opaque banks face more early withdrawal than transparent banks if government interest rates are sufficiently elevated.

The left half of both panels, region i in blue, corresponds to the mechanism argued by Dang et al. (2017)—opacity preserves bank liquidity by preventing information sensitive early withdrawal. Our model generalizes this intuition when banks are not only subject to illliquidity but also to insolvency; that is, they face distinct possibilities of interim and ex-post failure. We show that by withholding all information from depositors, banks are able to improve chances of staying both liquid and solvent. When interest rates are low, keeping depositors uninformed prevents information-driven panic runs and allows a reasonably low long-term return on deposits, concurrently. Figure 5 exhibits both of these outcomes. The left panel shows that information-driven early withdrawals are less frequent with an unobservable portfolio compared to a transparent portfolio. The right panel illustrates that the bank is further able to attract cheaper deposits with an unobservable portfolio.

The critical observation is that this argument is relevant only in *low interest rate environments.* In fact when interest rates are *moderately high*, in region ii, opacity leads to excessive fragility of the financial sector. The right panel of Figure 5 manifests that for intermediate levels of interest rates, the bank optimally chooses an opaque portfolio—the red portion of the plot. In this region, the probability of early withdrawals is higher than the probability of early withdrawal in a (hypothetical) transparent portfolio with optimally chosen portfolio allocation. The reason why the bank chooses to expose herself to excessive early failure is that in return, she can offer a lower long-term return to depositors if she stays liquid, as evident in region *ii* of the left panel of Figure 5.

In summary, when interest rates are bounded away from zero, an *intermediate* degree of opacity allows banks to earn more profits by finetuning the amount of information that they convey to depositors. They provide depositors with sufficiently low information that they do not require excessively high rate of return, but sufficiently high that they have enough confidence in the bank to lend to her. In this case, bank opacity does not guarantee stability of the financial sector. In fact, with moderate interest rates, banks use opacity as a tool to shift risk forward: They are willing to suffer excessive information-driven early withdrawals in exchange for offering a low long-term deposit rate and staying solvent conditional on weathering the deposit withdrawals in the short run.

6 Conclusion

This paper develops a theory of opacity as a deliberate choice in banking. In our model, the bank selects not only which assets to hold, but also how much information to reveal to depositors through its portfolio composition and allocation. When the portfolio is opaque—mixing transparent and unobservable assets—allocation affects both the return distribution and depositor beliefs, creating a trade-off between illiquidity and insolvency.

We show that the bank exploits this trade-off strategically. Under certain funding conditions, she chooses an opaque, under-diversified portfolio that raises the likelihood of early liquidation. This fragility is intentional: by increasing the risk of runs, the bank reduces deposit rates and improves solvency. Opacity becomes a tool to substitute short-term failure for long-term risk, maximizing her expected surplus at the depositors' expense.

By endogenizing the information structure, the model reveals how fragility can be optimally engineered, not merely avoided. This perspective opens new avenues for studying how institutions use opacity, signal precision, and information control to shape their funding environment—particularly in the presence of systemic risk, regulation, or competition.

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Appendix

A Portfolio Return Distribution

Recall bank's portfolio return $V(\phi)$,

$$V(\phi) = \phi R_1 + (1-\phi)R_2,$$

where R_1 and R_2 are independent random variables uniformly distributed in $[0,1]$, and $\phi \in [0,1]$.

In this section, we characterize the return distribution. Then, we show how changing ϕ affects the distribution of $V(\phi)$, focusing on how it alters the portfolio's risk profile.

$V(\phi)$ has the cumulative distribution function $H(z;\phi)$,

$$H(z;\phi) = \begin{cases} \frac{z^2}{2\phi(1-\phi)} & z < \phi \\ (1-\phi)^{\{-1\}}\left(z - \frac{\phi}{2}\right) & \phi \le z \le 1-\phi \\ 1 - \frac{(1-z)^2}{2\phi(1-\phi)} & z > 1-\phi \end{cases} \tag{A.1}$$

and the probability density function,

$$h(z;\phi) = \begin{cases} \frac{z}{(1-\phi)\phi} & z < \phi \\ \frac{1}{1-\phi} & \phi \le z \le 1-\phi \\ \frac{1-z}{(1-\phi)\phi} & z > 1-\phi \end{cases} \tag{A.2}$$

Note that $H(z;\phi)$ distribution is symmetric around $\phi = \frac{1}{2}$, so we only need to consider the case $0 \le \phi \le \frac{1}{2}$. The case $\frac{1}{2} \le \phi \le 1$ follows by symmetry.

Since $E[R_i] = \frac{1}{2}$, the expected return $E[V(\phi)]$ is always $\frac{1}{2}$, independent of ϕ. However, ϕ determines how spread out $V(\phi)$ is around this mean, thus controlling the portfolio's "risk" in a second-order stochastic dominance sense[9]. In our environment, a higher-variance portfolio is considered "riskier" precisely because it entails larger swings away from the mean.

Concretely, when $\phi = 0$ or $\phi = 1$, the return $V(\phi)$ reduces to a single Uniform$[0, 1]$ project, producing the highest variance and hence the most risk under SOSD. Conversely, when $\phi = 1/2$, the distribution of $V(\phi)$ becomes triangular, with probability mass concentrated near the midpoint and less spread into the tails — this is the least risky allocation. Intermediate values of ϕ yield partially diversified returns with correspondingly intermediate degrees of spread.

Thus, by comparing how dispersed each portfolio's distribution is around its (common) mean, we use second-order stochastic dominance to formalize the intuitive claim that corner allocations (no diversification) are riskier than balanced allocations (perfect diversification).

Fig. 6 shows the CDF and the PDF of the portfolio return $V(\phi)$ for different levels of ϕ. Clearly, the shape of distribution reflects the degree of diversification (or lack thereof) in the bank's portfolio.

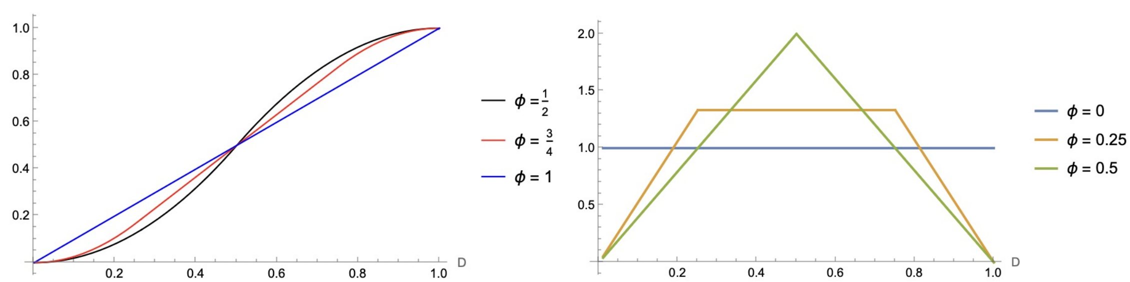

Figure 6: CDF and PDF of bank's portfolio return $V(\phi)$

As ϕ moves away from $\frac{1}{2}$ (green curves) toward 0 (blue curves), the variance of returns increases. It is minimized at $\phi = \frac{1}{2}$ with variance of $\frac{1}{12}$, and maximized at $\phi = 0$ with variance of $\frac{1}{24}$. The variance illustrates how the portfolio risk profile changes as ϕ moves from one extreme to the other.

[9]Under SOSD, a distribution X is considered riskier than Y if it can be seen as a mean-preserving spread of Y — i.e., it has the same mean but greater variance (or more generally, more probability mass in the tails).

B Transparent Portfolio

In this section, we provide the results for the benchmark case of the transparent portfolio (Subsection 3.1).

First, we show how the bank's expected payoff changes as a function of the deposit rate D and the portfolio allocation ϕ. Here we prove Lemma 1.

Second, we characterize the equilibrium outcome. We start by solving for the optimal deposit rate D^* and depositors' optimal withdrawal decision $\omega^*(\mathbf{s})$. Then, we solve for the optimal portfolio allocation $(\phi^*, 1-\phi^*)$ (see the solution concept Subsection 2.3).

Lastly, we show that the equilibrium allocation $\phi^*(r)$ minimizes the deposit rate, proving Proposition 1.

B.1 Proof of Lemma 1

Consider bank's expected payoff

$$\begin{aligned}&\mathbb{E}\left[(V(\phi)-D)|V(\phi)\geq D\right]\cdot\Pr\left[V(\phi)\geq D\right]\\&=\int_D^1(z-D)\frac{\partial H(z;\phi)}{\partial z}dz\end{aligned}\tag{5}$$

where $V(\phi)$ follows the cumulative distribution function $H(z;\phi)$ (Section A).

We find that, for a fixed portfolio allocation ϕ, Eq. (5) is *decreasing* in the deposit rate D. We also find that, for a fixed D, Eq. (5) is *decreasing* in ϕ.

To prove these results, we need to consider three different cases based on the relationship between the values of D and ϕ. That's due to the piece-wise feature of the return distribution $H(z;\phi)$.

Case 1: $\mathbf{D} < \boldsymbol{\phi}$ Eq. (5) is given by

$$
\begin{aligned}
f_1 &\equiv \int_D^{\phi} (z-D)\frac{z}{(1-\phi)\phi}\,dz + \int_{\phi}^{1-\phi} (z-D)\frac{1}{1-\phi}\,dz + \int_{1-\phi}^{1} (z-D)\frac{1-z}{(1-\phi)\phi}\,dz \\
&= \frac{D^3}{6\phi - 6\phi^2} - D + \frac{1}{2}
\end{aligned}
$$

Note that $D \leq \frac{1}{2}$. The derivative with respect to D delivers

$$
\frac{\partial}{\partial D} f_1 = \frac{D^2}{2\phi - 2\phi^2} - 1 < 0
$$

which holds for $D < \phi \leq \frac{1}{2}$.

The derivative with respect to ϕ yields

$$
\frac{\partial}{\partial \phi} f_1 = \frac{D^3(2\phi - 1)}{6(\phi - 1)^2\phi^2} < 0
$$

which holds for $D < \phi \leq \frac{1}{2}$.

Hence the expected payoff when $\phi > D$ is decreasing in ϕ for a fixed D, and decreasing in D for a fixed ϕ.

Case 2: $\boldsymbol{\phi \leq D \leq 1 - \phi}$ Eq. (5) is given by

$$
\begin{aligned}
f_2 &\equiv \int_d^{1-\phi} (z-D)\frac{1}{1-\phi}\,dz + \int_{1-\phi}^{1} (z-D)\frac{1-z}{(1-\phi)\phi}\,dz \\
&= \frac{3(D-1)\phi + 3(D-1)^2 + \phi^2}{6 - 6\phi}
\end{aligned}
$$

Note that $\phi \leq D \quad \& \quad \phi \leq 1 - D$.

The derivative with respect to D delivers

$$
\frac{\partial}{\partial D} f_2 = -\frac{2D + \phi - 2}{2(\phi - 1)} < 0
$$

which holds for $\phi < \frac{1}{2}$ and $\phi \leq D \leq 1-\phi$.

The derivative with respect to ϕ yields

$$\frac{\partial}{\partial \phi} f_2 = \frac{3(D-1)D - (\phi-2)\phi}{6(\phi-1)^2} < 0$$

The derivative changes signs at the threshold $\phi_T(D) = 1 - \sqrt{3D^2 - 3D + 1}$. If $\phi > \phi_T$, $\frac{d\Psi}{d\phi}$ is positive, while if $\phi < \phi_T$, $\frac{d\Psi}{d\phi}$ is negative.

Note that $\phi_T > D$ if $D < \frac{1}{2}$ and $\phi_T > 1 - D$ if $D > \frac{1}{2}$. Since $\phi \leq D$, if $D < \frac{1}{2}$ then $\phi < \phi_T$. Also, since $\phi \leq 1 - D$, if $D > \frac{1}{2}$ then $\phi < \phi_T$. Thus, the derivative with respect to ϕ is negative.

Hence the expected payoff when $\phi \leq D \leq 1-\phi$ is decreasing in ϕ for a fixed D, and decreasing in D for a fixed ϕ.

Case 3: $\boldsymbol{D > 1 - \phi}$ Eq. (5) is given by

$$\begin{aligned} f_3 &\equiv \int_d^1 (z - D)\frac{1-z}{(1-\phi)\phi}\, dz \\ &= \frac{(D-1)^3}{6(\phi-1)\phi} \end{aligned}$$

Note that $D \geq \frac{1}{2}$. The derivative with respect to D delivers

$$\frac{\partial}{\partial D} f_3 = -\frac{(1-D)^2}{2(1-\phi)\phi} < 0$$

which holds for $D < 1$ and $\phi \leq \frac{1}{2}$.

The derivative with respect to ϕ yields

$$\frac{\partial}{\partial \phi} f_3 = -\frac{(1-D)^3(1-2\phi)}{6(1-\phi)^2\phi^2} < 0$$

which also holds for $D < 1$ and $\phi \leq \frac{1}{2}$.

Thus the expected payoff when $D < 1-\phi$ is decreasing in ϕ for a fixed D, and decreasing in D for a fixed ϕ.

All Cases: We conclude from Cases 1, 2 and 3 together that the bank's expected payoff is decreasing in D for a fixed ϕ. The following lemma parallels Lemma 1 for the effect of ϕ in Eq. (5).

Lemma B.1. *For any fixed deposit rate D, the bank's expected payoff in Eq.(5) is decreasing in ϕ.*

B.2 Optimal Deposit Rate and Withdraw Probability

The following lemma characterizes depositors' optimal withdraw probability at date 1, $\omega^* = H(D, \phi)$, as well as the bank's optimal deposit rate, D^*, as a function of the depositors' reservation value r, and bank's diversification level ϕ.

Lemma B.2. *Let the bank's portfolio composition be fixed as a transparent portfolio. For any early redemption value $r \in \left(0, \frac{1}{2}\right)$ and any portfolio allocation $(\phi, (1-\phi))$ that bank hold, the probability, ω^*, that depositors withdraw the bank at date 1, as well as the optimal deposit rate, D^*, under full information are as follows:*

1. $r < \frac{1}{4}$.
   (a) $\phi \leq \frac{2r+2}{5}$, *then* $D^* = \frac{1}{2}(r+1) - \frac{1}{4}\phi \in [\phi, (1-\phi)]$ *and* $\omega^* = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$.
   (b) $\phi > \frac{2r+2}{5}$, *then* $D^* = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} \in (r, \phi)$ *and* $\omega^* = \frac{\left(\frac{1}{3}r+\frac{1}{3}\sqrt{r^2-6\phi^2+6\phi}\right)^2}{2\phi(1-\phi)}$.

2. $r \geq \frac{1}{4}$.
   (a) $\phi \leq \frac{2}{3}(1-r)$, *then* $D^* = \frac{1}{2}(r+1) - \frac{1}{4}\phi \in [\phi, (1-\phi)]$ *and* $\omega^* = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$.
   (b) $\phi > \frac{2}{3}(1-r)$, *then* $D^* = \frac{2}{3}r + \frac{1}{3} \in ((1-\phi), 1)$ *and* $\omega^* = \left(1 - \frac{\left(\frac{2}{3}-\frac{2}{3}r\right)^2}{2\phi(1-\phi)}\right)$.

Proof. Since $\phi \leq \frac{1}{2}$, then $\phi \leq 1-\phi$. Thus there are three relevant cases.

Case 1: $D < \phi$. The expected payoff of each depositors is given by

$$W_B = D\left(1 - \frac{D^2}{2\phi(1-\phi)}\right) + r\frac{D^2}{2\phi(1-\phi)}.$$

The first order condition with respect to D yields

$$\frac{1}{2\phi\left(\phi-1\right)}\left(3D^2 - 2rD - 2\phi\left(1-\phi\right)\right) = 0,$$

which has the following solutions

$$\begin{aligned} D_{1a} &= \frac{1}{3}r - \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} < 0 \\ D_{1b} &= \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} > 0 \end{aligned}$$

For D to be a feasible solutions, it must verify that $r < D < \phi$. We have that

$$\phi > \frac{2r+2}{5} \Rightarrow D_{1b} < \phi.$$

At the same time, $D_{1b} < \phi$ is sufficient to imply that $D_{1b} > r$, as

$$\phi > \frac{2r+2}{5} \Rightarrow \phi > \frac{1}{2} - \frac{1}{2}\sqrt{1-2r^2} \Rightarrow D > r$$

Case 2: $D > (1-\phi)$. The expected payoff of each depositors is given by

$$W_I = D\left(1 - \left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right)\right) + r\left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right).$$

The first order condition with respect to D yields

$$\frac{1}{2\phi\left(\phi-1\right)}\left(D-1\right)\left(2r-3D+1\right) = 0,$$

which admits the following solution

$$D_2 = \frac{2}{3}r + \frac{1}{3}.$$

For D to be a feasible solutions, it is sufficient that it verify that $D > (1-\phi)$, as $(1-\phi) > r$. We have that

$$D_2 > (1-\phi) \Longleftrightarrow \phi > \frac{2}{3}(1-r)$$

Case 3: $D \in [\phi, (1-\phi)]$. The expected payoff of each depositors is given by

$$W_I = D\left(1 - \frac{1}{1-\phi}\left(D - \frac{\phi}{2}\right)\right) + r\frac{1}{1-\phi}\left(D - \frac{\phi}{2}\right).$$

The first order condition with respect to D yields

$$\frac{1}{2(\phi-1)}(\phi - 2r + 4D - 2) = 0,$$

which admits the following solution

$$D_3 = \frac{1}{2}(r+1) - \frac{1}{4}\phi.$$

For D to be a feasible solutions, it is sufficient that it verify that $D_3 \leq (1-\phi)$, $D_3 \geq \phi$, and $D_3 > r$. We have that

$$D_3 \geq \phi \Leftrightarrow \frac{2}{5}(r+1) \geq \phi$$

and

$$D_3 \leq 1-\phi \Leftrightarrow \frac{2}{3}(1-r) \geq \phi.$$

Note that since $2(1-r) > \phi$, it follows that $D_3 > r$ as well.

$\square$

For the ease of exposition, we next summarize the optimal debt rate $D^*(r,\phi)$ and optimal withdraw probability $\omega^*(r,\phi)$ characterized in Lemma B.2:

$$D^*(r,\phi) = \begin{cases} \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} \in (r,\phi) & r < \frac{1}{4} \quad \& \quad \phi > \frac{2r+2}{5} \\ \frac{1}{2}(r+1) - \frac{1}{4}\phi \in [\phi, 1-\phi] & r < \frac{1}{4} \quad \& \quad \phi \leq \frac{2r+2}{5} \quad || \quad r \geq \frac{1}{4} \quad \& \quad \phi \leq \frac{2-2r}{3} \\ \frac{2}{3}r + \frac{1}{3} \in ((1-\phi), 1) & r \geq \frac{1}{4} \quad \& \quad \phi > \frac{2-2r}{3} \end{cases} \tag{B.1}$$

$$\omega^*(r,\phi) = \begin{cases} \frac{\left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2-6\phi^2+6\phi}\right)^2}{2\phi(1-\phi)} & r < \frac{1}{4} \quad \& \quad \phi > \frac{2r+2}{5} \\ \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right) & r < \frac{1}{4} \quad \& \quad \phi \leq \frac{2r+2}{5} \quad || \quad r \geq \frac{1}{4} \quad \& \quad \phi \leq \frac{2-2r}{3} \\ \left(1 - \frac{\left(\frac{2}{3} - \frac{2}{3}r\right)^2}{2\phi(1-\phi)}\right) & r \geq \frac{1}{4} \quad \& \quad \phi > \frac{2-2r}{3} \end{cases} \tag{B.2}$$

Consider the effect of the portfolio allocation ϕ. For the deposit rate, $D^*(r,\phi) = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}$ is increasing in ϕ, while $D^*(r,\phi) = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$ is decreasing in ϕ.

For the withdraw probability, $\omega^*(r,\phi) = \frac{\left(\frac{1}{3}r+\frac{1}{3}\sqrt{r^2-6\phi^2+6\phi}\right)^2}{2\phi(1-\phi)}$ and $\omega^*(r,\phi) = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$ are both decreasing in ϕ. While $\omega^*(r,\phi) = \left(1 - \frac{\left(\frac{2}{3}-\frac{2}{3}r\right)^2}{2\phi(1-\phi)}\right)$ is increasing in ϕ.

Thus, define the threshold $\bar{\phi}(r) = \min\{\frac{2+25}{5}, \frac{2-2r}{3}\}$. The following lemma states that $\bar{\phi}(r)$ determines how D and ω change as a function of ϕ.

Lemma B.3. *There exists a threshold $\bar{\phi}(r)$ such that for any $\phi > \bar{\phi}(r)$ the deposit rate D is increasing in ϕ, and for any $\phi \leq \bar{\phi}(r)$ the deposit rate D is decreasing in ϕ.*

Let $r \geq \frac{1}{4}$. For any $\phi > \bar{\phi}(r)$ the probability of withdraw ω is increasing in ϕ, and for any $\phi \leq \bar{\phi}(r)$ the probability of withdraw is decreasing ω in ϕ.

Let $r < \frac{1}{4}$. The probability of withdraw ω is decreasing in ϕ.

B.3 Equilibrium Level of Diversification

The next lemma characterizes the bank's optimal level of diversification ϕ^* with a transparent portfolio.

Lemma B.4. *Suppose the bank holds a transparent portfolio.*

There exists a value $\bar{r}_T = \frac{7}{18} \approx 0.3889$ *such that for any* $r < \bar{r}_T$*, the bank diversifies according to* $\phi^*(r) = \min\left\{\frac{2+2r}{5}, \frac{2-2r}{3}\right\}$*. If* $r \geq \bar{r}_T$*, the bank does not diversify* $\phi^* = 0$*.*

Proof. We need to calculate the optimal ϕ^* for the bank. That is, ϕ^* that maximizes

$$W_B = \int_D^1 \left[(z-D)\frac{\partial H(z,\phi)}{\partial z}\right] dz.$$

As the optimal deposit rate Eq. (B.1) changes depending on ϕ and r, we need to consider various cases.

Case 1: $D^* < \phi$. $r < \frac{1}{4}$ & $\phi > \frac{2r+2}{5}$. From Lemma B.2, we have that $D^* = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}$. Substituting into the objective function of the social planner, we obtain that

$$\begin{aligned} W_B \;=\; & \int_{D^*}^{\phi} \left[(z-D^*)\frac{\partial}{\partial z}\left(\frac{z^2}{2\phi(1-\phi)}\right)\right] dz + \int_{\phi}^{1-\phi} \left[(z-D^*)\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z-\frac{\phi}{2})\right)\right] dz + \\ & + \int_{1-\phi}^{1} \left[(z-D^*)\frac{\partial}{\partial z}\left(1 - \frac{(1-z)^2}{2\phi(1-\phi)}\right)\right] dz \end{aligned}$$

or

$$W_B = \frac{1}{2} - \left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}\right) + \frac{1}{6}\frac{\left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}\right)^3}{\phi\,(1-\phi)}$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi}W_B = \frac{2\phi - 1}{\sqrt{r^2 - 6\phi^2 + 6\phi}}\left(1 + \frac{1}{162\phi^2}\frac{\left(r + \sqrt{r^2 - 6\phi^2 + 6\phi}\right)^2}{(\phi - 1)^2}\left(r\sqrt{r^2 - 6\phi^2 + 6\phi} + 3\phi^2 - 3\phi + r^2\right)\right)$$

which has a unique solution of $\phi = \frac{1}{2}$. Note however that the second order condition with respect to ϕ evaluated at $\phi = \frac{1}{2}$ yields

$$\begin{aligned}
\left.\frac{\partial^2}{\partial \phi^2} W_B\right|_{\phi=\frac{1}{2}} &= \left.\frac{2r^2+3}{\left(r^2-6\phi^2+6\phi\right)^{\frac{3}{2}}}\right|_{\phi=\frac{1}{2}} + r\left.\left(\frac{\partial}{\partial \phi}\frac{(2\phi-1)}{162\phi^2}\frac{\left(r+\sqrt{r^2-6\phi^2+6\phi}\right)^2}{(\phi-1)^2}\right)\right|_{\phi=\frac{1}{2}} \\
&= \left.\frac{2r^2+3}{\left(r^2-6\phi^2+6\phi\right)^{\frac{3}{2}}}\right|_{\phi=\frac{1}{2}} + r\left.\left(\frac{\left(r+\sqrt{r^2-6\phi^2+6\phi}\right)^2}{(1-\phi)}\frac{1}{81\phi^3}\right)\right|_{\phi=\frac{1}{2}} > 0
\end{aligned}$$

which implies that the solution $\phi = \frac{1}{2}$ of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we need $W_B\left(\frac{2(1+r)}{5}, D^*, r\right)$ and $W_B\left(\frac{1}{2}, D^*, r\right)$. We show that

$$W_B\left(\frac{2(1+r)}{5}, D^*, r\right) - W_B\left(\frac{1}{2}, D^*, r\right) > 0.$$

Thus,

$$\phi^* = \frac{2(1+r)}{5}.$$

Notice that the bank's expected payoff is everywhere decreasing in this region. That's because for any $\phi < \frac{1}{2}$, $\frac{\partial}{\partial \phi} W_B < 0$. Thus this justify why the optimal ϕ^* is the lowest possible value (lower bound). Indeed, for any $r < \frac{1}{4}$ (which holds), $\frac{2(1+r)}{5} < \frac{1}{2}$.

Case 2: $\boldsymbol{D^* > 1-\phi}$. $r \geq \frac{1}{4}$ & $\phi > \frac{2}{3}(1-r)$. Lemma B.2, we have that $D^* = \frac{2}{3}r + \frac{1}{3}$. Substituting into the objective function of of bank, we obtain that

$$W_B = \int_{D^*}^{1}\left((z-D^*)\frac{\partial}{\partial z}\left(1-\frac{(1-z)^2}{2\phi(1-\phi)}\right)\right)dz$$

or

$$W_B = \frac{1}{6\phi(\phi-1)}\left(\frac{2}{3}r - \frac{2}{3}\right)^3$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial\phi}W_B = \frac{4}{81\phi^2\left(\phi-1\right)^2}\left(2\phi-1\right)\left(1-r\right)^3 \leq 0.$$

which has a unique solution of $\phi = \frac{1}{2}$. Note however that the second order condition with respect to ϕ evaluated at $\phi = \frac{1}{2}$ yields

$$\begin{aligned}\left.\frac{\partial^2}{\partial\phi^2}W_B\right|_{\phi=\frac{1}{2}} &= \left.\frac{8}{81\phi^3\left(1-\phi\right)^3}\left(1-r\right)^3\left(3\phi^2-3\phi+1\right)\right|_{\phi=\frac{1}{2}} \\ &= \frac{1}{81}(-128)(r-1)^3 > 0\end{aligned}$$

which implies that the solution $\phi = \frac{1}{2}$ of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we need $W_B\left(\frac{2(1-r)}{3}, D^*, r\right)$ and $W_B\left(\frac{1}{2}, D^*, r\right)$. We show that

$$W_B\left(\frac{2\left(1-r\right)}{3}, D^*, r\right) - W_B\left(\frac{1}{2}, D^*, r\right) > 0.$$

Thus,

$$\phi^* = \frac{2\left(1-r\right)}{3}.$$

Note bank's expected payoff is always decreasing in this case.

Case 3: $\boldsymbol{D^* \in [\phi, 1-\phi]}$. $r < \frac{1}{4}$ & $\phi \leq \frac{2r+2}{5}$, and $r \geq \frac{1}{4}$ & $\phi \leq \frac{2}{3}\left(1-r\right)$.

From Lemma B.2, we have that $D^* = \frac{1}{2}\left(r+1\right) - \frac{1}{4}\phi$. Substituting into the objective function of bank, we obtain that

$$W_B = \int_{D^*}^{1-\phi}\left[(z-D^*)\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z-\frac{\phi}{2})\right)\right]dz + \int_{1-\phi}^{1}\left[(z-D^*)\frac{\partial}{\partial z}\left(1-\frac{(1-z)^2}{2\phi(1-\phi)}\right)\right]dz$$

or

$$W_B = \frac{1}{96\left(1-\phi\right)}\left(12\left(r-1\right)^2 + 12\phi\left(r-1\right) + 7\phi^2\right)$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_B = \frac{1}{96\left(\phi-1\right)^2}\left(12r^2 - 12r - 7\phi^2 + 14\phi\right).$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial \phi^2} W_B = \frac{1}{48\left(1-\phi\right)^3}\left(12r^2 - 12r + 7\right) > 0$$

which implies that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank i's objective we need to compare the value of W_B evaluated at the corners. In particular, for $r < \frac{1}{4}$ we need to compare $W_B\left(\frac{2r+2}{5}, D^*, r\right)$ and $W_B\left(0, D^*, r\right)$, while for $r \geq \frac{1}{4}$ we need to compare $W_B\left(\frac{2(1-r)}{3}, D^{*/T}, r\right)$ and $W_B\left(0, D^*, r\right)$.

Consider first $r < \frac{1}{4}$. Then

$$W_B\left(\frac{2r+2}{5}, D^*, r\right) - W_B\left(0, D^*, r\right) = -\frac{1}{240r - 360}\left(30r^3 + 7r^2 - 16r + 7\right) > 0.$$

Thus

$$\phi^* = \frac{2r+2}{5}.$$

Consider next $r \geq \frac{1}{4}$. Then

$$W_B\left(\frac{2\left(1-r\right)}{3}, D^*, r\right) - W_B\left(0, D^*, r\right) = -\frac{1}{72\left(2r+1\right)}\left(18r-7\right)\left(r-1\right)^2.$$

Thus, if $r < \bar{r}_T = \frac{7}{18}$,

$$\phi^* = \frac{2\left(1-r\right)}{3},$$

and, if $r > \bar{r}_T$,

$$\phi^* = 0.$$

Notice that the bank's objective is non-monotonic in ϕ. Setting the first order condition with respect to ϕ to 0 yields $\phi_0 = 1 - \frac{\sqrt{12r^2-12r+7}}{\sqrt{7}}$. For $\phi < \phi_0$, $\frac{\partial}{\partial\phi}W_B < 0$ and so bank's expected payoff is decreasing in ϕ. For $\phi > \phi_0$, $\frac{\partial}{\partial\phi}W_B > 0$ and so bank's expected payoff is increaing in ϕ.

However, both ϕ^* for $r < \frac{1}{4}$ and for $r \geq \frac{1}{4}$ are greater than ϕ_0. This corresponds to the region in which the payoff is increasing in ϕ, justifying why the optimal ϕ^* is the highest possible value (upper bound).

All Cases: The derivations of all cases together imply that for $r < \bar{r}_T$ bank's expected payoff is maximized at $\phi^*(r) = \min\left\{\frac{2r+2}{5}, \frac{2-2r}{3}\right\}$. More precisely, for $r < \frac{1}{4}$, $\phi^*(r) = \frac{2r+2}{5}$ (Cases 1 and 3); for $\frac{1}{4} \leq r \leq \bar{r}_T$, $\phi^*(r) = \frac{2(1-r)}{3}$ (Cases 2 and 3); and for $r > \bar{r}_T$, $\phi^* = 0$ (Case 3),

$$\phi^*(r) = \begin{cases} \dfrac{2r+2}{5} & 0 < r \leq \frac{1}{4} \\ \dfrac{2-2r}{3} & \frac{1}{4} < r < \bar{r}_T \\ 0 & r \geq \bar{r}_T \end{cases} \tag{B.3}$$

Note that for any portfolio allocation $(\phi, 1-\phi)$ that is an equilibrium, the portfolio allocation $(1-\phi, \phi)$ is also an equilibrium. This follows from the symmetry of the bank' portfolio return distribution in the full information case. We will work with $(1-\phi, \phi)$ to make it comparable with the analysis with incomplete information. Thus, bank's expected payoff is maximized at $\phi^* = \max\left\{\frac{3-2r}{5}, \frac{21+2r}{3}\right\}$ if $r \leq \bar{r}_T$ and at $\phi^* = 1$ if $r > \bar{r}_T$.

$\square$

With the optimal portfolio allocation in hand, we can find the equilibrium deposit rate $D^*(r) = D^*(\phi^*(r), r)$ and probability of withdraw $\omega^*(r) = \omega^*(\phi^*(r), r)$. We substitute $\phi^*(r)$ into $D^*(\phi^*(r), r)$. When $\phi^*(r) = \frac{2-2r}{3}$, then $D^*(r) = \frac{2r+1}{3}$. When $\phi^*(r) = \frac{2+2r}{5}$, $D^*(r) = \frac{2+2r}{5}$. Lastly, when when $\phi^* = 0$, $D^*(r) = \frac{r+1}{2}$.

Similarly, we substitute $\phi^*(r)$ into $\omega^*(\phi^*(r), r)$. When $\phi^*(r) = \frac{(2-2r)}{3}$, then $\omega^*(r) = \frac{2r^2+r}{3}$. When $\phi^*(r) = \frac{(2r+2)}{5}$, $\omega^*(r) = \frac{3+r-2r^2}{25}$. Lastly, when when $\phi^* = 0$, $\omega^*(r) = \frac{r+1}{2}$.

The next result is a corollary of Lemma B.4.

Lemma B.5. *Consider the equilibrium with a transparent portfolio.*

For $r < \frac{1}{4}$*,* $D^*(r) = \phi^*(r)$*.*

For $r \in [\frac{1}{4}, \bar{r}_T)$*,* $D^*(r) = 1 - \phi^*(r)$ *.*

For $r > \bar{r}_T$*,* $D^*(r) \in (\frac{1}{2}, \frac{3}{4})$ *and* $\phi^* = 0$*.*

B.4 Proof of Proposition 1

The result in the proposition follows from the results in Lemma B.2 and Lemma B.4.

Lemma B.4 immediately implies that $\phi^*(r) > 0$ only for $r < \bar{r}_T$, and $\phi^* = 0$ otherwise. In other words, the bank diversifies ($\phi^* \neq \{0, 1\}$) if $r < \bar{r}_T$. This proves the second statement in Proposition 1.

We next need to prove the first statement in Proposition 1: for $r < \bar{r}_T$, $\phi^*(r)$ minimizes the deposit rate D. That is,

$$\phi^*(r) = \arg\min_{\phi} D^*(\phi, r)$$

We use the results in Lemma B.2 and Lemma B.4. Let $r < \frac{1}{4}$. First, consider when $\phi \leq \frac{2+2r}{5}$. In this case, $D^*(\phi, r) = \frac{1}{2}(r+1) - \frac{1}{4}\phi$ is decreasing in ϕ. Thus by choosing the upper bound $\phi^* = \frac{2+2r}{5}$ the bank minimizes D^*. When $\phi > \frac{2+2r}{5}$, the deposit rate is $D^*(\phi, r) = \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} + \frac{r}{3}$ which is increasing in ϕ. Thus by choosing the lower bound $\phi^* = \frac{2+2r}{5}$ the bank minimizes D.

Next let $r \in [\frac{1}{4}, \bar{r}_T)$. Consider when $\phi \leq \frac{2-2r}{3}$. In this case, the deposit rate is again $D^*(\phi, r) = \frac{1}{2}(r+1) - \frac{1}{4}\phi$. By choosing the upper bound $\phi^* = \frac{2-2r}{3}$ the bank minimizes D. If $\phi > \frac{2-2r}{3}$, the deposit rate is $D^*(r) = \frac{2r}{3} + \frac{1}{3}$ which does not depend on ϕ.

Thus, for any $r < \bar{r}_T$, the bank chooses a portfolio allocation ϕ^* that minimizes $D^*(\phi)$.

C Unobservable Portfolio

In this section, we provide the results for the benchmark case of the unobservable portfolio (Subsection 3.2).

First, we characterize the equilibrium outcome. We start by solving for the optimal deposit rate D^* and depositors' optimal withdrawal decision $\omega^*(\mathbf{s})$. Then, we solve for the optimal portfolio allocation $(\phi^*, 1-\phi^*)$ (see the solution concept Subsection 2.3).

Second, we show that the equilibrium allocation $\phi^*(r)$ minimizes the deposit rate, proving Proposition 2.

C.1 Optimal Deposit Rate and Withdraw Probability

Lemma C.6. *Let the bank's portfolio composition be fixed as an unobservable portfolio. For any early redemption value $r \in \left(0, \frac{1}{2}\right)$ and any portfolio allocation $(\phi, (1-\phi))$ that bank hold, the optimal deposit rate, D^*, as well as the probability, ω^*, that depositors withdraw the bank at date 1 are as follows:*

1. $r < 0.25$
   (a) *If* $\phi \leq \frac{2}{5}$, *then* $D^* = \frac{2-\phi}{4} \in (\phi, 1-\phi)$ *and* $\omega^* = 0$
   (b) *If* $\phi \in \left(\frac{2}{5}, \frac{1}{2}\right]$ *then* $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$ *and* $\omega^* = 0$
   (c) *Else, otherwise there is no D for which the depositors are willing to continue the bank, i.e* $\omega^* = 1$.
2. $r \in [0.25, \frac{4}{15})$
   (a) *If* $\phi \in (\frac{2}{5}, \frac{1}{2})$, *then* $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$ *and* $\omega^* = 0$
   (b) *Else, otherwise there is no D for which the depositors are willing to continue the bank, i.e* $\omega^* = 1$.
3. $r \in [\frac{4}{15}, \sqrt{\frac{2}{27}})$

(a) *If* $\phi \in \left(\frac{1}{2} - \frac{\sqrt{2-27r^2}}{2\sqrt{2}}, \frac{1}{2}\right)$, *then* $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$ *and* $\omega^* = 0$

(b) *Else, otherwise there is no* D *for which the depositors are willing to continue the bank, i.e* $\omega^* = 1$.

4. *For* $r \geq \sqrt{\frac{2}{27}}$ *there is no* D *for which the depositors are willing to continue the bank, i.e* $\omega^* = 1$.

Proof. Since $\phi \leq \frac{1}{2}$, then $\phi \leq 1 - \phi$. Thus there are three relevant cases.

Case 1: $D < \phi$. The expected payoff of each investor is given by

$$W_I = D\left(1 - \frac{D^2}{2\phi(1-\phi)}\right).$$

The first order condition with respect to D yields

$$\frac{\partial}{\partial D}\left(D\left(1 - \frac{D^2}{2\phi(1-\phi)}\right)\right) = -\frac{1}{2\phi\left(1-\phi\right)}\left(2\phi^2 - 2\phi + 3D^2\right) = 0$$

which has the following solutions

$$\begin{aligned} D_{1a} &= \sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)} > 0 \\ D_{1b} &= -\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)} < 0 \end{aligned}$$

Second order consdition implies

$$\frac{\partial}{\partial D}\left(-\frac{1}{2\phi\left(1-\phi\right)}\left(2\phi^2 - 2\phi + 3D^2\right)\right) = -\frac{3}{\phi}\frac{D}{1-\phi} < 0$$

For D to be a feasible solutions, it must verify that $D < \phi$ and $D(1 - H(D, \phi)) > r$. We have that

$$\begin{aligned} D_{1a} &\leq \phi \Leftrightarrow \frac{2}{3}\phi\left(1-\phi\right)-\phi^{2}=\frac{1}{3}\phi\left(2-5\phi\right)\leq 0 \\ \text{or } \phi &\geq \frac{2}{5}. \end{aligned}$$

At the same time

$$\begin{aligned} D_{1a}(1-H(D_{1a},\phi)) &> r \Longleftrightarrow \\ \frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)} &> r \end{aligned}$$

which is true for any $\phi \in \left(\frac{1}{2}-\frac{1}{4}\sqrt{2}\sqrt{2-27r^{2}}, \frac{1}{4}\sqrt{2}\sqrt{2-27r^{2}}+\frac{1}{2}\right)$ which are the positive roots of

$$\frac{8}{27}\phi\left(1-\phi\right)-r^{2}=0$$

for any $r<\sqrt{\frac{2}{27}}$. We have that $\frac{1}{4}\sqrt{2}\sqrt{2-27r^{2}}+\frac{1}{2}>\frac{1}{2}$ and

$$\frac{1}{2}-\frac{1}{4}\sqrt{2}\sqrt{2-27r^{2}}<\frac{2}{5} \text{ for } r<\frac{4}{15}.$$

Thus, if $r<\frac{4}{15}$ and $\phi \in \left(\frac{2}{5},\frac{1}{2}\right)$, then we have a solution as

$$D_{1a}=\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)}.$$

If $r \in \left(\frac{4}{15},\sqrt{\frac{2}{27}}\right)$ and $\phi \in \left(\frac{1}{2}-\frac{1}{4}\sqrt{2}\sqrt{2-27r^{2}},\frac{1}{2}\right)$ then we have a solution as

$$D_{1a}=\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)}.$$

If $r>\sqrt{\frac{2}{27}}$then

$$\frac{8}{27}\phi\left(1-\phi\right)-r^{2}<0$$

or

$$D_{1a}(1 - H(D_{1a}, \phi)) < r$$

Since D_{1a} maximizes $D(1 - H(D, \phi))$, then there is no D for which investors are willing to continue the bank.

Case 2: $D \geq (1 - \phi)$. The expected payoff of each investor is given by

$$W_I = D\left(1 - \left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right)\right).$$

The first order condition with respect to D yields

$$-\frac{1}{2\phi\left(\phi - 1\right)}\left(3D^2 - 4D + 1\right) = 0$$

which admits the following solution

$$D_{2a} = \frac{1}{3} \text{ and } D_{2b} = 1.$$

Note that D_{2b} is actually an inflexion point, and that W_I is decreasing between $\left(\frac{1}{3}, 1\right)$. For D_{2a} to be a feasible solutions, it is sufficient that it verify that $D > (1 - \phi)$. However note that $(1 - \phi) > \frac{1}{2}$. This implies that W_I achieves a maximum when $D^* = (1 - \phi)$.

We need to verify

$$D\left(1 - \left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right)\right) > r$$

or

$$(1-\phi)\left(1 - \left(1 - \frac{(1-(1-\phi))^2}{2\phi(1-\phi)}\right)\right) = \frac{1}{2}\phi > r \Longleftrightarrow \phi > 2r$$

If $r > \frac{1}{4}$, then this condition is violated. For $r < \frac{1}{4}$ note that

$$2r < \frac{1}{4}\sqrt{2}\sqrt{2 - 27r^2} + \frac{1}{2}$$

which implies that for $\phi \in \left(2r, \left(\frac{1}{4}\sqrt{2}\sqrt{2-27r^2}+\frac{1}{2}\right)\right)$ we have two candidate solutions

$$D_{2a} = (1-\phi)$$

and

$$D_{1a} = \sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)}$$

We need to compare the payoff of investors

$$\begin{aligned} W_I\left(D_{1a}\right) &= \sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)}\left(1-\frac{\left(\sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)}\right)^2}{2\phi(1-\phi)}\right) \\ &= \frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)} \end{aligned}$$

while

$$W_I\left(D_{2a}\right) = \frac{1}{2}\phi$$

$$\begin{aligned} W_I\left(D_{2a}\right) &< W_I\left(D_{1a}\right) \Longleftrightarrow \\ \frac{1}{2}\phi - \frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)} &< 0 \end{aligned}$$

$$\left(\frac{1}{2}\phi\right)^2 - \left(\frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)}\right)^2 = \frac{1}{4}\phi - \frac{8}{27}\,(1-\phi) = \frac{59}{108}\phi - \frac{32}{108} < 0 \Longleftrightarrow \phi < \frac{32}{59}$$

which is true, since $\phi < \frac{1}{2} < \frac{32}{59}$.

Case 3: $D \in [\phi, (1-\phi)]$. The expected payoff of each investor is given by

$$W_I = D\left(1-\frac{1}{1-\phi}\left(D-\frac{\phi}{2}\right)\right).$$

The first order condition with respect to D yields

$$\frac{1}{2(\phi-1)}(\phi+4D-2)=0$$

which admits the following solution

$$D_3=\frac{1}{4}(2-\phi).$$

For D to be a feasible solutions, it is sufficient that it verify that $D_3 \leq (1-\phi)$, $D_3 \geq \phi$, and $D_3(1-H(D_3,\phi)) > r$. We have that

$$\frac{1}{4}(2-\phi) > \phi \Leftrightarrow \frac{2}{5} > \phi$$

and

$$D_3 \leq 1-\phi \Leftrightarrow \frac{1}{4}(2-\phi) \leq 1-\phi \Leftrightarrow \phi < \frac{2}{3}$$

and

$$\begin{aligned} D\left(1-\frac{1}{1-\phi}\left(D-\frac{\phi}{2}\right)\right) &> r \Leftrightarrow \\ \frac{1}{4}(2-\phi)\left(1-\frac{1}{1-\phi}\left(\frac{1}{4}(2-\phi)-\frac{\phi}{2}\right)\right) &= -\frac{(\phi-2)^2}{16\phi-16} > r \end{aligned}$$

which is always true for $r < \frac{1}{4}$. If $r > \frac{1}{4}$, there is no D for which the investor is willing to continue the bank.

Note again that $\phi \in \left(2r, \frac{2}{5}\right)$ we have two candidate solutions

$$D_{2a}=(1-\phi)$$

and

$$D_3=\frac{1}{4}(2-\phi).$$

We need to compare the payoff of investors

$$\begin{aligned} W_I(D_3) &= \frac{1}{4}(2-\phi)\left(1-\frac{1}{1-\phi}\left(\frac{1}{4}(2-\phi)-\frac{\phi}{2}\right)\right) \\ &= \frac{(\phi-2)^2}{16(1-\phi)} \end{aligned}$$

while

$$W_I(D_{2a}) = \frac{1}{2}\phi$$

$$\begin{aligned} W_I(D_{2a}) &< W_I(D_3) \Longleftrightarrow \\ \frac{(\phi-2)^2}{16(1-\phi)} - \frac{1}{2}\phi &= \frac{1}{16(1-\phi)}(3\phi-2)^2 > 0 \end{aligned}$$

Thus, for $\phi < \frac{2}{5}$, the solution is

$$D_3 = \frac{1}{4}(2-\phi).$$

□

All together:

$$D^*(\phi, r) = \begin{cases} \frac{2-\phi}{4} & r < 0.25 \quad \& \quad \phi \le \frac{2}{5} \\ \sqrt{\frac{2}{3}}\sqrt{(1-\phi)\phi} & \left(r < 0.25 \quad \& \quad \frac{2}{5} < \phi \le \frac{11}{20}\right) \mid\mid \left(0.25 \le r < \frac{4}{15} \quad \& \quad \frac{2}{5} < \phi < \frac{1}{2}\right) \mid\mid \\ & \left(\frac{4}{15} \le r < \sqrt{\frac{2}{27}} \quad \& \quad \frac{1}{2} - \frac{\sqrt{2-27r^2}}{2\sqrt{2}} < \phi < \frac{1}{2}\right) \end{cases}$$

and $\omega^*(\phi, r) = 0$ whenever $D^*(\phi, r)$ exists.

C.2 Equilibrium Level of Diversification

Lemma C.7. *Suppose the bank can only hold an unobservable portfolio.*

There exists $\bar{r}_{U_{low}} = \frac{4}{15}$ *and* $\bar{r}_U = \sqrt{\frac{2}{27}}$*, with* $\bar{r}_{U_{low}} < \bar{r}_U$*, such that if* $r < \bar{r}_{U_{low}}$*, then* $\phi^* = \frac{2}{5}$*. If* $\bar{r}_{U_{low}} \le r < \bar{r}_U$*, then* $\phi^* = \frac{1}{2} - \frac{\sqrt{2}}{4}\sqrt{2-27r^2}$*. If* $r \ge \bar{r}_U$*, then* ϕ^* *is undetermined.*

Notice that if r is too high ($r \geq \bar{r}_U$), then the bank does not invest in any risky project. That's because, in this case, there exists no D for which the depositors are willing to continue funding the bank until date 2 (Lemma C.6).

Proof. We need to calculate the optimal ϕ^* for the bank. That is, ϕ^* that maximizes

$$W_B = \int_D^1 \left[(z - D) \frac{\partial H(z, \phi)}{\partial z} \right] dz.$$

As the optimal depositor rate D^* changes depending on ϕ and r, we need to consider various cases, following Lemma C.6.

Case 1a $\quad r < 0.25$ and $\phi \leq \frac{2}{5}$

From Lemma C.6, we have that $D^* = \frac{(2-\phi)}{4} \in (\phi, 1-\phi)$. Substituting into the objective function of bank, we obtain that

$$W_B = \int_{D^*}^{1-\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi} (z - \frac{\phi}{2}) \right) \right] dz + \int_{1-\phi}^{1} \left[(z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

or

$$W_B = \frac{12 - 12\phi + 7\phi^2}{96(1-\phi)}$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_B = -\frac{7(\phi - 2)\phi}{96(\phi - 1)^2} > 0$$

which implies that the payoff of the bank is always increasing, and hence bank chooses

$$\phi^* = \frac{2}{5}$$

This implies

$$D^* = \frac{2}{5}$$

and it holds that $D^* \geq \phi^*$.

Note that the expected payoff of the depositors is

$$D^* \left(1 - \frac{1}{1-\phi^*}(D^* - \frac{\phi^*}{2})\right) = \frac{4}{15} > r \quad \forall r < 0.25$$

and bank's payoff is

$$W_B(\phi^*) = \frac{13}{10}$$

Case 1b $r < 0.25$ and $\phi \in (\frac{2}{5}, \frac{1}{2}]$

From Lemma C.6, we have that $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$. Substituting into the objective function of bank, we obtain that

$$W_B = \int_{D^*}^{\phi} \left[(z - D^*)\frac{\partial}{\partial z}\left(\frac{z^2}{2\phi(1-\phi)}\right)\right] dz + \int_{\phi}^{1-\phi} \left[(z - D^*)\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z - \frac{\phi}{2})\right)\right] dz$$
$$+ \int_{1-\phi}^{1} \left[(z - D^*)\frac{\partial}{\partial z}\left(1 - \frac{(1-z)^2}{2\phi(1-\phi)}\right)\right] dz$$

or

$$W_B = \frac{1}{2} - \frac{8}{9}\sqrt{\frac{2}{3}}\sqrt{(1-\phi)\phi}$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_B = \frac{4\sqrt{\frac{2}{3}}(2\phi - 1)}{9\sqrt{(1-\phi)\phi}}$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial^2 \phi} W_B = \frac{2\sqrt{\frac{2}{3}}}{9(-((\phi-1)\phi))^{3/2}} > 0$$

which implies that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we compare

$$W_B\left(\frac{2}{5}\right) - W_B\left(\frac{1}{2}\right) = 0.144444 - 0.137113 > 0$$

Hence bank chooses

$$\phi^* = \frac{2}{5}$$

This implies

$$D^* = \frac{2}{5}$$

which is the same solution as Case 1a.

Note that the expected payoff of the depositors is

$$D^*\left(1 - \frac{(D^*)^2}{2\phi^*(1-\phi^*)}\right) = \frac{4}{15} > r \quad \forall r < 0.25$$

and bank's payoff is

$$W_B(\phi^*) = \frac{13}{10}$$

Case 2 $\quad r \in [0.25, \frac{4}{15}]$ and $\phi \in (\frac{2}{5}, \frac{1}{2})$

This is the same as Case 1b:

$$\phi^* = \frac{2}{5}$$
$$D^* = \frac{2}{5}$$

Case 3 $\quad r \in [\frac{4}{15}, \frac{\sqrt{2}}{\sqrt{27}})$ and $\phi \in (1 - \frac{1}{4}\sqrt{2}\sqrt{2 - 27r^2}, \frac{1}{2})$

From Lemma C.6, we have that $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$. This is the same as Case 1b with different intervals for r and ϕ.

We know that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we compare

$$W_B\left(\frac{1}{2}\right) - W_B\left(1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}\right) = 0.137113 - 0.16667\left(3 - 8\sqrt{r^2}\right)$$

Note that $W_B\left(1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}\right)$ is decreasing in r, and at the r threshold values we have

$$W_B\left(1 - \frac{1}{4}\sqrt{2}\sqrt{2 - 27\cdot\left(\frac{4}{15}\right)^2}\right) = 0.144444$$
$$W_B\left(1 - \frac{1}{4}\sqrt{2}\sqrt{2 - 27\cdot\left(\frac{\sqrt{2}}{\sqrt{27}}\right)^2}\right) = 0.137113$$

Hence for any $r < \frac{\sqrt{2}}{\sqrt{27}}$, $W_B\left(\frac{1}{2}\right) - W_B\left(1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}\right) < 0$ and bank chooses

$$\phi^* = 1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}$$

This implies

$$D^* = \frac{3r}{2}$$

Note that the expected payoff of the depositors is

$$D^*\left(1 - \frac{(D^*)^2}{2\phi^*(1-\phi^*)}\right) = r$$

and the bank's payoff is

$$W_B(\phi^*) = \frac{1}{6}(3 - 8r)$$

$\square$

Combining Lemma C.6 and Lemma C.7, the equilibrium $D^*(r) = D(\phi^*(r), r)$ and $\phi^*(r)$ are functions of r only,

$$(D^*(r), \phi^*(r)) = \begin{cases} \left(\frac{2}{5}, \frac{2}{5}\right) & if \quad r < \frac{4}{15} \\ \left(\frac{3r}{2}, \frac{1}{2} - \frac{\sqrt{2-27r^2}}{2\sqrt{2}}\right) & if \quad \frac{4}{15} \leq r < \sqrt{\frac{2}{27}} \\ \text{Undefined} & if \quad r \geq \sqrt{\frac{2}{27}} \end{cases}$$

with $D^*(r) \geq \phi^*(r)$.

C.3 Proof of Proposition 2

The proposition follows from the results in Lemma C.6 and Lemma C.7.

Let $r < \frac{1}{4}$. First, consider when $\phi \leq \frac{2}{5}$ (Case 1a). The optimal debt rate $D^* = \frac{2-\phi}{4} \in (\phi, 1-\phi)$ is decreasing in ϕ. By choosing the upper bound $\phi^* = \frac{2}{5}$ the bank minimizes the deposit rate D^*. Second, consider when $\phi \in (\frac{2}{5}, \frac{1}{2}]$ (Case 1b). The optimal debt rate $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)}$ is increasing in ϕ. Hence, by choosing the lower bound $\phi^* = \frac{2}{5}$ the bank minimizes the deposit rate D.

Next, let $r \in (\frac{1}{4}, \bar{r}_{U_{low}}]$ and $\phi \in (\frac{2}{5}, \frac{1}{2})$ (Case 2). In this case, the optimal debt rate is also $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)}$. By choosing the lower bound $\phi^* = \frac{2}{5}$ the bank minimizes the deposit rate D.

Lastly, let $r \in [\bar{r}_{U_{low}}, \bar{r}_U)$ and $\phi \in (1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}, \frac{1}{2})$ (Case 3). In this case, the optimal debt rate is also $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)}$. By choosing the lower bound $\phi^* = 1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}$ the bank minimizes the deposit rate D.

Thus, for any $r < \bar{r}_U$, the bank chooses a portfolio allocation ϕ^* that minimizes $D^*(\phi)$.

This proves the first statement in Proposition 2.

The second statement in Proposition 2 is straightforward. If $r > \bar{r}_U$, there exists no deposit rate D for which the probability of withdrawal ω^* is not equal to 1. Hence, ϕ^* is indeterminate. In other words, $\omega^* = 0$ whenever there exists a deposit rate $D^* > 0$.

D Opaque Portfolio

In this section, we provide the results for the case of an opaque portfolio, discussed in section 4. This is the case when the bank holds one transparent project and one unobservable project. Without loss of generality, we let ϕ be the allocation in the transparent project and $1-\phi$ in the unobservable one.

First, we describe the environment in detail. Second, we characterize the equilibrium outcome. We start by solving for the optimal deposit rate D^* and depositors' optimal withdrawal decision $\omega^*(\mathbf{s})$. Then, we solve for the optimal portfolio allocation $(\phi^*, 1-\phi^*)$. Lastly, we prove Example 1 discussed in Section 4.

D.1 The Environment

With an opaque portfolio, the depositors can partially infer the bank's portfolio return as they observe $\mathbf{s} = (R_1, \varnothing)$. Depositors find it optimal to continue funding the bank if the amount they expect to receive at date 2, given that their signal is $\mathbf{s}$, $D \cdot \Pr\left(D \leq V\left(\phi\right)|\mathbf{s}\right)$, is larger than the reservation value r, which they obtain when they liquidate the bank at date 1. Depositors' decision to continue funding the bank depends thus on how high the deposit rate is, as well as on how high the probability that the they get repaid. Indeed, if the realization of R_1 is sufficiently high that $D < \phi R_1$, then the depositors gets repaid D with certainty. However, if $D \geq \phi R_1$, then the depositors gets repaid D only if $R_2 \geq \frac{D-\phi R_1}{1-\phi}$, which occurs with probability $\left(1 - G\left(\frac{D-\phi R_1}{1-\phi}\right)\right)$. Otherwise, the the bank goes into default at date 2, and the depositor gets 0. Thus, everything else equal, the higher the face value of debt is, the lower the probability that the depositors get repaid when $D \geq \phi R_1$.

This implies that depositors find it optimal to continue funding the bank when

$$\Pr\left(R_2 \geq \max\left\{\frac{D-\phi R_1}{1-\phi}, 0\right\}\middle| R_1\right) \geq \frac{r}{D},$$

or when their signal is sufficiently large.

It follows that, for each portfolio allocation, $\phi \in (0,1)$, and each deposit rate, D, set at date 0, the optimal withdraw decision can be characterized by a threshold strategy

$$\omega^*(\mathbf{s}) = \begin{cases} 1 & \text{if } R_1 < \bar{\omega} \\ 0 & \text{if } R_1 \geq \bar{\omega} \end{cases}, \tag{D.1}$$

where

$$\bar{\omega} = \max\left\{\min\left\{1, \frac{D}{\phi} - \frac{1-\phi}{\phi}G^{-1}\left(1 - \frac{r}{D}\right)\right\}, 0\right\}. \tag{D.2}$$

Thus, $\bar{\omega}$ can be interpreted as the probability that depositors withdraw their money from the bank at date 1, under the assumption that the return of the projects is uniformly distributed[10]. That is, it represents the withdraw probability $\omega^*(\mathbf{s})$. For the sake of notation, we use ω^* and $\bar{\omega}$ interchangeably for an opaque portfoio.

At this point, it is useful to restate depositors' expected payoff and the bank expected payoff (See Definition 1) in terms of $\bar{\omega}$. We use these specifications in the subsequent proofs in this section.

Depositors' expected payoff is given by

$$W_I = D \cdot \mathbb{E}_{R_1}\left(\Pr\left(R_2 \geq \frac{D^* - \phi R_1}{1-\phi}\right) \middle| R_1 \geq \bar{\omega}\right) \Pr\left(R_1 \geq \bar{\omega}\right) + r \cdot \Pr\left(R_1 < \bar{\omega}\right). \tag{D.3}$$

The first term on the right-hand side of (D.3) represents depositors' expected payoff provided they continues funding the bank. The second term on the right-hand side of (D.3) represents depositors' expected payoff provided they withdraw from the bank at date 1.

For each ϕ, and $\bar{\omega}$ and D^* associated with that ϕ, the bank 's expected payoff is given by

$$W_B = \mathbb{E}_{R_1}\left\{\mathbb{E}_{R_2}\left\{\left(\phi R_1 + (1-\phi)R_2 - D^*\right) \middle| R_2 \geq \frac{D^* - \phi R_1}{(1-\phi)}\right\} \middle| R_1 \geq \bar{\omega}\right\} \Pr\left(R_1 \geq \bar{\omega}\right).$$

[10] Note that this threshold reveals that depositors' withdraw decision does not depend monotonically on D.

Isolating D^*, and substituting the depositors' payoff, we can write the above as

$$W_B = \mathbb{E}_{R_1}\left\{\phi R_1 + (1-\phi)\,\mathbb{E}_{R_2}\left\{R_2 \middle| R_2 \geq \frac{D^* - \phi R_1}{(1-\phi)}\right\} \middle| R_1 \geq \bar{\omega}\right\} \Pr\left(R_1 \geq \bar{\omega}\right) \quad \text{(D.4)}$$

$$-D^* \cdot \mathbb{E}_{R_1}\left(\Pr\left(R_2 \geq \frac{D^* - \phi R_1}{1-\phi}\right) \middle| R_1 \geq \bar{\omega}\right) \Pr\left(R_1 \geq \bar{\omega}\right). \quad \text{(D.5)}$$

The first term in (D.4) represents the total surplus that is expected to be realized when depositors continue to fund the bank . The last term is simply the payoff that depositors expect to receive if they continue the the bank. Thus, in equilibrium, the bank chooses ϕ such that the expected marginal total surplus equals the marginal depositors' payoff.

The optimal allocation ϕ^* solves

$$\phi^* = \arg\max \left\{ \int_{\bar{\omega}}^{1} \int_{\max\left\{\frac{D^* - \phi R_1}{1-\phi}, 0\right\}}^{1} \left(\phi R_1 + (1-\phi) R_2 - D^*\right) dR_2 dR_1 \right\}.$$

D.2 Optimal Deposit Rate and Withdraw Probability

Making use of (D.1), we obtain that the deposit rate satisfies the first order condition

$$\frac{1}{D} = -\frac{\frac{\partial}{\partial D}\mathbb{E}_{R_1}\left(\Pr\left(R_2 \geq \frac{D^* - \phi R_1}{1-\phi}\right) \middle| R_1 \geq \bar{\omega}\right) \Pr\left(R_1 \geq \bar{\omega}\right)}{\mathbb{E}_{R_1}\left(\Pr\left(R_2 \geq \frac{D^* - \phi R_1}{1-\phi}\right) \middle| R_1 \geq \bar{\omega}\right) \Pr\left(R_1 \geq \bar{\omega}\right)}. \quad \text{(D.6)}$$

The left handside of Equation (D.6) represents the marginal benefit for an investors expressed as a percentage increase in the face value of debt. The right hand side of Equation (D.6) can be seen as the marginal cost for an investors represented as a percentage decrease in the expected probability of repayment associated with an increase in the face value of debt. An equivalent interpretation is that, in equilibrium, the elasticity of the expected probability of repayment with respect to the face value of debt is -1.

We distinguish two main cases when $\bar{\omega} < 1$ and the investors is willing to continue his bank with positive probability: $D^* \leq \phi$ and $D^* > \phi$. When $D^* \leq \phi$, then there exists values of R_1 such that bank can repay the investors from the realization of its own project. Thus, the signal that investors receives at date 1 can be sufficiently informative about whether he will be repaid at date 2. When $D^* > \phi$, it is necessary that bank j's project has a sufficiently good realization for bank to be able to repay the debt to the investors at date 2. In this case, the signal that investors receives at date 1 is less informative, as the investors is uncertain about whether he will be repaid or not even if R_1 is high.

In the first case, when in equilibrium $D^* < \phi$, the first order condition (D.6) becomes

$$\int_{\bar{\omega}}^{\frac{D}{\phi}} \left(1 - \frac{D - \phi R_1}{1-\phi}\right) dR_1 + \int_{\frac{D}{\phi}}^{1} 1 dR_1 + D \int_{\bar{\omega}}^{\frac{D}{\phi}} \frac{\partial}{\partial D}\left(1 - \frac{D - \phi R_1}{1-\phi}\right) dR_1 + D \int_{\bar{\omega}}^{\frac{D}{\phi}} \frac{\partial}{\partial D} 1 dR_1 = 0.$$

Integrating, we obtain that the deposit rate must satisfy the following equation

$$\left(D^* - \bar{\omega}\phi\right)\left(3D^* - \bar{\omega}\phi\right) = 2\phi\left(1-\phi\right)\left(1-\bar{\omega}\right), \tag{D.7}$$

for any $\phi \in (0, 1)$.

In the second case, when in equilibrium $D^* > \phi$, the first order condition (D.6) becomes

$$\int_{\bar{\omega}}^{1} \left(1 - \frac{D - \phi R_1}{1-\phi}\right) dR_1 + D \int_{\bar{\omega}}^{1} \frac{\partial}{\partial D}\left(1 - \frac{D - \phi R_1}{1-\phi}\right) dR_1 = 0.$$

Integrating, we obtain that the deposit rate must satisfy the following equation

$$D^* = \frac{1}{4}\phi\bar{\omega} - \frac{1}{4}\phi + \frac{1}{2}, \tag{D.8}$$

for any $\phi \in (0, 1)$.

We now provide the general characterization of the depositors' optimal decision, i.e. the probability of withdraw ω^* and the deposit rate D^*, as a function of the depositors' reservation value, r, and the bank's portfolio allocation ϕ.

Lemma D.8. *For any early redemption value* $r \in \left(0, \frac{1}{2}\right)$ *and any portfolio allocation* $(\phi, (1-\phi))$ *that bank hold, the probability,* ω^**, that investors withdraw their funding* 1*, as well as the optimal deposit rate,* D^**, with an opaque portfolio are as follows.*

For any r*,*

1. *if* $0 \leq \phi \leq \min\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\}$*, then* $D^* = \frac{2-\phi}{4}$ *and* $\bar{\omega} = 0$*.*
2. *if* $\frac{2}{5} \leq \phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$*, then* $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ *and* $\bar{\omega} = 0$*.*
3. *if* $\max\{\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right), 1 - \frac{1}{4r}\} < \phi \leq \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$ *then* $D^* = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$ *and* $\bar{\omega} = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$*,* $0 < \bar{\omega} < 1$*.*
4. *if* $\max\{\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right), \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\} < \phi \leq 1$*, then* D^* *is the largest root of equation*

$$-4D^3 + D^2(2r + \phi + 1) + r^2(\phi - 1) = 0,$$

   and $\bar{\omega} = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$*,* $0 < \bar{\omega} < 1$*.*
5. *if* $0 \leq \phi < 1 - \frac{1}{4r}$*, then* $\bar{\omega} = 1$*.*

Proof. Start from equations (D.2) and (D.6), letting $G \sim U[0,1]$. We will consider $\bar{\omega} = 0$, $\bar{\omega} = 1$ and $0 < \bar{\omega} < 1$ separately. Moreover, we need to consider the following cases separately: $D^* < \phi$, and $D^* \geq \phi$. The distinction is that in the former case, investors take into account that even if the unobservable part of the bank's portfolio, $(1-\phi)R_2$ returns zero, for sufficiently high realizations of R_1, $\frac{D^*}{\phi} \leq R_1 \leq 1$, investors will get paid if they choose to continue the bank.

1. No early withdraw, $\bar{\omega} = 0$.

1. $\frac{D^*}{\phi} > 1$. In this case, investors payoff simplifies to

$$W = D \int_0^1 \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz.$$

The first order condition is

$$\frac{4D + \phi - 2}{2(\phi - 1)\phi} = 0,$$

which implies

$$D^* = \frac{1}{4}(2 - \phi).$$

The second order condition holds (SOC< 0), thus the above D^* is a maximum. $D^* > \phi$ requires $0 \leq \phi \leq \frac{2}{5}$, while $\bar{\omega} = 0$ requires $\phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, which leads the first case.

2. $\frac{D^*}{\phi} < 1$. In this case, investors payoff simplifies to

$$W = \int_0^{\frac{D}{\phi}} \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz + D(1 - \frac{D}{\phi})$$

The first order condition is

$$\frac{3D^2 + 2(\phi - 1)\phi}{2(\phi - 1)\phi} = 0,$$

which implies

$$D^* = \sqrt{\frac{2}{3}\phi(1 - \phi)}.$$

The second order condition holds (SOC< 0), thus the above D^* is a maximum. $D^* < \phi$ requires $\phi \geq \frac{2}{5}$, while $\bar{\omega} = 0$ requires $\phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, which leads the second case.

2. Some early withdraw, $0 < \bar{\omega} < 1$**.**

$$\bar{\omega} = \frac{D}{\phi} - \frac{1-\phi}{\phi}\left(1 - \frac{r}{D}\right)$$

We again separately consider two cases:

1. $\frac{D^*}{\phi} > 1$. In this case, investors payoff simplifies to

$$W = D\int_{\bar{\omega}}^{1}\left(1 - \frac{D-\phi z}{1-\phi}\right)dz + r\bar{\omega}$$

   Substituting for $\bar{\omega}$ and taking first order condition implies

$$\frac{\left(D(1-3D) + r(1-\phi)\right)\left(r(1-\phi) - D(1-D)\right)}{2\phi(1-\phi)D^2} = 0,$$

   This is a quadratic equation with four roots: $D_1 = \frac{1}{6}\left(1 - \sqrt{1+12r(1-\phi)}\right) < 0$. $D_{2,4} = \frac{1}{2}\left(1 \pm \sqrt{4r\phi - 4r + 1}\right)$, and $\bar{\omega}(D_2) = \bar{\omega}(D_4) = 1$. Thus, the only relevant face value is $D_3 = \frac{1}{6}\left(1 + \sqrt{1+12r(1-\phi)}\right)$. Note that $\bar{\omega}(D) < 1$ only if $D_2 < D < D_4$, thus the optimal face value can be in this interval. Moreover, $D_2 < D_3 < D_4$.

   Next, the second order condition is given by

$$\frac{\frac{r^2(1-\phi)^2}{D^3} + 3D - 2}{(1-\phi)\phi}$$

   Letting SOC$= 0$ leads to a quadratic equation. Only two of the roots are in between D_2 and D_3, and $D_2 < D_1^{soc} < D_3 < D_2^{soc} < D_4$. Thus the second order condition changes sign twice on the interval $[D_2, D_4]$. Moreover, the second derivative evaluated at D_2 and D_4 is

$$\frac{1 \pm \left(\sqrt{1-4r(1-\phi)}\right)(1-4r(1-\phi))}{2r\phi(1-\phi)^2} > 0,$$

   which implies that D_2 and D_4 are local minima, and that the second derivative is negative at D_3, thus D_3 is a (local) maximum. Since first order condition is positive between D_2 and D_3, and negative between D_3 and D_4, D_3 is the global maximum in the interval $[D_2, D_4]$. Thus we have

$$\begin{aligned} D^* &= \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right) \\ \bar{\omega} &= \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right). \end{aligned}$$

Lastly, $D^* > \phi$ requires $\phi < \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$, $\bar{\omega} < 1$ requires $\phi > 1 - \frac{1}{4r}$ and $\bar{\omega} > 0$ requires $\phi > \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$. This leads the third case. Moreover, $\bar{\omega}$ cannot exceed 1, thus $\bar{\omega} = 1$ if $\phi \geq 1 - \frac{1}{4r}$, which leads the fifth case.

2. $\frac{D^*}{\phi} < 1$. Here we need to consider two sub-cases

   (a) $\bar{\omega} < \frac{D^*}{\phi} \Leftrightarrow r < D^* < \phi$. In this case, investors payoff simplifies to

$$W = \int_{\bar{\omega}}^{\frac{D}{\phi}} \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz + D(1 - \frac{D}{\phi}) + r\bar{\omega},$$

   Substituting for $\bar{\omega}$ and taking the first order condition implies

$$\frac{-4D^3 + D^2(1 + \phi + 2r) - r^2(1-\phi)}{2D^2\phi} = 0.$$

   The numerator is a cubic function in D, with $\Delta = -432r^4(1-\phi)^2 + 4r^2(1-\phi)(1+\phi+2r)^3$, thus $\Delta < 0$ implies $(1+\phi+2r)^3 - 108r^2(1-\phi) < 0$. For any pair (r, ϕ) that satisfy $\Delta < 0$,

$$\phi < \tilde{\phi}(r) = \max\{\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right), \frac{1}{6}\left(-r + \sqrt{r(r+10) + 1} + 1\right)\},$$

   and (r, ϕ) is covered by one of the first 3 cases. Thus when $\phi > \tilde{\phi}(r)$, $Delta > 0$ and the cubic first order condition has 3 distinct real roots, $D_1 < D_2 < D_3$.

$D_1 < 0$, so it is not the solution. Moreover, note that the derivative of investors surplus approaches $-\infty$ as $D \to 0$ from above, and as $D \to \infty$.

Next, the second order condition is given by

$$\frac{r^2(1-\phi) - 2D^3}{D^3\phi},$$

which has one root: $D^{soc} = \left(\frac{r^2(1-\phi)}{2}\right)^{\frac{1}{3}}$, and it is positive iff $D < D^{soc}$. Moreover, $D_2 < D^{soc} < D_3$, thus D_2 is a local minimum while D_3 is a local maximum. Thus either D_3 is the optimal face value, or the minimum feasible D, which in this case is $D = r$. Comparing the two values leads $W(\phi, r, D_3) > W(\phi, r, r)$, $\forall\ \phi > \tilde{\phi}(r)$. Thus we have

$$\begin{aligned} D^* &= D_3 \\ \bar{\omega} &= \frac{1}{\phi}\left(D^* - (1-\phi)\left(1 - \frac{r}{D^*}\right)\right). \end{aligned}$$

Lastly, note that if $D^* = r$, first order condition implies that $\phi = r$. However, $r < \tilde{\phi}(r)$, which in turn implies that whenever $\frac{D^*}{\phi} < 1$, $D^* > r$ and thus the next case is never relevant.

(b) $\bar{\omega} > \frac{D^*}{\phi} \Leftrightarrow D^* < \min\{r, \phi\}$. As argued above, this case does not arise in equilibrium.

Lastly, if $D^* = \phi$, first order condition implies $\phi = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$, and $\bar{\omega} > 0$ implies $\phi > \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. This leads to the forth case.

Always liquidate.

$$\frac{D^*}{\phi} > 1$$

□

D.3 Equilibrium Level of Diversification

The next lemma characterizes the bank's optimal portfolio allocation $(\phi^*, 1-\phi^*)$ with an opaque portfolio.

Lemma D.9. *There exists $\bar{r}_O \approx 0.477$ such that for any $r < \bar{r}_O$, the bank holds in equilibrium a diversified portfolio, $\phi^*(r) \in (0,1)$. If $r \geq \bar{r}_O$, the bank invests only in the transparent project, $\phi^*(r) = 1$.*

Proof. We need to calculate the optimal ϕ for the bank. That is, ϕ that maximizes

$$W_B = \int_{\bar{\omega}}^{1} \int_{\max\left\{\frac{D^* - \phi R_1}{1-\phi}, 0\right\}}^{1} \left(\phi R_1 + (1-\phi) R_2 - D^* (\phi)\right) dR_2 dR_1$$

As the optimal deposit rate changes depending on ϕ and r, we need to consider various cases, as defined in lemma D.8.

Case 1 : $0 \leq \phi \leq \min\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\}$.

Here $D^* = \frac{2-\phi}{4}$ and $\bar{\omega} = 0$. Substitute in the planner objective function to get

$$W = \frac{7\phi^2 + 12(1-\phi)}{96(1-\phi)}.$$

Since $\bar{\omega} = 0$, the optimal face value and bank profit are independent of r. Observe that $\frac{dW_B}{d\phi} > 0$ and $\frac{d^2 W_B}{d\phi^2} > 0$, thus the bank profit function is increasing and convex in this region. It follows that if the equilibrium level of opacity is in this region we will have

$$\phi^* = \min\left\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right\},$$

or

$$\phi^* = \begin{cases} \frac{2}{5} & \text{if } 0 < r < \bar{r}_{O_1} \\ \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) & \text{if } \bar{r}_{O_1} < r < \frac{1}{4} \end{cases}$$

where $\bar{r}_{O_1} = \frac{2}{15}$.

Case 2 : $\frac{2}{5} \leq \phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$.

Here $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ and $\bar{\omega} = 0$. Substitute in the bank objective function to get

$$W_B = \frac{1}{2} - \frac{8}{9}\sqrt{\frac{2}{3}(1-\phi)\,\phi}.$$

Since $\bar{\omega} = 0$, again the optimal face value and bank profit are independent of r. Observe that $\frac{dW_i}{d\phi} = 0$ at $\phi = \frac{1}{2}$ and $\frac{d^2W}{d\phi^2} > 0$, thus the objective function is convex, and the maximum is attained on one of the corners, i.e. $\phi^* = \frac{2}{5}$ or $\phi^* = \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. Direct comparison of the bank profit on the two boundaries reveals that $W(\frac{2}{5}) \geq W\left(\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)\right)$ for $0 < r < \bar{r}_{O_1}$. It follows that if the equilibrium level of opacity is in this region we will have

$$\phi^* = \frac{2}{5}.$$

Case 3 : $\max\left\{\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right), 1 - \frac{1}{4r}\right\} < \phi \leq \frac{1}{6}\left(1 - r + \sqrt{r(r+10) + 1}\right)$.

Here, $D^* = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$ and $\bar{\omega} = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$, $0 < \bar{\omega} < 1$. Substitute in the bank objective function to get

$$W_B = \frac{144r(1-\phi) - (42r(1-\phi) + 23)\sqrt{12r(1-\phi) + 1} + 31}{162(1-\phi)\phi}.$$

It is more convenient to consider $r < \frac{1}{4}$ and $r > \frac{1}{4}$ separately.

When $\bar{r}_{O_1} < r < \frac{1}{4}$, $\frac{d^2W}{d\phi^2} > 0$. Since the bank objective is convex, maximum is attained at one of the two boundaries. Direct comparison reveals that $W\left(\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right) >$

$W\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right)$. On the other hand, when $\frac{1}{4}<r<\frac{1}{2}$, $\frac{dW_i}{d\phi}>0$. Since the bank objective is increasing in ϕ, maximum bank profit is attained at maximum relevant ϕ, i.e. $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$.

Thus if the equilibrium level of bank opacity is in this region, we have

$$\phi^* = \begin{cases} \frac{2}{3}\left(2(1-2r)-\sqrt{(4r)^2-4r+1}\right) & \text{if } \bar{r}_{O_1}<r<\frac{1}{4} \\ \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right) & \text{if } \frac{1}{4}<r<\frac{1}{2} \end{cases}$$

Case 4 : $\max\{\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right), \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\}<\phi\leq 1$.

Here, D^* is the largest root of equation

$$-4D^3+D^2(2r+\phi+1)+r^2(\phi-1)=0,$$

and $\bar{\omega}=\frac{1}{\phi}\left(D^*-(1-\phi)(1-\frac{r}{D^*})\right)$, $0<\bar{\omega}<1$. Bank profit is given by

$$W_B^{\text{case 4}}=W=\frac{3D^{*5}-3D^{*4}(\phi+1)+D^{*3}\left(\phi^2+\phi+1\right)-r^3(1-\phi)^2}{6D^{*3}\phi}.$$

We will use $W_B^{\text{case 4}}$ for the objective function in this region since we use it to define the equilibrium thresholds.

We consider two cases separately, when $r<\bar{r}_{O_1}$ and when $r>\bar{r}_{O_1}$.

- $\mathbf{r}<\bar{\mathbf{r}}_{\mathbf{O_1}}$:

  In this region, $\frac{d^2W^{\text{case 4}}}{d\phi^2}>0$, $\forall\ \phi>\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$, thus the objective function is convex and the maximum is attained at one of the two boundaries. The upper boundary is $\phi=1$ and the lower boundary is $\phi=\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$.

The bank profit at the two boundaries is given by

$$W_B\left(\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)\right)=$$
$$\frac{1}{2}-\frac{8}{45}\sqrt{2\left(\sqrt{1-2r(3r+2)}+1\right)+r\left(3r+6\sqrt{1-2r(3r+2)}+2\right)},$$
$$W_B(1)=\frac{(1-r)^2}{8}.$$

Where the first expression uses continuity of bank objective function on the boundary $\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$, and case 2 above. Direct comparison of the two expressions reveals that the former expression is always larger than the latter. Thus in this range

$$\phi^*=\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$$

- $\mathbf{r>\bar{r}_{O_1}}$:

  Consider the first order condition

  $$\frac{dW_B^{\text{case 4}}(\phi,r)}{d\phi}=0$$

  The first order condition has either one or two solutions for $r\in(\frac{2}{15},\frac{1}{2})$. Let ϕ_1 denote the larger solution. ϕ_1 exists for all $r\in(\frac{2}{15},\frac{1}{2})$, $\phi_1>\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$, and $\frac{d^2W_B^{\text{case 4}}(\phi,r)}{d\phi^2}>0$, i.e. ϕ_1 is a minimum.

  Let ϕ_2 denote the smaller solution (if it exists). ϕ_2 exists only if $r>\hat{r}\in(\frac{2}{15},\frac{1}{5})$, and $\frac{d^2W_B^{\text{case 4}}(\phi,r)}{d\phi^2}<0$, i.e. ϕ_1 is a maximum. However, ϕ_2 is not always larger than $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$, thus it is not always a relevant solution. Moreover, $\frac{d\phi_2}{dr}>0$.

  Let $\phi^*_{FOC}=\phi_2$, and let $\bar{r}_O^2$ denote the level of $r\in(\frac{1}{4},\frac{1}{2})$ such that $\phi^*_{FOC}(r)=\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$. Thus for $r>\bar{r}_{O_2}$, ϕ^*_{FOC} is an interior (local) maximum.

Given the above argument, for $r \in (\bar{r}_{O_1}, \bar{r}_{O_2})$, bank objective function is either decreasing or convex (or both) for $\phi \in \left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right), 1\right)$. Thus the maximum is attained at one of the two boundaries. The bank profit at the two boundaries is given by

$$W_B\left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\right) =$$
$$\frac{46\sqrt{2r\left(r - \sqrt{r(r+10)+1} + 5\right) + 1} + 2r\left(r - \sqrt{r(r+10)+1} + 5\right)\left(7\sqrt{2r\left(r - \sqrt{r(r+10)+1} + 5\right) + 1} - 24\right) - 62}{9\left(-r + \sqrt{r(r+10)+1} - 5\right)\left(-r + \sqrt{r(r+10)+1} + 1\right)},$$

and

$$W_B(1) = \frac{(1-r)^2}{8},$$

where the first expression uses continuity of bank objective function on the boundary $\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$, and case 3 above. Direct comparison of the two expressions reveal that the former expression is larger than the latter when $r \in (\bar{r}_{O_1}, \bar{r}_{O_2})$. Thus in case 4, in this range

$$\phi^* = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$$

Next, let $\bar{r}_O$ denote $r \in (\bar{r}_{O_2}, 1)$ such that $W_B^{\text{case 4}}(\phi^*_{FOC}, r) = \frac{(1-r)^2}{8} = W_B(1)$. For any $r \in (\bar{r}_{O_2}, \bar{r}_O)$, bank surplus is first concave and then convex over the interval $\phi \in \left(\frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right), 1\right)$, with an interior (local) maximum and a larger interior (local) minimum. Thus the global maximum is obtained at either the local maximum, ϕ^*_{FOC}, or at the upper boundary $\phi = 1$. Direct comparison of the corresponding levels of objective functions reveals that $W_B^{\text{case 4}}(\phi^*_{FOC}(r), r) > \frac{(1-r)^2}{8} = W_B(1)$ for $r \in (\bar{r}_{O_2}, \bar{r}_O)$.

Finally, for $r > \bar{r}_O$, the objective function is larger at the corner $\phi = 1$ compared to the interior local maximum, thus $\phi^* = 1$.

Putting the cases together,

$$\phi^* = \begin{cases} \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right) & \text{if } 0 < r < \bar{r}_{O_1} \\ \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right) & \text{if } \bar{r}_{O_1} < r < \bar{r}_{O_2} \\ \phi^*_{FOC} & \text{if } \bar{r}_{O_2} < r < \bar{r}_O \\ 1 & \text{if } \bar{r}_O < r < \frac{1}{2} \end{cases}$$

Case 5 : $0 \leq \phi < 1 - \frac{1}{4r}$.

Here $\bar{\omega} = 1$, thus $W_B = 0$.

Comparison across cases. For each r, we compare the optimum across cases. Again it is easiest to treat 3 ranges separately.

1. $\mathbf{0 < r < \bar{r}_{O_1}}$:

   Here we compare the maximum across cases 1, 2, and 4. Case 1 shows that when $0 < r < \frac{2}{15}$, maximum is attained at $\phi = \frac{2}{5}$. Case 2 shows that $\phi = \frac{2}{5}$ is also optimal in that range. Thus within cases 1 and 2, $\phi = \frac{2}{5}$ is optimal.

   Case 4 argues that if $r < \bar{r}_{O_1}$, maximum bank profit is attained at $\phi = \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$. However, case 2 shows that $W_B(\frac{2}{5}) \geq W_B\left(\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)\right)$. Thus the maximum is attained at $\phi = \frac{2}{5}$.

2. $\mathbf{\bar{r}_{O_1} < r < \frac{1}{4}}$:

   Here we compare the maximum across cases 1, 3, and 4. Case 1 shows that when $\frac{2}{15} < r < \frac{1}{4}$, maximum is attained at $\phi = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2-4r+1}\right)$, which also maximizes bank profit over the region covered by case 3. The latter implies
   $W_B\left(\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2-4r+1}\right)\right) \geq W_B(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$.
   Since $\phi = \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$ maximizes bank profit over the region covered by Case 4, the maximum is attained at $\phi = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2-4r+1}\right)$.

3. $\mathbf{\frac{1}{4} < r < \frac{1}{2}}$:

   Here we compare the maximum across cases 3 and 4. Case 3 shows that $W_B\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right) \geq W_B\left(1-\frac{1}{4r}\right)=0$. Comparing with case 4 in this region, and using continuity of the bank profit function at the boundary $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$ yields

$$\phi^* = \begin{cases} \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right) & \text{if } \frac{1}{4} < r < \bar{r}_{O_2} \\ \phi^*_{FOC} & \text{if } \bar{r}_{O_2} < r < \bar{r}_O \\ 1 & \text{if } \bar{r}_O < r < \frac{1}{2} \end{cases}$$

Putting all the regions together leads the final result.

$$\phi^* = \begin{cases} \frac{2}{5} & \text{if } 0 < r < \bar{r}_{O_1} \\ \frac{2}{3}\left(2(1-2r)-\sqrt{(4r)^2-4r+1}\right) & \text{if } \bar{r}_{O_1} < r < \frac{1}{4} \\ \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right) & \text{if } \frac{1}{4} < r < \bar{r}_{O_2} \\ \phi^*_{FOC} & \text{if } \bar{r}_{O_2} < r < \bar{r}_O \\ 1 & \text{if } \bar{r}_O < r < \frac{1}{2} \end{cases} \tag{D.9}$$

where ϕ^*_{FOC} is the solution to $\frac{dW_B^{\text{case 4}}(\phi,r)}{d\phi}=0$ with $\frac{d^2W_B^{\text{case 4}}(\phi,r)}{d\phi^2}|_{\phi^*_{FOC}} < 0$. $\bar{r}_{O_2}$ is the value of $r \in (\frac{1}{4},\frac{1}{2})$ such that $\phi^*_{FOC}(r) = \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right)$, $\bar{r}_{O_2} \approx 0.324$ and $\bar{r}_O > \bar{r}_{O_2}$ is the value of $r \in (\frac{1}{4},\frac{1}{2})$ such that $W_B^{\text{case 4}}\left(\phi^*_{FOC}, r\right) = \frac{(1-r)^2}{8} = W_B(1)$, $\bar{r}_O \approx 0.477$.

□

D.4 Proof Example 1

To explain Example 1 we start by proving how D^* and ω^* behave as ϕ changes. To do so we use the results in Lemma D.8 and Lemma D.9.

Face value of debt: Consider the optimal face value of debt D^* (Lemma D.8). We need to look into different cases. For $r < \bar{r}_{O_1}$, D^* can take two values. First, $D^* = \frac{2-\phi}{4}$, which is decreasing in ϕ. The optimal portfolio is $\phi^* = \frac{2}{5}$, the upper bound in this case. Thus bank's choice minimizes D. Second, $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ which is increasing in ϕ. The optimal portfolio is $\phi^* = \frac{2}{5}$, the lower bound in this case. Thus bank's choice minimizes D.

For $\bar{r}_{O_1} < r < \frac{1}{4}$, $D^* = \frac{1}{6}\left(\sqrt{12r(1-\phi)+1}+1\right)$ which is decreasing in ϕ. The optimal portfolio is $\phi^* = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$, the lower bound in this case. Thus bank's choice **does not** minimize D.

For $\frac{1}{4} < r < \bar{r}_{O_2}$, also $D^* = \frac{1}{6}\left(\sqrt{12r(1-\phi)+1}+1\right)$ which is decreasing in ϕ. The optimal portfolio is $\phi^* = \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$, the upper bound in this case. Thus bank's choice minimizes D.

For $\bar{r}_{O_2} < r < \bar{r}_O$, $D^* = D_3$ (the largest root of $-4D^3 + D^2(2r+\phi+1) + r^2(\phi-1)$ in Case 4 of Lemma D.8) which is increasing in ϕ[11]. The optimal portfolio is $\phi^* = \phi^*_{FOC}$, a local maximum in this case. Thus, the bank's choice **does not** minimize D.

And $r > \bar{r}_O$, $D^* = D_3$ again which is increasing in ϕ. The optimal portfolio is $\phi^* = 1$, the upper bound in this case. Thus, the bank's choice **does not** minimize D.

All cases together deliver the following lemma.

Lemma D.10. *For $r < \bar{r}_{O_1}$, bank's optimal portfolio allocation minimizes D. For $\bar{r}_{O_1} < r < \frac{1}{4}$, bank's optimal portfolio allocation does not minimize D. For $\frac{1}{4} < r < \bar{r}_{O_2}$, bank's optimal portfolio allocation minimizes D. For $r \geq \bar{r}_{O_2}$, bank's optimal portfolio allocation does not minimize D.*

Probability of withdraw: Consider ω^* (Lemma D.8). We need to look into different cases.

For $r < \bar{r}_{O_1}$, $\omega^* = 0$. Thus any choice of the bank minimizes the withdraw probability.

For $\bar{r}_{O_1} < r < \frac{1}{4}$, $\omega^* = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$, which is increasing in ϕ. The

[11] We prove this statement below Lemma D.13.

optimal portfolio is $\phi^* = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$, the lower bound in this case. Thus bank's choice minimizes ω.

For $\frac{1}{4} < r < \bar{r}_{O_2}$, also $\omega^* = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$ which is increasing in ϕ. The optimal portfolio is $\phi^* = \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$, the upper bound in this case. Thus bank's choice **does not** minimize ω.

For $\bar{r}_{O_2} < r < \bar{r}_O$, $\omega^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$ where $D^* = D_3$ (Case 4 of Lemma D.8). ω^* is increasing in $D^*(\phi)$, with $D^*(\phi)$ being increasing in ϕ, and ω^* is decreasing in ϕ for $D > r$. The total derivative of ω^* with respect to ϕ is negative[12]. The optimal portfolio is $\phi^* = \phi^*_{FOC}$, a local maximum in this case, which is smaller than the upper bound 1. Thus, the bank's choice **does not** minimize ω.

Lastly, for $r > \bar{r}_O$, $\omega^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$ again decreasing in ϕ. The optimal portfolio is $\phi^* = 1$, the upper bound in this case. Thus bank's choice minimizes ω.

All cases together deliver the following lemma.

Lemma D.11. *For $r < \bar{r}_{O_2}$, bank's optimal portfolio allocation minimizes ω. For $\bar{r}_{O_2} < r < \bar{r}_O$, bank's optimal portfolio allocation does not minimize ω. For $r > \bar{r}_O$, bank's optimal portfolio allocation minimizes ω.*

Result in Example Lemma D.10 and Lemma D.11 together deliver the result depicted in Figure 2 in Example 1. The next lemma formalizes it.

Lemma D.12. *When the portfolio composition is opaque, for r such that $\bar{r}_{O_2} < r < \bar{r}_O$, the optimal portfolio allocation ϕ^* (Lemma D.9) does not minimize neither the deposit rate D nor the probability of withdraw ω.*

Behavior of D and ω for $r > \bar{r}_{O_2}$: We now prove the result used above that, for high enough values of r, the debt rate D^* is increasing in ϕ and the probability of withdraw ω^* is decreasing in ϕ. Specifically, this is Case 4 of Lemma D.9 when $r > \bar{r}_{O_2}$. The optimal

[12]We prove this statement below in Lemma D.14.

portfolio allocation is $\phi^* = \{\phi^*_{FOC}, 1\}$; the equilibrium debt rate is given by $D^* = D_3$ and the equilibrium withdraw probability by $\omega^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1-\frac{r}{D^*}\right)$.

Lemma D.13. *Consider $D^*(\phi, r) = D_3$ which is the largest real root of the cubic equation*

$$P(D; \phi, r) := -4D^3 + D^2(1 + \phi + 2r) - r^2(1 - \phi) = 0,$$

and assume that (ϕ, r) satisfies the Case 4 of Lemma D.9 conditions:

$$r \in (\bar{r}_{O_2}, 0.5), \quad \phi \in \left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right), 1\right).$$

Then $D^*(\phi, r)$ is strictly increasing in ϕ.

Proof. The function D_3 is implicitly defined by $P(D^*, \phi, r) = 0$, and the partial derivatives of P are:

$$\begin{aligned}
\frac{\partial P}{\partial \phi} &= D^2 + r^2 > 0, \\
\frac{\partial P}{\partial D} &= -12D^2 + 2D(1 + \phi + 2r).
\end{aligned}$$

By the implicit function theorem, the derivative of D_3 with respect to ϕ is

$$\frac{dD_3}{d\phi} = -\frac{\partial P / \partial \phi}{\partial P / \partial D}.$$

From Lemma D.9, we know that D_3 is a strict local maximum in the range of (ϕ, r) considered. This ensures that $\frac{\partial P}{\partial D}(D^*, \phi, r) < 0$.

Since the numerator is positive and the denominator is negative, we conclude that:

$$\frac{dD^*}{d\phi} > 0.$$

Hence, $D^*(\phi, r)$ is strictly increasing in ϕ over the domain specified. □

Lemma D.14. *Consider*

$$\bar{\omega}(\phi, r) = \frac{1}{\phi}\left(D_3 - (1-\phi)\left(1 - \frac{r}{D_3}\right)\right),$$

where $D_3 = D^*(\phi, r)$ is the largest real root of

$$-4D^3 + D^2(1 + \phi + 2r) - r^2(1-\phi) = 0.$$

Assume that (ϕ, r) satisfies the Case 4 of Lemma D.9 conditions:

$$r \in (\bar{r}_{O_2}, 0.5), \quad \phi \in \left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right), 1\right).$$

Then $\bar{\omega}(\phi, r)$ is strictly decreasing in ϕ.

Proof. We analyze the derivative of $\bar{\omega}$ with respect to ϕ, using the chain rule and the fact that $D^* = D_3$ depends on ϕ.

We rewrite:

$$\bar{\omega}(\phi, r) = \frac{1}{\phi}\left(D^* - 1 + \phi + \frac{r(1-\phi)}{D^*}\right) = \frac{D^* - 1}{\phi} + 1 + \frac{r(1-\phi)}{\phi D^*}.$$

Differentiating:

$$\frac{\partial \bar{\omega}}{\partial \phi} = -\frac{1}{\phi^2}\left(D^* - (1-\phi)\left(1 - \frac{r}{D^*}\right)\right) + \frac{1}{\phi}\left(\frac{dD^*}{d\phi} + \left(1 - \frac{r}{D^*}\right) - \frac{(1-\phi)r}{D^{*2}} \cdot \frac{dD^*}{d\phi}\right).$$

We analyze the sign of this expression in the domain.

- The first term is strictly negative. Since $D^* > r$ and $\phi > 0$, the expression inside the parentheses is positive, so the whole term is negative.
- The second term is positive, but each of its components is bounded. Specifically,

$\frac{dD^*}{d\phi} > 0$ by Lemma D.13, but the correction term

$$-\frac{(1-\phi)r}{D^{*2}} \cdot \frac{dD^*}{d\phi}$$

reduces the overall magnitude of the derivative.

- Over the domain $\phi \in (0.63, 1)$ and $r \in (\bar{r}_{O_2}, 0.5)$, the negative first term dominates. As $\phi \to 1$, the term $\frac{r(1-\phi)}{\phi D^*}$ vanishes, and $\bar{\omega} \to D^*/\phi \to 1$, but with a strictly negative slope.

Hence, the total derivative is strictly negative:

$$\frac{\partial \bar{\omega}}{\partial \phi} < 0,$$

and $\bar{\omega}(\phi, r)$ is strictly decreasing in ϕ in the Case 4 domain. □

E Equilibrium with Fixed Portfolio Composition

In this section, we summarize summarize the implications of our model when the bank's portfolio composition is fixed and her only choice is how much to allocate to each project (ϕ^*), which we analyze in in the previous Appendix B, Appendix C and Appendix D. This is instructive as a preamble to the full equilibrium characterization presented in the subsequent Appendix F.

There are three endogenous portfolio allocation choices: when the bank holds a transparent portfolio, an unobservable portfolio, and an opaque portfolio (which consists of a mixture of a transparent and an opaque project). From the perspective of the depositors, these cases imply that they have perfect information, no information, and partial information about the bank's portfolio returns. The next proposition summarizes the equilibrium in each of the three possible portfolio composition.

Proposition E.1. *Let the bank's portfolio composition be fixed. The equilibrium is unique conditional on a portfolio composition:*

- *the transparent portfolio equilibrium exists for all r,*
- *the opaque portfolio equilibrium exists for all r,*
- *the unobservable portfolio equilibrium exists for $r < \bar{r}_U$, where $\bar{r}_{>}\frac{1}{4}$.*

The next corollary extends the result in Proposition E.1 and focus on bank's optimal portfolio allocation choice with a fixed portfolio composition.

Corollary 1. *Conditional on a portfolio composition, the bank's optimal portfolio allocation $(\phi^*, 1-\phi^*)$ is such that:*

- *For $r < \frac{1}{4}$, the transparent portfolio and the opaque portfolio are imperfectly diversified $(\phi^* \in (0,1))$.*

- *There exists $r_{T1} > \frac{1}{4}$ that makes the transparent portfolio perfectly diversified ($\phi^* = \frac{1}{2}$). And there exists $r_{O1} > r_{T1} > \frac{1}{4}$ that makes the opaque portfolio perfectly diversified.*

- *For $r > r_2 > r_{O1}$, the transparent portfolio and the opaque portfolio are not diversified ($\phi^* = 1$).*

- *The unobservable portfolio is always imperfectly diversified ($\phi^* \in (0, 1)$).*

Proof. See Section B, Section C and Section D. □

Figure 7 depicts the results in Proposition E.1 and Corollary 1.

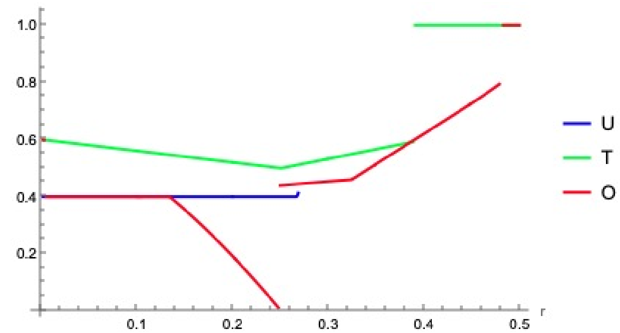

Figure 7: Optimal portfolio allocation ϕ^* conditional on a portfolio composition.

For the transparent and opaque portfolios, the equilibrium with no diversification has the bank holding the transparent project, i.e. ($\phi^* = 1$). The reason is that this occurs for high enough values of r, which is the depositors' outside option. By holding a unique transparent project, the bank maintains funding from depositors by having them perfectly informed on portfolio returns.

For the unobservable portfolio, if r is high enough there is no debt rate that would make depositors not to liquidate the bank early at date $t = 1$, since they have no information on returns. Being uninformed is also the reason behind the optimal choice of diversification in this case. By diversifying, the bank reduces a portion of the risk inherent to portfolio returns, and thus maintains depositors' funding.

F Optimal Diversification and Opacity

In this section, we look into the full-fledge problem when the bank is free to choose both the composition and allocation of her portfolio, discussed in Section 4. This is the case when the bank can choose to hold transparent and/or unobservable projects, in any proportion.

Intuitively, solving the model means asking what is the range of r values that makes the bank i) go fully opaque?; ii) go fully transparent?; or iii) hold exactly one unobservable and one transparent project?

F.1 Proof of Proposition 3

We need to characterize the equilibrium when the bank optimally chooses the levels of diversification and opacity. This involves two steps. First, we hold fixed the portfolio composition, and solve for bank's optimal portfolio allocation. This implies finding ϕ^* for when the bank holds: i) only transparent project(s) with $\mathbf{s} = (R_1, R_2)$; ii) only unobservable project(s) with $\mathbf{s} = (\varnothing, \varnothing)$, iii) both an unobservable and a transparent project with $\mathbf{s} = (R_1, \varnothing)$. The solution for this first step is in the previous Section B, Section C and Section D.

The second step, and the scope of this proof, is to solve for the bank's optimal choice of portfolio composition, given that she understands the optimal portfolio allocation associated with it. To do so, we compare the bank surplus,

$$W_B = \int_D^1 \left[(z - D) \frac{\partial H(z, \phi)}{\partial z} \right] dz.$$

when she holds the equilibrium transparent portfolio, the equilibrium unobservable portfolio, and the equilibrium opaque portfolio.

We start by characterizing the bank surplus in each portfolio composition equilibrium.

Transparent Portfolio The equilibrium is characterized by the portfolio allocation and deposit rate as follows (subsection B.3):

- For $r < \frac{1}{4}$, $\phi^*(r) = \frac{2+2r}{5}$ and $D^*(r) = \phi^*(r)$
- For $r \in [\frac{1}{4}, \bar{r}_T)$, $\phi^*(r) = \frac{2-2r}{3}$ and $D^*(r) = 1 - \phi^*(r)$
- For $r > \bar{r}_T$, $\phi^*(r) = 0$ and $D^*(r) > \phi^*(r)$.

Thus bank surplus is given by

$$W_B = \int_{D^*}^{1-\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi}(z - \frac{\phi}{2}) \right) \right] dz + \int_{1-\phi}^{1} \left[(z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

Substituting for D^*,

$$W_B = \frac{1}{96\,(1-\phi)} \left(12\,(r-1)^2 + 12\phi\,(r-1) + 7\phi^2 \right)$$

We need to consider three cases: $r < \frac{1}{4}$, $\frac{1}{4} \leq r < \bar{r}_T$, and $r \geq \bar{r}_T$

$\mathbf{r < \frac{1}{4}}$**:** The portfolio allocation is $\phi^*(r) = \frac{2+2r}{5}$. Substituting it into W_B,

$$W_B = \frac{28r^2 - 34r + 13}{90 - 60r}$$

$\mathbf{\frac{1}{4} \leq r < \bar{r}_T}$**:** The portfolio allocation is $\phi^*(r) = \frac{2-2r}{3}$. Substituting it into W_B,

$$W_B = \frac{2(1-r)^2}{18r + 9}$$

$\mathbf{r > \bar{r}_T}$**:** The portfolio allocation is $\phi^* = 0$. Substituting it into W_B,

$$W_B = \frac{12(r-1)^2}{96}$$

All Cases : All cases together yield the bank surplus

$$W_B^T = \begin{cases} \frac{28r^2-34r+13}{90-60r} & r < \frac{1}{4} \\ \frac{2(1-r)^2}{18r+9} & \frac{1}{4} \leq r < \bar{r}_T \\ \frac{1}{8}(r-1)^2 & r \geq \bar{r}_T \end{cases}$$

Unobservable Portfolio The equilibrium is characterized by the portfolio allocation and deposit rate as follows (subsection C.2):

- For $r < \bar{r}_{U_{low}}$, $\phi^*(r) = \frac{2}{5}$ and $D^*(r) = \phi^*(r)$
- For $r \in [\bar{r}_{U_{low}}, \bar{r}_U)$, $\phi^*(r) = \frac{3r}{2}$ and $D^*(r) = \frac{1}{2} - \frac{\sqrt{2-27r^2}}{2\sqrt{2}}$

We need to consider two cases: when $r < \bar{r}_{U_{low}}$ and when $\bar{r}_{U_{low}} \leq r < \bar{r}_U$.

$\mathbf{r} < \bar{\mathbf{r}}_{\mathbf{U_{low}}}$**:** Bank surplus is given by

$$W_B = \int_{D^*}^{1-\phi} \left[(z - D^*)\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z - \frac{\phi}{2})\right)\right] dz + \int_{1-\phi}^{1} \left[(z - D^*)\frac{\partial}{\partial z}\left(1 - \frac{(1-z)^2}{2\phi(1-\phi)}\right)\right] dz$$

Substituting for D^*,

$$W_B = \frac{12 - 12\phi + 7\phi^2}{96(1-\phi)}$$

Substituting for ϕ^*,

$$W_B = \frac{13}{90}$$

$\bar{\mathbf{r}}_{\mathbf{U_{low}}} \leq \mathbf{r} < \bar{\mathbf{r}}_{\mathbf{U}}$**:** Bank surplus is given by

$$W_B = \int_{D^*}^{\phi} \left[(z - D^*)\frac{\partial}{\partial z}\left(\frac{z^2}{2\phi(1-\phi)}\right)\right] dz + \int_{\phi}^{1-\phi} \left[(z - D^*)\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z - \frac{\phi}{2})\right)\right] dz$$
$$+ \int_{1-\phi}^{1} \left[(z - D^*)\frac{\partial}{\partial z}\left(1 - \frac{(1-z)^2}{2\phi(1-\phi)}\right)\right] dz$$

Substituting for D^*,

$$W_B = \frac{1}{2} - \frac{8}{9}\sqrt{\frac{2}{3}}\sqrt{(1-\phi)\phi}$$

and for ϕ^*,

$$W_B = \frac{1}{6}\left(3 - 8r\right)$$

All Cases : All cases together yield the bank surplus

$$W_B = \begin{cases} \frac{13}{90} & r < \bar{r}_{U_{low}} \\ \frac{1}{2} - \frac{16\sqrt{(r+1)\left(1-\frac{2(r+1)}{5}\right)}}{9\sqrt{15}} & \bar{r}_{U_{low}} \leq r < \bar{r}_U \\ \frac{1}{6}(3-8r) & r \geq \bar{r}_U \end{cases}$$

Opaque Portfolio The equilibrium characterization is more complex. For the sake of exposition, we refer to Subsection D.3 for the details. For the proof, the following is enough.

We need to consider five cases.

$\mathbf{r < \frac{2}{15}}$: Portfolio allocation is $\phi^* = \frac{2}{5}$ and bank surplus

$$W_B = \frac{13}{90}$$

$\mathbf{\frac{2}{15} < r < \frac{1}{4}}$: Portfolio allocation is $\phi^* = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$ and bank surplus

$$W_B = \frac{23\sqrt{4r\left(8r+2\sqrt{(2r-1)^2}-1\right)+1}+2r\left(8r+2\sqrt{(2r-1)^2}-1\right)\left(7\sqrt{4r\left(8r+2\sqrt{(2r-1)^2}-1\right)+1}-24\right)-31}{36\left(4r+\sqrt{(2r-1)^2}-2\right)\left(8r+2\sqrt{(2r-1)^2}-1\right)}$$

$\mathbf{\frac{1}{4} < r < r_z \approx 0.324}$**:** Portfolio allocation is $\phi^* = \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$ and bank surplus

$$W_B = \frac{46\sqrt{2r\left(r - \sqrt{r(r+10)} + 5 + 1\right)}\left(7\sqrt{2r\left(r - \sqrt{r(r+10)} + 5 + 1\right)} + 1 - 24\right) - 62}{9\left(-r + \sqrt{r(r+10)} + 1 - 5\right)\left(-r + \sqrt{r(r+10)} + 1 + 1\right)}$$

$\mathbf{r_z < r < r_h \approx 0.477}$**:** Portfolio allocation is $\phi^* = \phi^*_{FOC}$ and bank surplus

$$W_B = \frac{(1-r)^2}{8}$$

$\mathbf{r > r_h}$**:** Portfolio allocation is $\phi^* = 1$ and bank surplus

$$W_B = \frac{(1-r)^2}{8}$$

All Cases: All cases together yields the bank surplus

$$W_B = \begin{cases} \frac{13}{90} & r < \bar{r}_{O_1} \\ \frac{23\sqrt{4r+\sqrt{(2r-1)^2}-1+2r\left(8r+2\sqrt{(2r-1)^2}-1\right)}\left(7\sqrt{4r+\sqrt{(2r-1)^2}-1}-23\right)-20}{36\left(4r+\sqrt{(2r-1)^2}-2\right)\left(8r+2\sqrt{(2r-1)^2}-1\right)} & \bar{r}_{O_1} \le r < \frac{1}{4} \\ \frac{46\sqrt{2}\sqrt{r\left(r-\sqrt{r(r+10)}+6\right)}\left(7\sqrt{2}\sqrt{r\left(r-\sqrt{r(r+10)}+6\right)}-23\right)-62}{9\left(-r+\sqrt{r(r+10)}-4\right)\left(-r+\sqrt{r(r+10)}+2\right)} & \frac{1}{4} \le r < \bar{r}_{O_2} \\ \frac{1}{8}(1-r)^2 & r \ge \bar{r}_{O_2} \end{cases}$$

With the surplus in hand, we proceed to compare them.

Bank Surplus Comparison We need to consider several cases depending on the depositors' outside value r.

First, let $r < \bar{r}_{O_1}$. It holds that

$$W_B^T \leq W_B^O = W_B^U$$

Thus the bank prefers the opaque or the unobservable portfolio against the transparent one.

Next let $\bar{r}_{O_1} \geq r < \frac{1}{4}$. It holds that

$$W_B^T < W_B^O \leq W_B^U$$

Thus the bank prefers the unobservable portfolio against the opaque and transparent ones.

Next let $\frac{1}{4} \leq r < \bar{r}_{U_{low}}$. It holds that

$$W_B^T < W_B^O \leq W_B^U$$

Thus the bank prefers the unobservable portfolio against the opaque and transparent ones.

Next let $\bar{r}_{U_{low}} \leq r < \bar{r}_U$. It holds that

$$W_B^T < W_B^O \leq W_B^U$$

Thus the bank prefers the unobservable portfolio against the opaque and transparent ones.

Next let $\bar{r}_U \leq r < \bar{r}_{O_2}$. It holds that

$$W_B^U < W_B^T < W_B^O$$

Thus the bank prefers the opaque portfolio against the unobservable and transparent ones.

Next let $\bar{r}_{O_2} \leq r < \bar{r}_T$. It holds that

$$W_B^U < W_B^T < W_B^O$$

Thus the bank prefers the opaque portfolio against the unobservable and transparent ones.

Next let $\bar{r}_T \leq r < \bar{r}_O$. It holds that

$$W_B^U < W_B^T < W_B^O$$

Thus the bank prefers the opaque portfolio against the unobservable and transparent ones.

Lastly let $r \geq \bar{r}_O$. It holds that

$$W_B^U < W_B^T = W_B^O$$

Thus the bank prefers the opaque or the transparent portfolio against the unobservable one.

We conclude the following. The bank chooses to hold the unobservable portfolio whenever it exists since it delivers the (weakly) higher surplus. This is the case when r is relatively low, i.e $r < \bar{r}_U$. From Lemma C.7, we know that $\phi^*(r) \in (0, 1)$ and thus the bank diversifies. For intermediate levels of r, i.e. $\bar{r}_U \leq r < \bar{r}_O$, the bank holds the opaque portfolio since it delivers a strictly higher surplus than the transparent portfolio. From Lemma D.9, we know that $\phi^*(r) \in (0, 1)$ and thus the bank diversifies. When r is too high, i.e. $r > \bar{r}_O$, the bank holds a transparent portfolio due to two reasons. First, the opaque and transparent portfolios deliver the same surplus. Second, from Lemma D.9 and Lemma B.4, we know that these portfolios are not diversified, $\phi^*(r) = 1$, and the bank only holds a single transparent project.